Advertising and Voter Data in Asymmetric Political Contests

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Abstract

We study a political contest where two candidates advertise on a platform to persuade voters to vote in their favor. Voters a priori favor one of the candidates. The extent of a candidate's favorability can be ascertained by a data intermediary who can decide to sell this information to one, both or neither of the candidates. We contrast the intermediary's incentives for selling information with the platform's incentives for maximizing candidates' advertising expenditures, and show that the two are always at conflict. Our findings suggest that tensions may exist between social-media platforms, which often generate data that an intermediary may collect, and an intermediary whose data sale choice can lower the platform's profit from advertisements. We characterize conditions under which the intermediary can influence the outcome of the contest.

Keywords: Platform, intermediary, information asymmetry, asymmetric contest

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1 Introduction

Recent revelations about the extent to which voter data can be collected have fueled growing concerns about the harvesting and use of voter data in political campaigns. These concerns are exacerbated by recent reports from the Federal Trade Commission which demonstrate the proliferation of data intermediaries—entities that collect information from a wide range of sources and form detailed individualized profiles of the citizenry, which, among other things, can be used to discern political preferences.[1] A concomitant increase in expenditures on political advertising (approaching $7 billion in the US in 2016, and, in particular, exceeding $1 billion on social-media platforms) has further amplified these concerns.[2] The literature on campaign spending includes earlier foundational works by Brams and Davis (1973), Snyder (1989), and Nagler and Leighley (1992). While these works predate the ascent of the digital economy and social-media platforms, tensions among data intermediaries, media platforms, and political campaigns have long persisted. However, they are heightened by the degree to which data intermediaries and media platforms are now interlinked, with those linkages encompassing voter interactions, voter profiling, and political advertising. Our aim in this paper is to provide an analysis of voter data flow between a data intermediary and candidates in a political contest.

With social media quickly becoming a core advertising platform for political campaigns, some policy commentators and consumer groups have urged governmental authorities to deal with privacy issues that arise in a variety of contexts, and the literature in the area has been rapidly expanding (Acquisti et al., 2016).[3] It has also been shown that information revealed about political preferences on social networks combined with other information aggregated by data intermediaries can help assess voter predisposition (Jernigan and Mistree, 2009). Our analysis assumes that voter data had already been collected, and we study the drivers behind a political candidate's access to this data. That is, our focus is on which political

[1]The Federal Trade Commission's 2014 report on data brokers (FTC, 2014), for example, states: "Of the nine data brokers, one data broker's database has information on 1.4 billion consumer transactions and over 700 billion aggregated data elements; another data broker's database covers 1 trillion dollars in consumer transactions; and yet another data broker adds 3 billion new records each month to its databases. Most importantly, data brokers hold a vast array of information on individual consumers. For example, one of the nine data brokers has 3000 data segments for nearly every U.S. consumer."

[2]`http://adage.com/article/media/2016-political-broadcast-tv-spend-20-cable-52/307346/`

[3]See, e.g., `https://nyti.ms/1VMEtHY`.

candidate gains access to election-pertinent data, and how that access can influence the profits of the advertising platform and the information intermediary, and the outcome of the election.

The model we study can be described as follows. Two candidates, Alice and Bob, are competing for the same political office. Before voters go to the polls, each candidate can buy campaign advertisements on a social-media platform. A priori, Alice is favored to win among voters. This is exhibited in a predisposition of voters towards voting for her. Bob, consequently, would need to spend more than Alice on campaigning in order to have a fair shot at winning. An intermediary owns data that reveals more precise information about Alice's favorability relative to Bob—information that can be helpful in determining candidates' advertising spending. The intermediary can decide whether to sell the data to both Alice and Bob or to sell the data exclusively to one of the candidates. The advertising platform, on the other hand, has its separate objective of maximizing the candidates' total advertising spending on the platform. Under this setup, we ask: How much should candidates spend on advertising? To which of the candidates should the intermediary sell access to its data, and for how much? What are the advertising platform's preferences over candidates' access to data? Will the existence of the intermediary improve or worsen the platform's payoff? Does the sale of data influence the outcome of the election and, if so, in what way?

To answer these questions in our framework, we assume that the price of advertisements can be ascertained up front.[4] A Tullock contest is used to determine a winner based on relative spending (Tullock, 1980), and voters' political predispositions, which are modeled in an overall population sense for simplicity, weigh the relative efficacy of campaign spending (i.e., Alice is the 'favored to win' in the context of Dixit, 1987). Candidates' spending decisions depend on what information they have about voters and what information they expect their opponent to have. In other words, when deciding how much money to spend on advertising, Alice must take into account not only her own access to information, but also Bob's, because Bob's spending will determine the marginal impact of Alice's own campaign dollars. Both candidates share a common prior about the extent of voters' predisposition towards voting for Alice, but neither knows it precisely. The intermediary seeks to maximize its profit from

[4]While evidence of price discrimination has been demonstrated in the literature (e.g., Moshary, 2015), pricing to official campaigns is regulated by the Federal Communications Commission. See, also, the Honest Ads Act proposal: `https://www.congress.gov/bill/115th-congress/senate-bill/1989/text`.

selling information about voters' precise predisposition parameter, whereas the advertising platform profits from candidates' advertising outlays.

We show that, in equilibrium, the intermediary and the platform are always at conflict with respect to candidates' information access. For instance, if the intermediary decides to sign a contract to sell the information exclusively to Bob, then the platform, in contrast, prefers that the intermediary either sells non-exclusively to both candidates, or instead signs an exclusive contract with Alice. If the intermediary decides to sign a non-exclusive contract to sell the information to both candidates, then the platform prefers that the intermediary instead signs an exclusive agreement with one of the candidates. We then proceed to identify conditions under which the intermediary's existence can influence the outcome of the contest.

Our findings have practical implications. Most prominently, either the intermediary or the platform always has an incentive for voter information to be shared exclusively with only one of the candidates. The incentives to grant access to voter information exclusively may affect the outcome of political contests by either hurting the winning chances of the favored candidate or help cement her victory. Furthermore, our analysis indicates that a social-media platform that is also used for advertising may have incentives to hinder an intermediary's access to its data, whether by encouraging regulation or by way of limiting the intermediary's access to its platform's data hose. In the market for data, such actions may have additional implications with respect to data concentration and data portability.

1.1 Related Literature

Tullock contests have been used for some time to study resource allocations in competitive settings including rent-controlled housing, grants, and lobbying efforts (Corchón, 2007). Justifications for using the Tullock formulation include axiomatic foundations (Skaperdas, 1996; Clark and Riis, 1998) and a strategic equivalency to a variety of other rent-seeking settings (Baye and Hoppe, 2003; Chowdhury and Sheremeta, 2011). Sources of opponent asymmetry in Tullock contests include heterogeneity in player valuations (Klumpp and Polborn, 2006; Fu et al., 2012), informational advantages (Einy et al., 2013), and organizer favoritism (Kirkegaard, 2012). The literature documents a direct effect on the party that receives an asymmetric advantage, and an indirect effect by way of the opposing party's response (Cohen et al., 2008). In our context, an intermediary balances these effects to determine a profit-maximizing way for granting an informational advantage to one of the players, which is contrasted with a platform that seeks to maximize total resource outlays.

If a contest favors one party too strongly, the resource expenditure by the underdog player can be reduced, which subsequently also reduces the favorite's incentive to spend (Clark and Riis, 2000; Denter et al., 2018). In the same vein, a head start by a weaker contestant may increase expected revenues (Kirkegaard, 2012). In our framework, we observe analogous effects in equilibrium; however, we focus less on exogenous asymmetry and more on the endogenous asymmetry that is driven by the intermediary's decision regarding whether candidates have exclusive versus non-exclusive access to its data.

This paper is related to the literature that studies the relationship between candidate behavior and contest outcomes, including electoral policies on the informational aspects of elections, implications of candidate behavior, and optimal campaign spending (Feddersen and Pesendorfer, 1999; Besley and Burgess, 2002; Gentzkow, 2006; Ferraz and Finan, 2008; Snyder Jr and Strömberg, 2010; Banerjee et al., 2011). We add to these works by examining candidates' optimal spending against the backdrop of tensions that may arise between an advertising platform and a data intermediary, specifically when information in the contest flows from the intermediary to the candidates, rather than between candidates and voters.

This paper also has ties to the literatures on intermediaries and exclusive contracting. Intermediaries and other sellers of action-pertinent data have been studied extensively in the consumer-recognition literature (recent examples include Conitzer et al., 2012; Kim and Wagman, 2015), but less so in the context of political elections. The literature on exclusive contracting (Rasmusen et al., 1991; Segal and Whinston, 2000; Simpson and Wickelgren, 2007) has recently considered intermediaries who sell access to individuals' data (Kim et al., 2019), but to our knowledge has yet to focus on political campaigns. While the Federal Communications Commission regulates advertising sales to political campaigns with the aim of facilitating equal prices (Karanicolas, 2012),[5] no law mandates that candidates must

[5] These regulations, however, may not apply to political action committees (PACs), where advertisement prices are not necessarily regulated. Moshary (2015), for instance, finds that, on average, stations charge PACs 40% higher prices for airtime relative to official campaigns, and that Republican PACs pay rates that are 14% higher on average relative to Democrat PACs. The literature has also documented correlation between television advertisement pricing and the watching demographic (Goettler, 1999; Bel and Domènech, 2009), and between readership tastes and newspaper ad pricing (Gentzkow and Shapiro, 2010).

have equal access to voter-pertinent information. Motivated by recent revelations about social-media platforms such as Facebook and intermediaries such as Cambridge Analytica, as well as by an apparent asymmetry in access to voter-pertinent information in recent US presidential elections, this paper aims to contribute to the literature by studying the dynamics behind voter data flow, the resultant conflict between an advertising platform and a data intermediary, and the impact on candidates' winning probabilities.

The remainder of the paper is organized as follows. Section 2 presents the model and a benchmark case where neither candidate has access to the intermediary's data. Sections 3 and 4 consider non-exclusive and exclusive access to the intermediary's data. Section 5 characterizes the equilibrium and demonstrates the conflict between the platform and the intermediary, and Section 6 examines the impact on candidates' winning probabilities. Section 7 incorporates different assumptions into the base framework and Section 8 concludes. Proofs are in the appendix.

2 Model

Consider two candidates, A and B, who are engaged in a contest, with c_A and c_B denoting their respective amounts of spending towards winning. The standard Tullock contest success function determines a winner according to relative spending, with an unfavorability handicap parameter $x \in [\underline{x}, \overline{x}]$ against candidate B, with $0 < \underline{x} < \overline{x} \leq 1$. Candidates share a common prior belief over x, represented by a distribution with a differentiable cumulative density function F and a probability density function f that is positive over its support $[\underline{x}, \overline{x}]$. That is, both candidates know that A is favored to win and that each dollar spent by B is at most $\overline{x} \leq 1$ and at least $\underline{x} > 0$ as effective as a dollar spent by A. The probability that A wins is given by $\Pi_A(c_A, c_B) = \frac{c_A}{c_A + xc_B}$, with the complement giving the probability that B wins, $\Pi_B(c_A, c_B) = 1 - \Pi_A(c_A, c_B)$. Candidates' common valuations of winning are denoted by V.[6] Candidates' respective separable utility functions are denoted by U_A and U_B, such that $U_A = V\frac{c_A}{c_A + xc_B} - c_A$ for candidate A and $U_B = V\frac{xc_B}{c_A + xc_B} - c_B$ for candidate B.

In addition to the candidates, we consider an intermediary that is able to collect and pro-

[6]The analysis pertaining to the case where candidates have unequal winning valuations is available upon request. With unequal winning valuations, the qualitative nature of the results remains unchanged.

cess data about voters' overall predisposition towards A at some fixed cost. Before incurring this cost, the intermediary first decides how to engage with candidates. We consider four informational regimes: both candidates have access to the intermediary's data, denoted by (D, D); neither has access, denoted by (ND, ND); only A has access, denoted by (D, ND); and only B has access, denoted by (ND, D). Given the reduced-form nature of our model, we refer to the intermediary's "data" and "information" interchangeably, and abstract from specific considerations of how the intermediary processes and provides data. One can envision the intermediary's operations as continuous, with the intermediary choosing whether to have a contractual relationship with one or both of the candidates. For brevity, we refer to such a relationship as "data access." For simplicity, we assume that the intermediary's cost of acquiring the data is 0; the results go through with positive costs provided they are sufficiently small for an interior equilibrium to exist.[7]

At the onset, the intermediary chooses one of the four informational regimes and prices access to its data accordingly, deciding whether to engage with one or both of the candidates, and this decision becomes common knowledge. Next, based on the intermediary's chosen regime and data access pricing, candidates decide whether to contract with the intermediary, as applicable. Should a candidate acquire access to the intermediary's data, the candidate then learns the realized value of x.[8] In the final stage, candidates choose their advertising resource outlays to expend on an advertising platform.[9] We model the platform as the non-strategic recipient of advertising expenditures chosen by the candidates. While the platform does not explicitly act as a strategic player, we identify its preferences over the four regimes.

In the proceeding, we first consider each of the informational regimes from the perspective of the candidates—their advertising outlays and resulting expected utilities. We solve for interior Bayesian-Nash equilibria of the game.

[7] The intermediary may also, as a matter of chosen policy, offer its services exclusively to a specific party. Doing so could make sense in a broader game context when one contractual relationship is likely to be preferred. For instance, exclusivity policies or their lack thereof may be used to convey ex-ante assurances about the secrecy or availability of any information acquired, and/or to avoid conflicts of interest.

[8] If the intermediary were to price its data after realizing x, its pricing decision itself amounts to a signaling game with candidates. While such a signaling game is interesting, it is not our focus here.

[9] For technical simplicity, we abstract from budget considerations. Our analysis does not preclude candidates raising the requisite advertising budgets to accommodate their spending decisions.

2.1 No Data Access

As a benchmark, we begin with the case (ND, ND), where neither candidate has access to the intermediary's information, whereby each candidate is choosing their advertising outlay based on their prior beliefs about x. Then candidates choose c_A and c_B to maximize their respective expected utilities

$$U_A : V \int_{\underline{x}}^{\overline{x}} \frac{c_A}{c_A + xc_B} f(x)\, dx - c_A dx \quad \text{and} \quad U_B : V \int_{\underline{x}}^{\overline{x}} \frac{xc_B}{c_A + xc_B} f(x)\, dx - c_B dx.$$

The corresponding first-order conditions are:

$$FOC_A^{ND,ND} : V \int_{\underline{x}}^{\overline{x}} \frac{xc_B}{(c_A + xc_B)^2} f(x)\, dx = 1, \quad FOC_B^{ND,ND} : V \int_{\underline{x}}^{\overline{x}} \frac{xc_A}{(c_A + xc_B)^2} f(x)\, dx = 1.$$

Candidates' equilibrium advertising outlays are determined by simultaneously solving the first-order conditions, giving:

$$c_A^{ND,ND} = V\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right] = c_B^{ND,ND}. \tag{1}$$

Hence, candidates' ad expenditures are positive and equal in equilibrium. Given these outlays, candidates' ex-post winning likelihoods are:

$$\Pi_A^{ND,ND}(x) = \frac{1}{1+x} = 1 - \Pi_B^{ND,ND}(x) \quad \text{for all } x \in [\underline{x}, \bar{x}]. \tag{2}$$

Candidates' expected utilities are thus given by $U_A^{ND,ND} = V\mathbb{E}\left[\left(\frac{1}{1+x}\right)^2\right]$ and $U_B^{ND,ND} = V\mathbb{E}\left[\left(\frac{x}{1+x}\right)^2\right]$ for A and B, respectively, demonstrating a clear disadvantage for candidate B.

3 Non-Exclusive Data Access

In this section we consider the non-exclusive data access case (D, D) where both candidates A and B know the realized value of voters' predisposition parameter, x, prior to choosing their ad expenditures. Candidates now choose $c_A(x)$ and $c_B(x)$ to maximize:

$$U_A : V\frac{c_A(x)}{c_A(x)+xc_B(x)} - c_A(x) \quad \text{and} \quad U_B : V\frac{xc_B(x)}{c_A(x)+xc_B(x)} - c_B(x).$$

Taking the first-order conditions of candidates' objective functions with respect to their chosen advertising outlays $c_A(x)$ and $c_B(x)$ gives:

$$FOC_A^{D,D} : V\frac{xc_B(x)}{\left(c_A(x)+xc_B(x)\right)^2} = 1 \quad \text{and} \quad FOC_B^{D,D} : V\frac{xc_A(x)}{\left(c_A(x)+xc_B(x)\right)^2} = 1,$$

which yields equilibrium advertising outlays of

$$c_A^{D,D}(x) = V\frac{x}{(1+x)^2} = c_B^{D,D}(x). \tag{3}$$

Hence, candidates' advertising expenditures are equal for any realization of x under the non-exclusive data regime. These expenditures are positive and increasing in $x \in [\underline{x}, \overline{x}]$. Given candidates' equilibrium outlays in the non-exclusive access and no-data access cases, we have the following result about the platform's preferences over the two access regimes.

Proposition 1 *The platform is ex-ante indifferent between non-exclusive data access and no-data access, with expected profits* $2\mathbb{E}\left[c_A^{D,D}\right] = 2\mathbb{E}\left[c_A^{ND,ND}\right]$.

Proposition 1 states that the platform is ex-ante agnostic about the operations of the intermediary, provided that the intermediary is restricted (for instance, as a matter of law or policy) to grant non-exclusive data access to candidates. Intuitively, by having non-exclusive access to the intermediary's data, candidates' relative positioning in the political contest are unchanged in expectation, which means that, ex-ante, their expected advertising outlays and the platform's profit are equal under the two regimes. From a welfare perspective, a policy that requires non-exclusive access to an intermediary's data may simply entail a monetary transfer from candidates to the intermediary—the extent of such transfer will be soon characterized—without influencing candidates' ex-ante chances of winning. It should further be noted that ex post, once x is realized, the platform may indeed have a preference for one of the two data-access regimes.

Candidates' corresponding ex-post winning probabilities are given by:

$$\Pi_A^{D,D}(x) = \frac{1}{1+x} = 1 - \Pi_B^{D,D}(x) \quad \text{for every } x, \tag{4}$$

resulting in utilities $U_A^{D,D}(x) = V \frac{1}{(1+x)^2}$ and $U_B^{D,D}(x) = V \left(\frac{x}{1+x}\right)^2$. The corresponding ex-ante winning probabilities are therefore:

$$\mathbb{E}\left[\Pi_A^{D,D}\right] = \mathbb{E}\left[\frac{1}{1+x}\right] = 1 - \mathbb{E}\left[\Pi_B^{D,D}\right]. \tag{5}$$

As can be seen from (4) and (5), with both candidates possessing access to the intermediary's data under the non-exclusive regime, candidate A continues to remain the "favorite" for every realization of x. Given candidates' winning probabilities and expected resource outlays, ex-ante expected utilities under non-exclusive data access are given by

$$\mathbb{E}\left[U_A^{D,D}\right] = V\mathbb{E}\left[\left(\frac{1}{1+x}\right)^2\right] \quad \text{and} \quad \mathbb{E}\left[U_B^{D,D}\right] = V\mathbb{E}\left[\left(\frac{x}{1+x}\right)^2\right]. \tag{6}$$

It follows from (6) that, in equilibrium, both candidates' expected utilities are positive; however, candidate A's expected utility is unambiguously higher than B's, for any distribution on x that places mass on realizations below 1. It is worth noting that candidates' expected winning likelihoods and utilities are the same in both the non-exclusive access and no-access regimes. This implies that unlike the platform, given any positive payment to the intermediary, candidates ex-ante strictly prefer the no-data regime.

4 Exclusive Data Access

We now consider the two asymmetric regimes, (D, ND) and (ND, D), where one of the candidates has exclusive access to the intermediary's data.

4.1 The Favorite Has Data

We begin with the case where candidate A, the favorite, learns the realized value of x, while candidate B only knows its prior distribution. Based on the sequential timing of the game, that candidate A knows x is common knowledge in the stage where candidates choose their resource outlays. Given this informational setting, candidate A chooses $c_A(x)$ and candidate B chooses c_B to maximize

$$U_A : V \frac{c_A(x)}{c_A(x) + xc_B} - c_A(x) \quad \text{and} \quad U_B : V \int_{\underline{x}}^{\overline{x}} \frac{xc_B}{c_A(x) + xc_B} f(x)\, dx - c_B.$$

The first-order conditions now yield:

$$FOC_A^{D,ND} \quad : \quad V \frac{xc_B}{(c_A(x) + xc_B)^2} = 1 \tag{7}$$

$$FOC_B^{D,ND} \quad : \quad V \int_{\underline{x}}^{\overline{x}} \frac{xc_A(x)}{(c_A(x) + xc_B)^2} f(x)\, dx = 1 \tag{8}$$

It follows from $FOC_A^{D,ND}$ that an interior optimum at any given x is obtained if and only if $V \geq xc_B$, whereby

$$c_A^{D,ND}(x) = \sqrt{Vxc_B^{D,ND}} - xc_B^{D,ND}. \tag{9}$$

Hence, given any winning valuation V, there exists a threshold $\tilde{x}^{D,ND} \in [\underline{x}, \overline{x}]$, such that candidate A expends resources at x if and only if $x \leq \tilde{x}^{D,ND}$. Intuitively, if the support of the favorability parameter x were unbounded, for a given V, c_B, and sufficiently high values of x, the marginal benefit of an advertising dollar spent by candidate A can be arbitrarily small. However, such a scenario cannot arise in equilibrium given that A is the favorite in the asymmetric contest, that is, given that $x \leq 1$.

Lemma 1 *Candidate A always spends a positive amount on advertising; that is, $\tilde{x}^{D,ND} = \overline{x}$ and $c_A^{D,ND}(x) > 0$ for all $x \in [\underline{x}, \overline{x}]$.*

Given a positive expenditure by candidate A, the equilibrium ex-ante expected expenditures are given by:

$$\mathbb{E}\left[c_A^{D,ND}\right] = V \left(\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1 + x\right]}\right)^2 = c_B^{D,ND} = \mathbb{E}\left[c_B^{D,ND}\right]. \tag{10}$$

Notably, these expenditures are equal, and result in the following ex-post (given any realized value of x) and ex-ante probabilities of winning:

$$\Pi_A^{D,ND}(x) = 1 - \sqrt{x}\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]} = 1 - \Pi_B^{D,ND}(x) \quad \text{and} \tag{11}$$

$$\mathbb{E}\left[\Pi_A^{D,ND}\right] = 1 - \frac{\left(\mathbb{E}\left[\sqrt{x}\right]\right)^2}{\mathbb{E}\left[1+x\right]} = 1 - \mathbb{E}\left[\Pi_B^{D,ND}\right]. \tag{12}$$

From (12), it is straightforward to see that $\mathbb{E}[\Pi_A^{D,ND}] > 0.5 > \mathbb{E}[\Pi_B^{D,ND}]$, which implies that when A has exclusive data access she always maintains her position as the favorite to win. Candidates' ex-ante expected utilities when A has exclusive data access are given by:

$$\mathbb{E}\left[U_A^{D,ND}\right] = V - \mathbb{E}\left[c_A^{D,ND}\right]\mathbb{E}\left[2+x\right] \qquad \text{and} \tag{13}$$

$$\mathbb{E}\left[U_B^{D,ND}\right] = \mathbb{E}\left[c_B^{D,ND}\right]\mathbb{E}\left[x\right], \tag{14}$$

where $\mathbb{E}[c_A^{D,ND}]$ and $\mathbb{E}[c_B^{D,ND}]$ are specified in (10). Since candidates' expected expenditures are equal but their winning probabilities are not, it follows that candidate A's expected utility is always greater than B's. It further follows from (13) and (14) that candidates' expected utilities are positive in equilibrium.

4.2 The Underdog Has Data

We now consider the (ND, D) case where candidate B, the underdog, learns the realized value of x, while candidate A only knows its prior distribution. Candidates now choose c_A and $c_B(x)$ so as to maximize

$$U_A : V\int_{\underline{x}}^{\overline{x}} \frac{c_A}{c_A + xc_B(x)} f(x)\,dx - c_A, \qquad \text{and} \qquad U_B : V\frac{xc_B(x)}{c_A + xc_B(x)} - c_B(x).$$

The corresponding first-order conditions are:

$$FOC_A^{ND,D}: \quad V\int_{\underline{x}}^{\overline{x}} \frac{xc_B(x)}{\left(c_A + xc_B(x)\right)^2} f(x)\,dx = 1 \tag{15}$$

$$FOC_B^{ND,D}: \quad V\frac{xc_A}{\left(c_A + xc_B(x)\right)^2} = 1. \tag{16}$$

It follows from $FOC_B^{ND,D}$ that an interior optimum at a given x is obtained if and only if $xV \geq c_A$, whereby

$$c_B^{ND,D}(x) = \frac{\sqrt{Vxc_A^{ND,D}} - c_A^{ND,D}}{x}. \tag{17}$$

Hence, given any valuation V, there exists a threshold $\tilde{x}^{ND,D} \in [\underline{x}, \overline{x}]$, such that $c_B(x) \geq 0$ if and only if $x \geq \tilde{x}^{ND,D}$. Such a threshold is intuitive because sufficiently low values of x can render the marginal benefit of any advertising expenditure by candidate B lower than its marginal cost. An alternative interpretation is that for any given $x \in [\underline{x}, \overline{x}]$, B's winning valuation must be sufficiently high in order for B to advertise. We have the following result.

Lemma 2 *There exists a threshold $\tilde{x}^{ND,D} \in [\underline{x}, \overline{x})$ such that candidate B's ad spend $c_B^{ND,D}(x)$ is positive if and only if $x \geq \tilde{x}^{ND,D}$. If the distribution satisfies $\left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}\right]}{\mathbb{E}\left[1+\frac{1}{x}\right]}\right)^2 < \underline{x}$, then $\tilde{x}^{ND,D} \equiv \underline{x}$ and candidate B's expenditure is always positive. Otherwise, $\tilde{x}^{ND,D} \in (\underline{x}, \overline{x})$ and is implicitly defined by $\tilde{x}^{ND,D} = \left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\right)^2$.*

Lemma 2 states that, upon learning x, candidate B may not spend a positive amount on advertising if x is exceedingly low. It is worthwhile noting that the condition in Lemma 2 for a positive ex-post resource outlay from B is in contrast with the case of non-exclusive access to data. When data is non-exclusive, candidates always make positive advertising expenditures irrespective of their winning valuations and voters' predisposition towards A. The condition in Lemma 2 also stands in contrast with Lemma 1. While the favorite candidate always expends a positive amount for every realization of x, that may not necessarily be the case for the underdog. The reason is that an informed candidate B, anticipating the resource outlay of an uninformed candidate A, may, for a sufficiently low realization of x, find the marginal benefit of any positive outlay to be below its cost. In those instances, candidate B's ex-post expenditure is 0, whereby an interior pure-strategy equilibrium does not hold.

From an ex-ante perspective, expected expenditures satisfy:

$$\mathbb{E}\left[c_B^{ND,D}\right] = V\left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\right)^2 = c_A^{ND,D} = \mathbb{E}\left[c_A^{ND,D}\right]. \tag{18}$$

Under these advertising expenditures, candidates' ex-post probabilities of winning given any realized value of x are specified by:

$$\Pi_A^{ND,D}(x) = \begin{cases} 1 & \text{for} \quad x \in \left[\underline{x}, \tilde{x}^{ND,D}\right) \\ \frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\sqrt{\frac{1}{x}} & \text{for} \quad x \in \left[\tilde{x}^{ND,D}, \bar{x}\right] \end{cases} = 1 - \Pi_B^{ND,D}(x). \tag{19}$$

The resulting ex-ante expected winning probabilities are

$$\mathbb{E}\left[\Pi_A^{ND,D}\right] = F\left(\tilde{x}^{ND,D}\right) + \frac{\left(\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]\right)^2}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]} = 1 - \mathbb{E}\left[\Pi_B^{ND,D}\right], \tag{20}$$

and candidates' expected utilities are given by:

$$\mathbb{E}\left[U_A^{ND,D}\right] = VF\left(\tilde{x}^{ND,D}\right) + \mathbb{E}\left[c_A^{ND,D}\right]\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right] \quad \text{and} \tag{21}$$

$$\mathbb{E}\left[U_B^{ND,D}\right] = V\left(1 - F\left(\tilde{x}^{ND,D}\right)\right) - \mathbb{E}\left[c_B^{ND,D}\right]\left(2 + \mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]\right), \tag{22}$$

where $\mathbb{E}\left[c_A^{ND,D}\right]$ and $\mathbb{E}\left[c_B^{ND,D}\right]$ are as specified in (18). The ex-post winning probabilities in (19) highlight that candidate B essentially 'gives up' upon learning that $x \in \left[\underline{x}, \tilde{x}^{ND,D}\right)$, whereby candidate A wins with certainty for realizations of x in this range. In a later section, it is further shown that there are parameter specifications under which for higher realizations of x, candidate B's ex-post winning likelihood exceeds 0.5; i.e., under exclusive data access, B may be more likely than A to win. The ex-ante winning probabilities in (20) account for these facts, and the resultant expected utilities in (21)-(22) are positive. The preceding allows us to complete the characterization of the platform's expected profits, as follows.

Proposition 2 *The platform's expected profit when the intermediary's data is exclusively accessed by A and by B are given by $2\mathbb{E}\left[c_A^{D,ND}\right]$ and $2\mathbb{E}\left[c_A^{ND,D}\right]$, respectively.*

Proposition 2 highlights the result that candidates' expected advertising expenditures are equal to each other in each of (but not across) the exclusive-access regimes. That is, if one candidate's expected spending increases, the other candidate increases their spending proportionally in equilibrium. This result mirrors similar findings in the contest literature and follows from candidates' spending choices being set to optimally respond to one another. However, the amount spent across the two exclusive regimes still differs depending on which candidate has access to the intermediary's data. Hence, the platform's expected profit is affected by the intermediary's choice of candidate for exclusive access.

5 Equilibrium Characterization

5.1 The Platform

To contrast the platform's preferences with the intermediary's, we first identify the necessary and sufficient conditions for the platform to prefer one data regime over another.

Proposition 3 *The following is satisfied in equilibrium:*

1. *The platform prefers non-exclusive access over exclusive access by candidate A if and only if* $\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}[1+x]} < \sqrt{\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right]}$.

2. *The platform prefers non-exclusive access over exclusive access by candidate B if and only if* $\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right]} < \sqrt{\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right]}$.

3. *The platform prefers exclusive access by candidate A to exclusive access by candidate B if and only if* $\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right]} < \frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}[1+x]}$.

Proposition 3 illustrates that the platform has a preference ranking over the different data-access regimes, driven by different expected profits. These conditions arise from a direct comparison of its profits and demonstrate that the platform's ranking is dependent on the distribution of voters' predisposition parameter x. For instance, when the distribution places sufficient mass on higher values of x, the platform prefers that candidate A exclusively accesses the intermediary's data. As intermediate and lower values of x become sufficiently likely, the platform's profit scales tip in favor of candidate B having (possibly exclusive) access to the intermediary's data. Hence, while the ratio of candidates' expected expenditures is constant across the four regimes, the expected expenditures themselves are not. Example 1 demonstrates different regime rankings by the platform for a specific distribution of x.

Example 1 *Consider the probability density function $f(x) = Beta(\alpha, \beta)$ with support $(0, 1)$ where α and β are shape parameters. Candidates' winning valuation is $V = 1$. When*

$\alpha = \beta = 1$, the distribution coincides with the uniform distribution $U(0,1)$. Under these parameter specifications, it can be shown that condition (1) in Proposition 3 is violated, while conditions (2) and (3) are met. Accordingly, it can be verified that the platform's profit is maximized when candidate A has exclusive access to the intermediary's data, whereas the platform's profit is lowest when candidate B has exclusive access. Alternatively, if we consider $\alpha = 1$ and $\beta = 3$, then it can be confirmed that all of the conditions in Proposition 3 are met. Hence, given this alternate distribution, the platform's profit is maximized when both candidates have access to the intermediary's data. In a similar manner, it can be shown that if $\alpha = 1$ and $\beta = 5$ then condition (1) of Proposition 3 is met, but conditions (2) and (3) are not. Accordingly, the platform's preferred regime is for candidate B to have exclusive access and its least preferred regime is for A to have exclusive access.

To gain some intuition for Proposition 3, it is helpful to examine candidates' ex-post advertising expenditures. Specifically, let us compare each candidate's ex-post expenditure under the non-exclusive symmetric data-access cases, given in (3), to their expenditure in the cases where only one candidate has access to the intermediary's data, i.e., where

$$c_A^{D,ND}(x) = V\frac{\sqrt{x}\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]}\left(1 - \frac{\sqrt{x}\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]}\right) \tag{23}$$

for candidate A, and

$$c_B^{ND,D}(x) = \begin{cases} 0 & \text{for} \quad x \in \left[\underline{x}, \tilde{x}^{ND,D}\right) \\ V\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\frac{1}{\sqrt{x}}\left(1 - \frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\frac{1}{\sqrt{x}}\right) & \text{for} \quad x \in \left[\tilde{x}^{ND,D}, \overline{x}\right] \end{cases} \tag{24}$$

for candidate B. This comparison yields the next result.

Proposition 4 *The following is satisfied in equilibrium:*

1. *There exists a cutoff $\underline{x}^{D,ND} \in [\underline{x}, \overline{x}]$ such that candidate A's ex-post expenditure is higher (lower) when she has exclusive access to the intermediary's data compared to non-exclusive access if and only if $x < \underline{x}^{D,ND}(x > \underline{x}^{D,ND})$.*

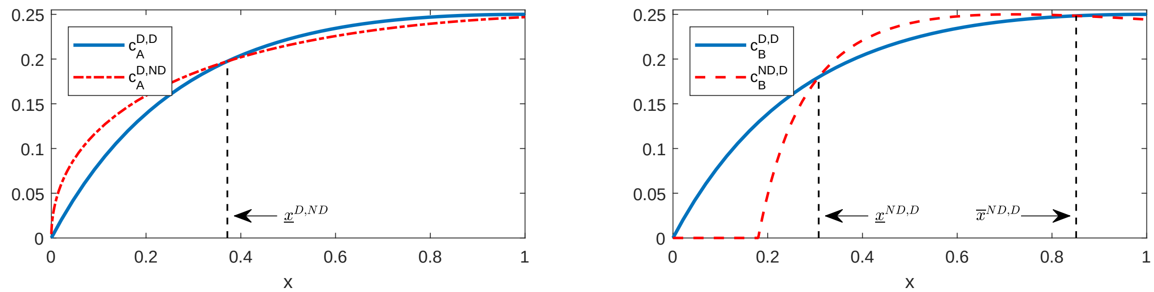
2. *There exists an interval $(\underline{x}^{ND,D}, \overline{x}^{ND,D}) \subseteq [\underline{x}, \overline{x}]$ such that candidate B's ex-post expenditure is higher (lower) when she has exclusive access to intermediary's data compared*

to non-exclusive access when $x \in (\underline{x}^{ND,D}, \overline{x}^{ND,D})$ $(x \notin (\underline{x}^{ND,D}, \overline{x}^{ND,D}))$.

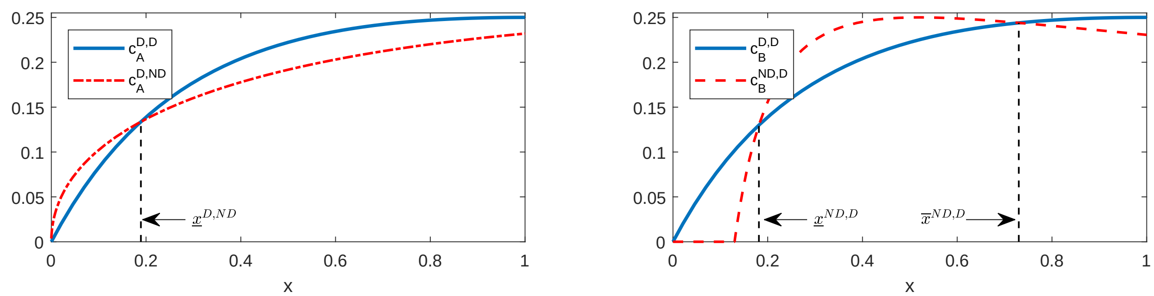
Proposition 4 states that in comparison to the non-exclusive data access regime, when candidates have exclusive access they may increase or decrease their advertising expenditures upon learning x. More specifically, when A has exclusive data access, A increases her outlay for sufficiently low realizations of x and decreases her expenditure otherwise. When B has exclusive data access, his expenditure increases over intermediate realizations of x and decreases otherwise. The findings in Proposition 4 revolve around the ex-post realization of voters' predisposition parameter, x, and thus on the distribution-dependent implications of having access to the intermediary's data. Figure 1 illustrates this by way of an example, depicting comparisons between candidates' ex-post expenditures under the exclusive and non-exclusive access regimes for the parametric specifications from Example 1. The figure demonstrates that as the underlying distribution of x changes, candidates change their advertising outlays, which in turn alters the platform's expected profit, and may thus change its ranking of the access regimes.

5.2 The Intermediary

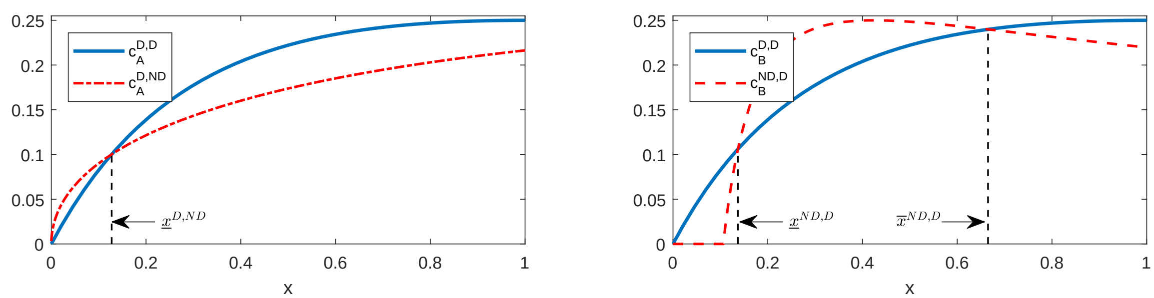
Folding the game back to the stage in which the intermediary contracts with candidates, we first determine the intermediary's pricing under each data-access regime. To do so, we seek the highest price a candidate would be willing to pay for data access as a function of the intermediary's chosen regime. For instance, if the intermediary offers non-exclusive contracts, candidate A's highest willingness to pay for access is given by $\mathbb{E}[U_A^{D,D}] - \mathbb{E}[U_A^{ND,D}]$. Alternatively, if the intermediary offers exclusive access, candidate A's highest willingness to pay is given by $\mathbb{E}[U_A^{D,ND}] - \mathbb{E}[U_A^{ND,D}]$. In each case, candidate A takes into account the fact that if she does not purchase access, she would be the sole candidate without access to the data. This is in line with the literature on exclusive contracting (e.g., Rasmusen et al., 1991; Segal and Whinston, 2000; Simpson and Wickelgren, 2007; Kim et al., 2019). In an analogous manner, candidate B's willingness to pay is determined by $\mathbb{E}[U_B^{D,D}] - \mathbb{E}[U_B^{D,ND}]$ and $\mathbb{E}[U_B^{ND,D}] - \mathbb{E}[U_B^{D,ND}]$ in the non-exclusive and exclusive data access cases, respectively. Combining these observations, it can be seen that the intermediary's profit under non-



(a) Candidates' advertising expenditures when $\alpha = 1, \beta = 1$



(b) Candidates' advertising expenditures when $\alpha = 1, \beta = 3$



(c) Candidates' advertising expenditures when $\alpha = 1, \beta = 5$

Figure 1: Advertising expenditures by candidates for different distributions of x.

exclusive access equals the following:

$$\left(\mathbb{E}\left[U_A^{D,D}\right] - \mathbb{E}\left[U_A^{ND,D}\right]\right) + \left(\mathbb{E}\left[U_B^{D,D}\right] - \mathbb{E}\left[U_B^{D,ND}\right]\right), \tag{25}$$

with two distinct revenue sources, one from each candidate, under non-exclusive access. The intermediary's profits from contracting exclusively with A and B are given by

$$\mathbb{E}\left[U_A^{D,ND}\right] - \mathbb{E}\left[U_A^{ND,D}\right] \tag{26}$$

and

$$\mathbb{E}\left[U_B^{ND,D}\right] - \mathbb{E}\left[U_B^{D,ND}\right]. \tag{27}$$

Notice that for the intermediary to operate and sell access to its data, its profit under just one of the regimes—and thus the corresponding willingness to pay by candidates—needs to be positive. Appendix B delineates a set of necessary conditions for (25)-(27) to be all positive, which we henceforth assume. The next result is obtained by comparing the intermediary's profit across the three regimes.

Proposition 5 *The following is satisfied in equilibrium:*

1. *The intermediary prefers exclusive access by candidate A over non-exclusive access if and only if* $\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}[1+x]} < \sqrt{\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right]}$.

2. *The intermediary prefers exclusive access by candidate B over non-exclusive access if and only if* $\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right]} < \sqrt{\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right]}$.

3. *The intermediary prefers exclusive access by candidate A to exclusive access by candidate B if and only if* $\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}[1+x]} < \frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right]}$.

Proposition 5 highlights the fact that, in a manner similar to the platform, the intermediary's preferences over the different regimes are also distribution-dependent.

Example 2 *Under the parameter specifications from Example 1, it can be shown that when* $\alpha = 1$ *and* $\beta \in \{1, 3\}$*, then condition (2) of Proposition 5 is met, condition (3) is violated and the intermediary prefers to contract exclusively with candidate B. Under* $\alpha = 1$ *and* $\beta = 5$*, in contrast, conditions (1) and (3) of Proposition 5 are met and the intermediary prefers to contract exclusively with A.*

5.3 The Conflict

For comparing the intermediary's ranking of the exclusive and non-exclusive data-access regimes to the platform's, it is helpful to adjust the profit expressions of the intermediary in equations (25)–(27) by adding $\mathbb{E}[U_A^{ND,D}] + \mathbb{E}[U_B^{D,ND}]$ to each one. Doing so maintains the ranking of the intermediary's profit alternatives, and while the expressions would no longer represent its actual profits, the adjusted ranking is significantly simpler to compare with the platform's ranking. In particular, under any of the access regimes, $\mathbb{E}[U_A] + \mathbb{E}[U_B]$ can be simplified to $(\mathbb{E}[\Pi_A]V - \mathbb{E}[c_A]) + (\mathbb{E}[\Pi_B]V - \mathbb{E}[c_B]) = V - \mathbb{E}[c_A] - \mathbb{E}[c_B]$. The intermediary's rating of the data-access regimes is then equivalent to the following:

$$\begin{aligned}
\text{Non-exclusive access} \;&:\; \mathbb{E}\left[U_A^{D,D}\right] + \mathbb{E}\left[U_B^{D,D}\right] &&= V - \mathbb{E}\left[c_A^{D,D}\right] - \mathbb{E}\left[c_B^{D,D}\right] \\
A \text{ has exclusive access} \;&:\; \mathbb{E}\left[U_A^{D,ND}\right] + \mathbb{E}\left[U_B^{D,ND}\right] &&= V - \mathbb{E}\left[c_A^{D,ND}\right] - \mathbb{E}\left[c_B^{D,ND}\right] \\
B \text{ has exclusive access} \;&:\; \mathbb{E}\left[U_B^{ND,D}\right] + \mathbb{E}\left[U_A^{ND,D}\right] &&= V - \mathbb{E}\left[c_A^{ND,D}\right] - \mathbb{E}\left[c_B^{ND,D}\right]
\end{aligned}$$

Combining the above with the results in Propositions 1 and 2, it follows that the intermediary and the platform rate the access regimes as follows:

1. Non-exclusive access:

$$\text{Intermediary rating: } V - 2\mathbb{E}\left[c_A^{D,D}\right] \qquad \text{Platform expected profit: } 2\mathbb{E}\left[c_A^{D,D}\right] \tag{28}$$

2. A has exclusive access:

$$\text{Intermediary rating: } V - 2\mathbb{E}\left[c_A^{D,ND}\right] \qquad \text{Platform expected profit: } 2\mathbb{E}\left[c_A^{D,ND}\right] \tag{29}$$

3. B has exclusive access:

$$\text{Intermediary rating: } V - 2\mathbb{E}\left[c_A^{ND,D}\right] \qquad \text{Platform expected profit: } 2\mathbb{E}\left[c_A^{ND,D}\right] \tag{30}$$

As is readily apparent, the ratings of the intermediary and the platform conflict. If the platform prefers the non-exclusive regime, so that $\mathbb{E}[c_A^{D,D}] > \max\{\mathbb{E}[c_A^{D,ND}], \mathbb{E}[c_A^{ND,D}]\}$, then $V - \max\{2\mathbb{E}[c_A^{D,ND}], 2\mathbb{E}[c_A^{ND,D}]\} > V - 2\mathbb{E}[c_A^{D,D}]$ holds, whereby the intermediary prefers to contract exclusively with one of the candidates, and vice versa. If under exclusive contracting the platform prefers that candidate A has access to data, so that $\mathbb{E}[c_A^{D,ND}] > \mathbb{E}[c_A^{ND,D}]$, then $V - 2\mathbb{E}[c_A^{ND,D}] > V - 2\mathbb{E}[c_A^{D,ND}]$, such that the intermediary prefers to contract exclusively with B, and vice versa. Overall, the most preferred alternative for the platform is the least preferred for the intermediary. That is, their ranking of the exclusive and non-exclusive access regimes are mirror opposites. The following proposition states this result formally.

Proposition 6 *The sum of the platform's and intermediary's profits is constant across the exclusive and non-exclusive data-access regimes. Further, their profit rankings of these regimes are mirror opposites.*

To gain some intuition for Proposition 6, note that for a candidate, the benefits from data access can be direct—by having access to the realization of x, they can more optimally choose their advertising spending, as well as indirect—by diminishing the advertising outlay of the opponent. The intermediary maximizes its profits when the willingness to pay for access to its data is highest. This occurs when the aggregate benefits of data to candidates are highest, which in turn happens when data, overall, saves candidates the highest amount of resources they would otherwise spend on advertising. The platform, on the other hand, seeks to maximize spending on advertising, and is thus in conflict with the intermediary.

Proposition 6 has several practical implications. First, it immediately follows that either the intermediary or the platform has an incentive for voter data to be exclusively shared with one of the candidates. Second, if the platform is the source for at least part of the intermediary's data, the platform may have incentives to hinder the intermediary's access, whether by encouraging regulations that restrict data transfer and portability or by way of limiting the intermediary's access to its data hose. Moreover, if the platform anticipates that the intermediary would prefer to grant data access exclusively to one of the candidates, then the platform, which reaps the same advertising revenue whether candidates have non-exclusive or no data access, would be better off preventing the intermediary from operating altogether. Third, the incentives to sell data exclusively, which under any specification of the model will be favored by either the platform or by the intermediary, may either hurt the winning chances of the favorite candidate or help cement the favorite's victory, as the next section demonstrates.

Since the platform's profit is the same under the non-exclusive data regime and the no-data regime, and the intermediary's profit is non-negative in the three regimes where it sells access to data (and strictly positive under the conditions in Appendix B), it follows that the sum of the platform's and intermediary's profits under the non-exclusive regime is at least as high as their sum under the no-data regime (where the intermediary's profit is 0). Furthermore, per Proposition 6, the sum of the intermediary and platform profits is constant in the three access regimes; hence, this sum is always greater or equal to the sum of their profits under the no-data regime. Therefore, an entity that vertically integrates the platform and the intermediary would strictly prefer to sell data access (under any of the three access regimes) to not doing so. This is because the joint entity can extract more rents from the candidates under any of the access regimes relative to the regime with no data access.

6 The Impact of Data Access

We now examine how the intermediary's data may influence the outcome of the contest. While one may look at candidates' ex-post and ex-ante utilities as a measure of this outcome, our preference is to examine their winning probabilities. Doing so allows us to assess whether the intermediary's data access choice has the potential to tilt or influence the outcome of the contest in favor of one of the candidates.

In the cases where both candidates either do or do not have access to the intermediary's data, from the expressions for $\Pi_A^{ND,ND}(x)$ and $\Pi_A^{D,D}(x)$ in (2) and (4), it follows that candidates' winning probabilities remain unchanged under these two access regimes. In contrast, in the two cases where exactly one of the candidates has exclusive access to the intermediary's data, we have the proceeding result.

Proposition 7 *The following is satisfied under the exclusive data-access regimes:*

1. *There exists a cutoff $\widehat{x}^{D,ND} \in [\underline{x}, \overline{x}]$ such that candidate A's ex-post winning likelihood is higher (lower) when she has exclusive access to the intermediary's data compared to non-exclusive access if and only if $x > \widehat{x}^{D,ND} (x < \widehat{x}^{D,ND})$.*

2. *There exists a cutoff $\widehat{x}^{ND,D} \in [\underline{x}, \overline{x}]$ such that candidate A's ex-post winning likelihood is higher (lower) when candidate B has exclusive access to the intermediary's data compared to non-exclusive access for $x < \widehat{x}^{ND,D}$ $(x > \widehat{x}^{ND,D})$.*

The results in Proposition 7 indicate that winning probabilities and data access are not necessarily positively linked. Ex post, exclusive data access, as represented by being the only candidate informed of the realization of voters' predisposition parameter, x, enables the candidate to know whether they are over- or under-spending on advertising, and adjust their expenditures accordingly. This adjustment, in turn, leads to either an increase or a decrease in the candidate's winning probability, as illustrated in Figure 2.

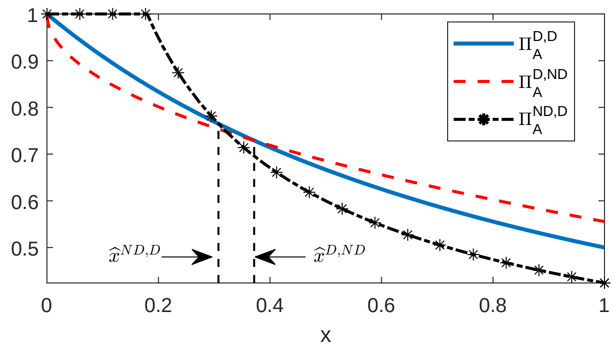


Figure 2: Candidate A's ex-post winning probability as a function of the realization of a uniformly distributed x on $(0, 1]$ and candidates' access to the intermediary's data.

The first part of Proposition 7 states that for low realizations of x, candidate A has a higher winning probability when her opponent has exclusive access to data. For high realizations of x, Candidate A's winning likelihood is higher when she has exclusive data access. This result follows directly from the informed candidate's response to learning x. Based on this finding, it is straightforward to see that ex ante, whether a candidate's probability of winning increases or decreases under the exclusivity regime relative to non-exclusive data access is dependent on the distribution of x, as the next result formalizes.

Proposition 8 *The following is satisfied under the exclusive data-access regimes:*

1. *There exists a cutoff $\mu^{D,ND}$ such that candidate A's ex-ante winning likelihood is higher when she has exclusive access to the intermediary's data compared to non-exclusive access if and only if $\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}[1+x]} < \mu^{D,ND}$.*

2. *There exists a cutoff $\mu^{ND,D}$ such that candidate A's ex-ante winning likelihood is higher when candidate B has exclusive access to the intermediary's data compared to non-exclusive access if and only if $\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]} > \mu^{ND,D}$.*

To gain intuition for the results in Proposition 8, let us conceptually begin from the no-access data regime and consider the case where candidate A exclusively learns that the realization of x is low. That means that candidate B, who does not know x, is overspending—given x, the marginal benefit of the last dollar spent by B is below its marginal cost. Since A is privately informed of x, in response, A increases her own spending to the point where her marginal benefit equals the marginal cost; however, due to the concavity of her marginal benefit from spending, the amount A chooses to spend still ends up being less than the amount that would "match" B's expenditure under the non-exclusive data access regime. Since A is interested in maximizing her expected utility rather than her winning likelihood, while A does increase her spending, overall she may still be less likely to win.

Similarly, when candidate B exclusively learns that the realization of x is high, he chooses to decrease his spending, knowing that candidate A, who does not know the realization of x, is underspending relative to its realization; however, candidate B only reduces his spending to the point where his marginal benefit from spending equals its marginal cost, which still results in a higher probability of B winning relative to non-exclusive data access.

Thus, a candidate may effectively choose to reduce his or her own probability of winning in order to better align their ad expenditure with the amount that is utility maximizing. Moreover, ex ante, whether a candidate is more or less likely to win when granted exclusive access to the intermediary's data is dependent upon whether the distribution over voters' predisposition parameter places more mass on lower or higher outcomes.

Example 3 *Under the parameter specifications from Example 1, with* $\alpha = 1$ *and* $\beta \in \{1, 3, 5\}$*, it can be verified that condition (1) of Proposition 8 is met, while condition (2) is not. Accordingly, ex ante, candidate A's winning likelihood is highest when she has exclusive access to the intermediary's data and lowest when candidate B has exclusive access.*

In light of candidates' advertising expenditures and ex-ante winning likelihoods as a function of their data access, it is straightforward to see that candidates' willingness to pay for data and the intermediary's profits vary across the access regimes. The following lemma provides necessary and sufficient conditions for the intermediary's profit to be positive.

Lemma 3 *The intermediary's profit is positive in equilibrium if and only if either of the following conditions is satisfied:*

$$\mathbb{E}\left[c_A^{D,ND}\right] < \frac{V - \min\left\{\left(VF\left(\tilde{x}^{ND,D}\right) + \mathbb{E}\left[c_A^{ND,D}\right]\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]\right), V\mathbb{E}\left[\left(\frac{1}{1+x}\right)^2\right]\right\}}{\mathbb{E}\left[2+x\right]} \quad (31)$$

$$\mathbb{E}\left[c_B^{ND,D}\right] < \frac{V\left(1 - F\left(\tilde{x}^{ND,D}\right)\right) - \min\left\{\mathbb{E}\left[c_B^{D,ND}\right]\mathbb{E}\left[x\right], V\mathbb{E}\left[\left(\frac{x}{1+x}\right)^2\right]\right\}}{2 + \mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]} \quad (32)$$

Conditions (31) and (32) provide upper bounds on candidates' expenditures under the exclusive access regimes. If the inequality in (31) is met, the intermediary's expected profit is positive when access is exclusively sold to candidate A; that is, candidate A is willing to pay a positive amount for exclusive data access, and similarly for (32) with respect to candidate B. If neither of these conditions is satisfied, the intermediary either does not sell data or provides non-exclusive access to both candidates, in which case its profit is given by:

$$\left(\mathbb{E}\left[U_A^{D,D}\right] - \mathbb{E}\left[U_A^{ND,ND}\right]\right) + \left(\mathbb{E}\left[U_B^{D,D}\right] - \mathbb{E}\left[U_B^{ND,ND}\right]\right)$$

However, as delineated in Section 3, candidates' ex-ante winning likelihoods and utilities are identical under both the non-exclusive access and no-access regimes. If there are costs associated with data collection, the intermediary stands to make negative profit from collecting the data in the first place. Hence, when neither (31) nor (32) are satisfied, the intermediary does not operate.

Taken together, the results in the preceding analysis give rise to the following corollary.

Corollary 1 *Under the conditions in Lemma 3, when*

$$\min\left\{\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]}, \frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\right\} < \sqrt{\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right]},$$

the intermediary grants exclusive data access to one of the candidates, thus altering candidates' winning likelihoods and potentially influencing the outcome of the election,

Corollary 1 follows from the intermediary's ranking of the three data-access regimes. For the parametric specifications in Example 1, the condition in the corollary is always met.

7 Alternate Assumptions

Our framework thus far demonstrates tensions that may prevail between a data intermediary's profit motives and the profit motives of an advertising platform—a platform that could, in part, be responsible for the data the intermediary possesses in the first place. This section considers alternate assumptions to those in the base setting.

7.1 Exclusionary Pricing

Our framework assumes common knowledge regarding the intermediary's chosen prices for access to its data. One may consider alternate settings where this assumption is not satisfied ex ante. To that end, suppose, first, that the intermediary sets the highest possible exclusionary price. A candidate would only be willing to pay this price if the intermediary could certify or commit to the arrangement that the candidate would receive exclusive access to the intermediary's data. If the intermediary refuses to certify a candidate's exclusive access, the candidate would know that access is not exclusive and would refuse to pay the data access price set by the intermediary.

Suppose, alternatively, that the intermediary sets a price that corresponds to somewhere between exclusionary and non-exclusionary pricing. Candidates would then know that access is non-exclusionary, else the intermediary would have set a higher price. Hence, candidates would refuse to pay a price that corresponds to somewhere in between exclusionary and non-exclusionary pricing, whereby the intermediary's price reveals its precise access regime.

Said another way, the common knowledge assumption in our framework regarding the intermediary's prices for data access is without significant loss of generality. This is because any uncertainty about whether the intermediary's chosen regime is exclusionary or non-exclusionary would be resolved in equilibrium through the intermediary's chosen pricing.

7.2 Strategic Platform

A common clause in contractual data relationships is for the data owner to retain the ultimate say over how their data is used. When an advertising social-media platform is the source of a significant portion of the intermediary's data, then the platform may take advantage of such clauses to specify that the intermediary must act in accordance with a ruleset that complies with the platform's own profit motives. The platform may, for instance, require that any downstream contractual agreements that pertain in any way to its data must first undergo a review by the platform. For example, under a similar condition to the one in Corollary 1, where $\max\{\frac{\mathbb{E}[\sqrt{x}]}{\mathbb{E}[1+x]}, \frac{\mathbb{E}[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}]}{1+\mathbb{E}[\frac{1}{x}|x\geq\tilde{x}^{ND,D}]}\} < \sqrt{\mathbb{E}[(\frac{\sqrt{x}}{1+x})^2]}$, it is in the platform's best interest to ensure that candidates have equal access to the data, which would render candidates' winning probabilities unaltered relative to the no-data case. That is, the platform could take advantage of a contractual agreement with an intermediary to ensure that both candidates end up with access to the data.

While incorporating such contractual features can indeed influence our equilibrium characterization, provided that at least some of the intermediary's data comes from sources outside of the platform, the intermediary would, however, under the condition in Corollary 1, still have strict incentives to contract with only one of the candidates, at least as it pertains to data from those other sources. Thus, the results would continue to hold to a degree, which means the intermediary still has the potential to influence the outcome of the contest.

Furthermore, if we envision the framework as having, in addition to a strategic intermediary, also a strategic platform with its own set of voter-pertinent data—data which may be valuable independently of the data possessed by the intermediary and which the platform can offer to candidates—then candidates' winning probabilities are guaranteed to be influenced in equilibrium relative to the no-data case. This is because either the intermediary or the platform will have strict incentives to share their data exclusively with one of the candidates.

7.3 Micro Targeting

Our framework groups voters into a single voter segment that may swing towards voting for either candidate as a function of an exogenous predisposition parameter and candidates' advertising spending. We studied this framework in order to isolate the effect of an intermediary possessing information that is pertinent to a political contest. While we have not focused on micro targeting, information revealed about political preferences on social media combined with other information aggregated by data intermediaries can help pinpoint individual voters' predispositions with some accuracy (Jernigan and Mistree, 2009). Each swing voter can then essentially become a battleground for a political contest.

While the present framework does not incorporate an intermediary who may possess individualized data on heterogeneous voters, our analysis can extend here if voters of a particular swing group or those who share similar characteristics are grouped together into a segment. This is the case provided candidates' campaigns do not face tight budgetary constraints for advertising to these segments. For instance, if an intermediary is able to identify similar voter types and discern their predisposition towards candidates in a swing locale of particular interest, then our findings may extend. Hence, our framework is a step towards a study of platforms and data intermediaries in the context of micro targeting.

8 Conclusion

We studied a political contest where two candidates spend ad dollars on an advertising platform in order to persuade voters to vote in their favor. While voters a priori favor one of the candidates, the precise extent of voters' predisposition can only be ascertained by an intermediary, which faces a choice of whether to sell access to such information to one or both of the candidates. We contrasted the intermediary's incentives for profiting from selling access with the platform's incentives for maximizing candidates' total ad spend, and showed that they have opposing rankings (and constant profit sums) in the three regimes in which access is sold either exclusively or non-exclusively. Our results indicate that there is always an incentive by either the intermediary or the platform for data to be exclusively shared with one of the candidates, which can influence the outcome of the contest, and that the platform has incentives to hinder the intermediary's ability to operate.

Our findings shed some light on potential tensions between a social media platform, which often helps generate at least some of the data that facilitates the intermediary's operations in the first place, and an intermediary whose choices can backfire on the platform because of choosing an access regime that lowers expected advertising revenues. This conflict between the platform and the intermediary further implies that the incentives of a platform, in terms of generating advertising revenue, and those of consumers, in terms of protecting their privacy, may in some cases be, in fact, aligned.

Future work can take on a number of directions. One direction is to consider an intermediary that is unable to commit to an access policy prior to acquiring data. For instance, the intermediary may first realize some private information about voters' predisposition and only then decide on an access regime. Importantly, one may also consider settings where the platform, as a strategic player, endogenously determines the extent of its data portability, which may influence the quantity and quality of the information that the intermediary possesses. One may also consider settings where candidates are competing over multiple voter groups and locations, where budget constraints play a prominent role, and where multiple intermediaries and platforms are vying for candidates' resources.

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A Proofs

Proof of Proposition 1. From the equilibrium resource outlays specified in (1) and (3), it immediately follows that, ex ante,

$$\mathbb{E}\left[c_A^{D,D}\right] = c_A^{ND,ND} = c_B^{ND,ND} = \mathbb{E}\left[c_B^{D,D}\right]$$

The result follows from the fact that the platform's profit is the sum of candidates' ad spend.

Proof of Lemma 1. Candidate A's first order condition of optimization $FOC_A^{D,ND}$ in (7) yields $c_A^{D,ND}(x)$ as given in (9). Substituting (9) back in $FOC_B^{D,ND}$ in (8) and re-arranging the terms of the resulting expression, we obtain

$$c_B^{D,ND} = \mathbb{E}\left[c_A^{D,ND}\right]. \tag{A–1}$$

As can be seen from (9), given c_B, $c_A(x)$ is positive if and only if $V > xc_B$ or $x < \frac{V}{c_B}$. Let $\tilde{x}^{D,ND} \equiv \max\left\{x \in [\underline{x}, \overline{x}] \,|x < \frac{V}{c_B}\right\}$. Plugging (A–1) back in (9), we obtain:

$$c_A(x) = \begin{cases} \sqrt{Vx\mathbb{E}\left[c_A^{D,ND}\right]} - x\mathbb{E}\left[c_A^{D,ND}\right] \text{ for } x \in \left[\underline{x}, \tilde{x}^{D,ND}\right] \\ 0 \text{ for } x \in \left(\tilde{x}^{D,ND}, \bar{x}\right] \end{cases}$$

Forming expectations by multiplying both sides by $f(x)$ and integrating with respect to x yields

$$\mathbb{E}\left[c_A^{D,ND}\right] = \sqrt{V\mathbb{E}\left[c_A^{D,ND}\right]}\mathbb{E}\left[\sqrt{x}|x \leq \tilde{x}^{D,ND}\right] - \mathbb{E}\left[x|x \leq \tilde{x}^{D,ND}\right]\mathbb{E}\left[c_A^{D,ND}\right].$$

Rearranging terms, we have

$$\mathbb{E}\left[c_A^{D,ND}\right] = V\left(\frac{\mathbb{E}\left[\sqrt{x}|x \leq \tilde{x}^{D,ND}\right]}{1 + \mathbb{E}\left[x|x \leq \tilde{x}^{D,ND}\right]}\right)^2, \tag{A–2}$$

which is positive as long as $\tilde{x}^{D,ND} > \underline{x}$. Substituting (A–2) in (A–1) gives

$$c_B^{D,ND} = V\left(\frac{\mathbb{E}\left[\sqrt{x}|x \leq \tilde{x}^{D,ND}\right]}{1 + \mathbb{E}\left[x|x \leq \tilde{x}^{D,ND}\right]}\right)^2 > 0,$$

which, in turn, can be substituted into (9) to arrive at the following necessary and sufficient condition for $c_A^{D,ND}(x)$ to be positive:

$$x\left(\frac{\mathbb{E}\left[\sqrt{x}|x\leq\tilde{x}^{D,ND}\right]}{1+\mathbb{E}\left[x|x\leq\tilde{x}^{D,ND}\right]}\right)^{2}<1. \tag{A–3}$$

Keeping in mind that support of x, $[\underline{x},\overline{x}]\subseteq(0,1]$ and $\frac{\mathbb{E}\left[\sqrt{x}|x\leq\tilde{x}^{D,ND}\right]}{1+\mathbb{E}\left[x|x\leq\tilde{x}^{D,ND}\right]}<0.5$, it is straightforward to see that the left hand side of (A–3) is always less than 1.[10] This implies that $c_A^{D,ND}(x)$ is positive for all realizations of x and $\tilde{x}^{D,ND}=\overline{x}$.

Proof of Lemma 2. Candidate B's first-order condition of optimization $FOC_B^{ND,D}$ in (16) yields $c_B(x)$ as given in (17). Substituting (17) back in $FOC_A^{ND,D}$ in (15) and re-arranging the terms of the resulting expression, we obtain

$$\mathbb{E}\left[c_B^{ND,D}\right]=c_A^{ND,D}. \tag{A–4}$$

As can be seen from (17), given c_A, $c_B\left(x\right)$ is positive if and only if $xV>c_A$ or $x>\frac{c_A}{V}$. Let $\tilde{x}^{ND,D}\equiv\min\left\{x\in\left[\underline{x},\overline{x}\right]|x>\frac{c_A}{V}\right\}$. Plugging (A–4) in (17), we obtain:

$$c_B^{ND,D}\left(x\right)=\left\{\begin{array}{c}0\text{ for }x\in\left[\underline{x},\tilde{x}^{ND,D}\right)\\ \frac{\sqrt{Vx\mathbb{E}\left[c_B^{ND,D}\right]}-\mathbb{E}\left[c_B^{ND,D}\right]}{x}\text{ for all }x\in\left[\tilde{x}^{ND,D},\bar{x}\right]\end{array}\right\}$$

Forming expectations by multiplying both sides by $f(x)$ and integrating with respect to x yields

$$\mathbb{E}\left[c_B^{ND,D}\right]\;=\;\sqrt{V\mathbb{E}\left[c_B^{ND,D}\right]}\mathbb{E}\left[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right]-\mathbb{E}\left[\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right]\mathbb{E}\left[c_B^{ND,D}\right].$$

Rearranging terms, we have

$$\mathbb{E}\left[c_B^{ND,D}\right]=V\left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right]}\right)^{2}, \tag{A–5}$$

which is positive for $\tilde{x}^{ND,D}<\overline{x}$. Substituting (A–5) in (A–4) above gives

$$c_A^{ND,D}=V\left(\frac{\mathbb{E}\left(\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right)}{1+\mathbb{E}\left(\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right)}\right)^{2},$$

which, in turn, can be substituted in (17) to arrive at the following necessary and sufficient

[10] Note that $\sqrt{x}<\frac{1+x}{2}$ holds for all $x\in(0,1)$ which implies that $\frac{\mathbb{E}\left[\sqrt{x}|x\leq\tilde{x}^{D,ND}\right]}{1+\mathbb{E}\left[x|x\leq\tilde{x}^{D,ND}\right]}<0.5$ for all $\tilde{x}^{D,ND}\in(0,1)$.

condition for $c_B^{ND,D}(x)$ to be positive:

$$x > \left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}} | x \geq \tilde{x}^{ND,D}\right]}{1 + \mathbb{E}\left[\frac{1}{x} | x \geq \tilde{x}^{ND,D}\right]} \right)^2 \tag{A–6}$$

We utilize (A–6) to identify the equilibrium value of $\tilde{x}^{ND,D}$. Differentiating, it is straightforward to verify that the right-hand side of (A–6) is decreasing in $\tilde{x}^{ND,D}$. Then either of the following possibilities may arise:

- If $\left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}} | x \geq \underline{x}\right]}{1+\mathbb{E}\left[\frac{1}{x} | x \geq \underline{x}\right]} \right)^2 = \left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}\right]}{\mathbb{E}\left[1+\frac{1}{x}\right]} \right)^2 \leq \underline{x}$, then $c_B^{ND,D}$ is positive for all x and $\tilde{x}^{ND,D} = \underline{x}$.
- Otherwise if $\left(\frac{\mathbb{E}\left(\frac{1}{\sqrt{x}}\right)}{\mathbb{E}\left(1+\frac{1}{x}\right)} \right)^2 > \underline{x}$, then $\tilde{x}^{ND,D} > \underline{x}$ solves

$$\tilde{x}^{ND,D} = \left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}} | x \geq \tilde{x}^{ND,D}\right]}{1 + \mathbb{E}\left[\frac{1}{x} | x \geq \tilde{x}^{ND,D}\right]} \right)^2$$

At $\overline{x}$, the right side of (A–6) becomes zero implying that $\tilde{x}^{ND,D} < \overline{x}$ always.

Proof of Proposition 2. Since the platform is interested in maximizing the sum of candidates' ad spend, the result follows immediately from (A–1), (A–2), (A–4) and (A–5).

Proof of Proposition 3. We prove, in order, each of the three claims in the proposition.
Proof of Claim 1: From Propositions 1 and 2, it can be seen that the platform prefers non-exclusive access over exclusive access by A if and only if $\mathbb{E}[c_A^{D,D}] > \mathbb{E}[c_A^{D,ND}]$. Equivalently, the following condition needs to hold:

$$\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x} \right)^2 \right] > \left(\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]} \right)^2$$

Rearranging the terms of the above expression, it is easy to see that $\mathbb{E}[c_A^{D,D}] > \mathbb{E}[c_A^{D,ND}]$ if and only if

$$\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]} < \sqrt{\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x} \right)^2 \right]}$$

Proof of Claim 2: From Propositions 1 and 2, it can be seen that the platform prefers non-exclusive access over exclusive access by B if and only if $\mathbb{E}[c_A^{D,D}] > \mathbb{E}[c_B^{ND,D}]$. Equivalently, the following condition needs to hold:

$$\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right] > \left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\right)^2,$$

Re-arranging the terms of the above expression, it is easy to see that $\mathbb{E}[c_A^{D,D}] > \mathbb{E}[c_A^{ND,D}]$ if and only if

$$\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]} < \sqrt{\mathbb{E}\left[\left(\frac{\sqrt{x}}{1+x}\right)^2\right]}$$

Proof of Claim 3: The proof follows from a direct comparison of the platform's profits provided in Proposition 2.

Proof of Proposition 4. The two parts of the proposition are proven in order:
Proof of Claim 1: This claim seeks to identify conditions under which $c_A^{D,ND}(x) > c_A^{D,D}(x)$ where $c_A^{D,D}(x)$ and $c_A^{D,ND}(x)$ are given in (3) and (23), respectively. Equivalently, conditions are identified under which the following holds:

$$\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]} > \sqrt{x}\left(\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]}\right)^2 + \frac{\sqrt{x}}{(1+x)^2} \tag{A–7}$$

Given that $0 < \frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}[1+x]} < 0.5$ is a constant and $0 < x \leq 1$, it is straightforward to see that (A–7) is met for values of x close to 0 and is violated for $x = 1$. From the Intermediate Value Theorem, it follows that there exists at least one x which satisfies (A–7). In order to find such x, we solve inequality (A–7) numerically. Through some tedious algebra, it can be shown that there exists $0 < x_*^A \leq 1$ such that (A–7) is satisfied for all $x \in \left(0, x_*^A\right)$ and violated otherwise. Then depending on the value of $\underline{x}$ and $\overline{x}$, one of the following three possibilities would arise:

1. $\underline{x} < \overline{x} < x_*^A$. In this case, $\underline{x}^{D,ND} \equiv \overline{x}$.

2. $\underline{x} \leq x_*^A \leq \overline{x}$. In this case, $\underline{x}^{D,ND} \equiv x_*^A$.

3. $x_*^A < \underline{x} < \overline{x}$. In this case, $\underline{x}^{D,ND} \equiv \underline{x}$.

Claim 1 follows immediately from the above. For the completeness of the statement made in Claim 1, we note that $c_A^{D,ND}(\underline{x}^{D,ND}) = c_A^{D,D}\left(\underline{x}^{D,ND}\right)$ if $\underline{x}^{D,ND} = x_*^A$, whereas $c_A^{D,ND}(\underline{x}^{D,ND}) > c_A^{D,D}\left(\underline{x}^{D,ND}\right)$ if $\underline{x}^{D,ND} < x_*^A$ and $c_A^{D,ND}(\underline{x}^{D,ND}) < c_A^{D,D}\left(\underline{x}^{D,ND}\right)$ if $\underline{x}^{D,ND} > x_*^A$.

Proof of Claim 2: This claim seeks to identify conditions under which $c_B^{ND,D}(x) > c_B^{D,D}(x)$ where $c_B^{D,D}(x)$ and $c_B^{ND,D}(x)$ are given in (3) and (24) respectively. From the expressions of $c_B^{ND,D}(x)$ and $c_B^{D,D}(x)$, it is straightforward to see that $c_B^{ND,D}(x)$ is strictly lower than $c_B^{D,D}(x)$ for all $x \in \left[\underline{x}, \tilde{x}^{ND,D}\right)$. Proceeding in a manner similar to that in the preceding claim, conditions are first identified under which the following holds:

$$\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\sqrt{x} - \left(\frac{x}{1+x}\right)^2 > \left(\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]}\right)^2 \tag{A–8}$$

Given that $0 < \frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right]} < 0.5$ is a constant and $0 < x \leq 1$, we use numerical methods to identify x which satisfy (A–8).[11] In particular, it can be shown that there exists cutoffs $0 < x_*^B < x_{**}^B \leq 1$ such that (A–8) is satisfied for all $x \in \left(x_*^B, x_{**}^B\right)$. Then depending on the values of $\tilde{x}^{ND,D}$ and $\overline{x}$, one of the following possibilities would arise:

1. $\tilde{x}^{ND,D} < \overline{x} < x_*^B < x_{**}^B$. In this case, $\underline{x}^{ND,D} = \overline{x}^{ND,D} \equiv \overline{x}$.
2. $\tilde{x}^{ND,D} \leq x_*^B < \overline{x} < x_{**}^B$. In this case, $\underline{x}^{ND,D} \equiv x_*^B$ and $\overline{x}^{ND,D} \equiv \overline{x}$.
3. $\tilde{x}^{ND,D} \leq x_*^B < x_{**}^B \leq \overline{x}$. In this case, $\underline{x}^{ND,D} \equiv x_*^B$ and $\overline{x}^{ND,D} \equiv x_{**}^B$.
4. $x_*^B < \tilde{x}^{ND,D} \leq x_{**}^B \leq \overline{x}$. In this case, $\underline{x}^{ND,D} \equiv \tilde{x}^{ND,D}$ and $\overline{x}^{ND,D} \equiv x_{**}^B$.
5. $x_*^B < x_{**}^B < \tilde{x}^{ND,D} < \overline{x}$. In this case, $\underline{x}^{ND,D} = \overline{x}^{ND,D} \equiv \tilde{x}^{ND,D}$.

Claim 2 follows immediately from the above analysis. For the completeness of the statement made in Claim 2, we note that $c_B^{ND,D}(\underline{x}^{ND,D}) = c_B^{D,D}(\underline{x}^{ND,D})$ for $\underline{x}^{ND,D} = x_*^B$, $c_B^{ND,D}(\underline{x}^{ND,D}) < c_B^{D,D}(\underline{x}^{ND,D})$ for $\underline{x}^{ND,D} = \overline{x}^{ND,D}$ and $c_B^{ND,D}(\underline{x}^{ND,D}) > c_B^{D,D}(\underline{x}^{ND,D})$ for $\overline{x}^{ND,D} > \underline{x}^{ND,D} > x_*^B$. In a similar vein, it can be stated that $c_B^{ND,D}(\overline{x}^{ND,D}) = c_B^{D,D}(\overline{x}^{ND,D})$ if $\overline{x}^{ND,D} = x_{**}^B$, $c_B^{ND,D}(\overline{x}^{ND,D}) < c_B^{D,D}(\overline{x}^{ND,D})$ if $\overline{x}^{ND,D} = \underline{x}^{ND,D}$ and $c_B^{ND,D}(\overline{x}^{ND,D}) > c_B^{D,D}(\overline{x}^{ND,D})$ if $\underline{x}^{ND,D} < \overline{x}^{ND,D} < x_{**}^B$.

Proof of Proposition 5. From (25) and (26), it can be seen that the intermediary prefers exclusive access by A over non-exclusive access if and only if $\mathbb{E}[U_A^{D,D}]+\mathbb{E}[U_B^{D,D}] < \mathbb{E}[U_A^{D,ND}]+U_B^{D,ND}$, or equivalently, $\mathbb{E}[(\frac{\sqrt{x}}{1+x})^2] > (\frac{\mathbb{E}[\sqrt{x}]}{\mathbb{E}[1+x]})^2$. By proceeding in a manner similar to that in the proof of Proposition 3, it can be seen that a necessary sufficient condition for this to hold is $\frac{\mathbb{E}[\sqrt{x}]}{\mathbb{E}[1+x]} < \sqrt{\mathbb{E}[(\frac{\sqrt{x}}{1+x})^2]}$. The remaining two claims can be proved in an analogous fashion.

Proof of Proposition 6. First, we prove that the sum of the platform's and intermediary's profits under non-exclusive access and exclusive access to A are same. Given Proposition 1

[11] Note that $\frac{1}{\sqrt{x}} \leq \frac{1}{2}(1+\frac{1}{x})$ which implies that $\frac{\mathbb{E}\left(\frac{1}{\sqrt{x}}|x\geq\tilde{x}^{ND,D}\right)}{1+\mathbb{E}\left(\frac{1}{x}|x\geq\tilde{x}^{ND,D}\right)} < 0.5$ for all $\tilde{x}^{ND,D} \in (0,1)$.

and (25), it can be seen that the sum of the platform's and intermediary's profits equals

$$\mathbb{E}\left[U_A^{D,D}\right] - \mathbb{E}\left[U_A^{ND,D}\right] + \mathbb{E}\left[U_B^{D,D}\right] - \mathbb{E}\left[U_B^{D,ND}\right] + 2\mathbb{E}\left[c_A^{D,D}\right]$$

Replacing $\mathbb{E}[U_A^{D,D}]+\mathbb{E}[U_B^{D,D}]$ with $V-2\mathbb{E}[c_A^{D,D}]$ and $\mathbb{E}[U_B^{D,ND}]$ with $V-\mathbb{E}[U_A^{D,ND}]-2\mathbb{E}[c_A^{D,ND}]$, it immediately follows that the sum of the platform's and intermediary's profits equals $\mathbb{E}[U_A^{D,ND}]-\mathbb{E}[U_A^{ND,D}]+2\mathbb{E}[c_A^{D,ND}]$ which is exactly equal to the sum of their profits when candidate A has exclusive access. In a similar manner, it can be shown that the the sum of the platform's and intermediary's profits under non-exclusive access equals the joint sum when candidate B has exclusive access. The conflict between the platform's and the intermediary's rankings over these regimes is immediate from (28) – (30).

Proof of Proposition 7. The two parts of the proposition are proven in order:
Proof of Claim 1: First, we identify the necessary and sufficient conditions under which $\Pi_A^{D,ND}(x) > \Pi_A^{D,D}(x)$. Given (4) and (11), it can be seen that $\Pi_A^{D,ND}(x) > \Pi_A^{D,D}(x)$ if and only if the following holds:

$$\frac{\sqrt{x}}{1+x} > \frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]}. \tag{A–9}$$

Given that $0 < \frac{\mathbb{E}[\sqrt{x}]}{\mathbb{E}[1+x]} < 0.5$ is a constant and $0 < \frac{\sqrt{x}}{1+x} \leq 0.5$ is a strictly increasing function of x, an application of the Intermediate Value Theorem implies that there exists a unique $0 < x_+^A < 1$ such that (A–9) is satisfied for all $x \in \left(x_+^A, 1\right]$. Depending on the value of $\underline{x}$ and $\overline{x}$, either of the following three possibilities might arise:

1. $\underline{x} < \overline{x} < x_+^A$. In this case, $\widehat{x}^{D,ND} \equiv \overline{x}$.

2. $\underline{x} \leq x_+^A \leq \overline{x}$. In this case, $\widehat{x}^{D,ND} \equiv x_+^A$.

3. $x_+^A < \underline{x} < \overline{x}$. In this case, $\widehat{x}^{D,ND} \equiv \underline{x}$.

For the completeness of the statement in Claim 1, we note that $\Pi_A^{D,ND}\left(\widehat{x}^{D,ND}\right) = \Pi_A^{D,D}\left(\widehat{x}^{D,ND}\right)$ if $\widehat{x}^{D,ND} = x_+^A$, $\Pi_A^{D,ND}\left(\widehat{x}^{D,ND}\right) < \Pi_A^{D,D}\left(\widehat{x}^{D,ND}\right)$ if $\widehat{x}^{D,ND} < x_+^A$ and $\Pi_A^{D,ND}(\widehat{x}^{D,ND}) > \Pi_A^{D,D}(\widehat{x}^{D,ND})$ if $\widehat{x}^{D,ND} > x_+^A$.

Proof of Claim 2: By proceeding in an analogous manner to Claim 1, we identify the necessary and sufficient conditions for $\Pi_A^{ND,D}(x) > \Pi_A^{D,D}(x)$ to hold. Given (4) and (19), it is straightforward to see that $\Pi_A^{ND,D}(x) > \Pi_A^{D,D}(x)$ for all $x \in \left[\underline{x}, \tilde{x}^{ND,D}\right)$. For $x \geq \tilde{x}^{ND,D}$, it can be seen that $\Pi_A^{ND,D}(x) > \Pi_A^{D,D}(x)$ if and only if

$$\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]} > \frac{\sqrt{x}}{1+x}. \tag{A–10}$$

Given that $0 < \frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]} < 0.5$ is a constant and $0 < \frac{\sqrt{x}}{1+x} \leq 0.5$ is a strictly increasing function of x, an application of the Intermediate Value Theorem implies that there exists a unique $0 < x_+^B \leq 1$ such that (A–10) is satisfied for all $x \in \left(0, x_+^B\right)$. Depending on the value of $\tilde{x}^{ND,D}$ and $\overline{x}$, either of the following three possibilities might arise:

1. $\tilde{x}^{ND,D} < \overline{x} < x_+^B$. In this case, $\widehat{x}^{ND,D} \equiv \overline{x}$.
2. $\tilde{x}^{ND,D} \leq x_+^B \leq \overline{x}$. In this case, $\widehat{x}^{ND,D} \equiv x_+^B$.
3. $x_+^B < \tilde{x}^{ND,D} < \overline{x}$. In this case, $\widehat{x}^{ND,D} \equiv \tilde{x}^{ND,D}$.

For completeness of the statement in Claim 2, we note that $\Pi_A^{ND,D}\left(\widehat{x}^{ND,D}\right) = \Pi_A^{D,D}\left(\widehat{x}^{ND,D}\right)$ if $\widehat{x}^{ND,D} = x_+^B$, $\Pi_A^{ND,D}\left(\widehat{x}^{ND,D}\right) > \Pi_A^{D,D}\left(\widehat{x}^{ND,D}\right)$ if $\widehat{x}^{ND,D} < x_+^B$, and $\Pi_A^{ND,D}\left(\widehat{x}^{ND,D}\right) < \Pi_A^{D,D}\left(\widehat{x}^{ND,D}\right)$ if $\widehat{x}^{ND,D} > x_+^B$.

Proof of Proposition 8. The two parts of the proposition are proven in order:
Proof of Claim 1: We identify the necessary and sufficient conditions under which $\mathbb{E}[\Pi_A^{D,ND}] > \mathbb{E}[\Pi_A^{D,D}]$. Given (5) and (12), a necessary and sufficient condition for $\mathbb{E}[\Pi_A^{D,ND}] > \mathbb{E}[\Pi_A^{D,D}]$ is given by:

$$\frac{\left(\mathbb{E}\left[\sqrt{x}\right]\right)^2}{\mathbb{E}\left[1+x\right]} < \mathbb{E}\left[\frac{x}{1+x}\right].$$

Re-arranging the terms of the above expression yields the following necessary and sufficient condition for $\mathbb{E}[\Pi_A^{D,ND}] > \mathbb{E}[\Pi_A^{D,D}]$ to hold.

$$\frac{\mathbb{E}\left[\sqrt{x}\right]}{\mathbb{E}\left[1+x\right]} < \frac{\mathbb{E}\left[\frac{x}{1+x}\right]}{\mathbb{E}\left[\sqrt{x}\right]} \equiv \mu^{D,ND}$$

Proof of Claim 2: Using (5) and (20) and proceeding in manner similar to preceding claim, it is straightforward to show that a necessary and sufficient condition for $\mathbb{E}[\Pi_A^{ND,D}] > \mathbb{E}[\Pi_A^{D,D}]$ is:

$$\frac{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]}{1+\mathbb{E}\left[\frac{1}{x}|x \geq \tilde{x}^{ND,D}\right]} > \frac{\mathbb{E}\left[\frac{1}{1+x}\right] - F\left(\tilde{x}^{ND,D}\right)}{\mathbb{E}\left[\frac{1}{\sqrt{x}}|x \geq \tilde{x}^{ND,D}\right]} \equiv \mu^{ND,D}$$

Proof of Lemma 3. These conditions are directly derived by simplifying the following:

$$\begin{aligned} \mathbb{E}\left[U_A^{D,ND}\right] &> \min\left\{\mathbb{E}\left[U_A^{ND,D}\right], \mathbb{E}\left[U_A^{ND,ND}\right]\right\} \quad \text{and} \\ \mathbb{E}\left[U_B^{ND,D}\right] &> \min\left\{\mathbb{E}\left[U_B^{D,ND}\right], \mathbb{E}\left[U_B^{ND,ND}\right]\right\}. \end{aligned}$$

Proof of Corollary 1. The result follows from a direct comparison of conditions provided in Proposition 5.

B Intermediary's Profit

Assuming the intermediary's cost of acquiring data is negligible, its profits are positive across all three data-access regimes (A has exclusive access, B has exclusive access, and A and B have non-exclusive access) if and only if the following conditions are met:

$$\begin{aligned}
\mathbb{E}\left[c_A^{ND,D}\right] &< \frac{\min\left\{V - \mathbb{E}\left[c_A^{D,ND}\right]\mathbb{E}\left[2+x\right], V\mathbb{E}\left[\left(\frac{1}{1+x}\right)^2\right]\right\} - VF\left(\tilde{x}^{ND,D}\right)}{\mathbb{E}\left[\frac{1}{x}|x > \tilde{x}^{ND,D}\right]} \quad \text{and} \\
\mathbb{E}\left[c_B^{D,ND}\right] &< \frac{\min\left\{V\left(1 - F\left(\tilde{x}^{ND,D}\right)\right) - \mathbb{E}\left[c_B^{ND,D}\right]\left(2 + \mathbb{E}\left[\frac{1}{x}|x > \tilde{x}^{ND,D}\right]\right), V\mathbb{E}\left[\left(\frac{x}{1+x}\right)^2\right]\right\}}{\mathbb{E}\left[x\right]}
\end{aligned}$$

These conditions are derived by simplifying the following two inequalities:

$$\begin{aligned}
\mathbb{E}\left[U_A^{ND,D}\right] &< \min\left\{\mathbb{E}\left[U_A^{D,ND}\right], \mathbb{E}\left[U_A^{D,D}\right]\right\} \quad \text{and} \\
\mathbb{E}\left[U_B^{D,ND}\right] &< \min\left\{\mathbb{E}\left[U_B^{ND,D}\right], \mathbb{E}\left[U_B^{D,D}\right]\right\}
\end{aligned}$$