Optimal Unilateral Carbon Policy*

Samuel Kortum† and David A. Weisbach‡

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Abstract

We derive the optimal unilateral policy in a general equilibrium model of trade and climate change where one region of the world imposes a climate policy and the rest of the world does not. A climate policy in one region shifts activities—extraction, production, and consumption—in the other region. The optimal policy trades off the costs of these distortions. The optimal policy can be implemented through: (i) a nominal tax on extraction at a rate equal to the global marginal harm from emissions, (ii) a tax on imports of energy and goods, and a rebate of taxes on exports of energy but not goods, both at a lower rate than the extraction tax rate, and (iii) a goods-specific export subsidy. The policy controls leakage by combining supply-side and demand-side taxes to control the price of energy in the non-taxing region. It exploits international trade to expand the reach of the climate policy. We calibrate and simulate the model to illustrate how the optimal policy compares to more traditional policies such as extraction, production, and consumption taxes and combinations of those taxes. The simulations show that combinations of supply-side and demand-side taxes are much better than simpler policies and can perform nearly as well as the optimal policy.

Keywords: carbon taxes, border adjustments, leakage, climate change
JEL Codes: F18, H23, Q54

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†Yale University, Department of Economics, samuel.kortum@yale.edu

‡The University of Chicago Law School, d-weisbach@uchicago.edu

1 Introduction

Global negotiations have given up trying to achieve a uniform approach to climate change, such as a harmonized global carbon tax. Instead, current negotiations focus on achieving uniform participation, with each country pursuing its own approach and its own level of emissions reductions. As a result, policies to control emissions of greenhouse gases vary widely by country, and are likely to continue to do so for the indefinite future.

Widely varying carbon policies potentially affect the location of extraction, production, and consumption, the effectiveness of the policies, and the welfare of people in various countries or regions. Often expressed in terms of leakage—the increase in emissions abroad because of carbon policies at home—these effects are of critical importance to the design of carbon policy and to its political feasibility. Uncontrolled leakage threatens the viability of unilateral carbon policies: When leakage is high, a unilateral carbon policy shifts production offshore while failing to control emissions. As a result, leakage was central to the design of the European Union Emissions Trading System, the Regional Greenhouse Gas Initiative, and California's carbon pricing system. To address leakage, the EU is considering an import tariff on carbon-intensive goods, known as the Carbon Border Adjustment Mechanism. One of the reasons that the United States did not ratify the Kyoto Protocol was concern about the lack of emissions policies in developing countries and the resulting trade effects. Unless concerns about the effects of differential carbon prices are addressed, it may be difficult to achieve significant reductions in global emissions.

Because of its prominence, there is a substantial prior literature on regional carbon taxes and leakage (reviewed below). The overwhelming majority of this work uses computable general equilibrium models to estimate leakage due to carbon policies that are chosen by the modeler. A smaller literature, which we build on, finds optimal regional pollution policies using analytic models of the problem. The first and primary paper in this line of literature is Markusen (1975), with subsequent work including Hoel (1994), Hoel (1996), and Keen and Kotsogiannis (2014). A key result found in Markusen and related models is that optimal regional policies balance taxes on the supply side and the demand side of the market.

Markusen's model includes only two sectors, which we interpret in the climate context as an extraction sector and a consumption sector. A key concern with leakage, however, is how carbon policies affect the location of production. Markusen's model is not able to capture these effects.

To study this issue, we add a production sector that manufactures goods, and the Dornbusch, Fisher, and Samuelson (1977; henceforth DFS) model of trade in those goods. Following Markusen, we assume one region (Home) imposes a pollution policy (here, on carbon emissions) and the rest of the world (Foreign) does not. As in Markusen, we have an extraction sector and trade in fossil fuels. DFS brings in production, trade, and consumption of goods produced with fossil fuels. Following Böhringer, Lange, and Rutherford (2014), we restrict policies adopted by Home to those that do not make Foreign worse off. Our solution strategy borrows from Costinot, Donaldson, Vogel, and Werning (2015; henceforth CDVW).

We solve the model to find the optimal allocation and then show how the allocation can be implemented through taxes and subsidies. Similar to Markusen, in our model, a planner seeking to optimize Home's welfare balances (i) the wedge between the planner's marginal valuation of extracting a unit of energy and the Foreign energy price (the *extraction wedge*) and (ii) the wedge between the planner's marginal valuation of energy when used and the Foreign energy price (the *consumption wedge*). In addition, the planner balances (iii) the wedge between the shadow cost of Home's exports of goods to Foreign and marginal utility to consumers in Foreign of those goods (the *export wedge*). Each of these wedges corresponds to an activity in Foreign that is not directly under the planner's control.

The taxes and subsidies that generate this policy in a decentralized equilibrium match up with these wedges: a tax on domestic extraction equal to the extraction wedge, a tax on energy embodied in domestic production and domestic consumption equal to the consumption wedge, and an export subsidy equal to the export wedge. These taxes and subsidies can be implemented via nominal taxes and border adjustments as follows: (i) a domestic carbon tax on the extraction of fossil fuels at the global marginal harm from emissions, i.e., at the full Pigouvian rate; (ii) a border tax on imports and a tax rebate for exports of fossil fuels, both at a rate equal to the consumption wedge (which we will call a "partial border adjustment" because it is at a lower rate than the underlying nominal extraction tax); (iii) a border tax on the energy content of imports at that same partial rate; and (iv) an export subsidy designed to expand low-carbon exports from Home to the rest of the world, set at the export wedge.[1] While the nominal extraction tax is equal to the Pigouvian rate, the partial border adjustment removes some of that tax, leaving the effective extraction tax equal to (minus) the extraction wedge.

We compare the optimal policy to more conventional policies, such as an extraction tax, a production tax, a consumption tax, and combinations of these taxes. To do this, we solve the model when the planner is constrained in the outcomes it can control. The planner's solution in each case follows the same logic as the optimal policy, setting the policy wedges to balance the marginal costs of different channels of leakage. Policies that combine taxes on both the supply and demand for energy inherit some of the good properties of the unilaterally optimal policy.

To explore the quantitative implications of our analysis, we calibrate the model and solve it numerically for both the optimal policy and the various constrained policies. In our core calibration, we assume that the OECD countries impose a carbon price and the rest of the world does not. Following the intuition just described, policies that combine taxes on the supply and demand for fossil fuels perform well in our simulations, considerably outperforming the more standard demand-side taxes on emissions from domestic production and those taxes combined with border adjustments, even approaching the outcomes of the optimal policy. As a result, these combinations of basic taxes may be desirable approaches for implementing a unilateral carbon policy. The combination of an extraction tax and a production tax would, in addition, be much easier to implement than several more conventional approaches.[2]

[1]The subsidy for exports is reminiscent of the export policy suggested by Fisher and Fox (2012) in that it offers export rebates without removing the domestic carbon tax. It differs from Fisher and Fox, however, in that the policy seeks to expand the export margin rather than merely maintain it.

[2]This tax can be imposed with a nominal tax on extraction combined with border adjustments

Our core model does not include renewable energy, and stimulating renewables is often seen as a central goal of carbon pricing. To examine this issue, we extend the analysis to show that including renewables only requires modest adjustments to the optimal policy. Not surprisingly, renewables are exempt from the tax on extraction, which therefore acts as an implicit subsidy.

The paper proceeds as follows. The remainder of this section provides additional motivation and reviews the relevant literature. Section 2 lays out the basic elements of the model. Section 3 solves the problem of a planner designing an optimal carbon policy for one region with the other region behaving as in the competitive equilibrium. In Section 4 we derive a set of taxes and subsidies that implement the optimal policy. Section 5 derives the taxes that Home would impose if it is constrained to using simpler policies. We explore the quantitative implications of the optimal policy in Section 6, using a calibrated version of the model. Section 7 extends the analysis to include a renewable energy sector. Section 8 concludes.

1.1 Prior Literature

As noted, there is a voluminous prior literature studying this problem, with the overwhelming majority of studies using computable general equilibrium models to simulate carbon taxes and border adjustments. By our count, there are over 50 CGE studies of the general problem of differential carbon prices in the peer-reviewed literature (and many more in the gray literature). Branger and Quirion (2014) perform a meta-analysis of 25 studies of differential carbon taxes (20 of which were CGE studies while 5 were partial equilibrium studies). These 25 studies had 310 different modeled scenarios.

CGE studies almost uniformly use leakage as their measure of the effects

(at a lower rate) on the imports and exports of energy, but not goods. As suggested by Metcalf and Weisbach (2009), an extraction tax would be easy to impose because there are a relatively small number of large extractors who would need to remit taxes. Border adjustments on energy would also be easy to impose because imports and exports of energy are already carefully tracked. As a result, the simulations suggest that the combination of an extraction tax and a production tax is a promising policy to explore. It is also likely that the extraction/production hybrid raises fewer concerns about WTO compatibility than do the optimal tax or conventional border adjustments imposed on goods.

of differential carbon prices. Leakage is commonly defined as the increase in emissions in non-taxing regions as a percentage of the reduction in emissions in the taxing region (hence, 100% leakage means the policy is totally ineffective in reducing global emissions). Leakage estimates fall within a relatively consistent range. Branger and Quirion's meta-study finds leakage rates between 5% and 25% with a mean of 14% without border adjustments. With border adjustments, leakage ranges from −5% to 15%, with a mean of 6%. Similarly, as summarized by Böhringer et al. (2012), the Energy Modeling Forum commissioned 12 modeling groups to study the effects of border adjustments on leakage using a common data set and common set of scenarios. They considered emissions prices in the Kyoto Protocol Annex B countries (roughly the OECD) that reduce global emissions by about 9.5%. Without border adjustments, leakage rates were in the range of 5% to 19% with a mean value of 12%. These studies find that border adjustments reduce leakage by about a third, with a range between 2% and 12% and a mean value of 8%. Elliott et al. (2013) replicated 19 prior studies within their own CGE model, finding leakage rates between 15% and 30% for a tax on Annex B countries that reduced global emissions by about 13%.[3]

Rather than a large CGE model, we use an analytic general equilibrium model of trade to study the problem. This approach allows us to uncover the underlying economic logic for why some policies perform better than others, as well as solve for the optimal policy. It means, however, that our quantitative analysis is more illustrative than definitive.

There are a number of studies that precede us in this approach. As noted, Markusen (1975) analyzes a two-country, two-good model in which production of one of the goods generates pollution that harms both countries. Writing before climate change was a widespread concern, he considers a simple pollutant, such as the release of chemicals into Lake Erie by polluters in the United States, which

[3] Other surveys of the leakage literature include Droge et al. (2009), Zhang (2012) and Metz et al. (2007). A few studies focus on the effects of carbon taxes on particular energy-intensive and trade-exposed sectors. For example, Fowlie et al. (2016) consider the effects of a carbon price on the Portland cement industry. They find that a carbon price has the potential to increase distortions associated with market power in that industry. Leakage compounds these costs. They find that border adjustments induce negative leakage because of how industry actors respond, and can generate significant welfare gains at high carbon prices.

harms Canada (as well as the United States). One of the countries imposes policies to address the pollution; the other is passive. Markusen finds that the optimal tax is a Pigouvian tax on the dirty good combined with a tariff (if the good is imported) or a subsidy (if it is exported). The optimal tariff or subsidy combines terms of trade considerations and considerations related to leakage and is generally lower than the Pigouvian tax.

Hoel (1996) further generalizes Markusen's analysis and produces similar results in the context of climate change and carbon taxes. Hoel also considers the case where the country imposing the carbon policy may not impose tariffs. In this case, the optimal policy will involve carbon taxes that vary by sector (even though the harms from emissions do not vary by sector). Other analytic models of the problem include Fowlie and Reguant (2020), Keen and Kotsogiannis (2014), Böringher, Lange and Rutherford (2014), Holladay et al (2018), Hemous (2016), Baylis et al. (2014), Jakob, Marschinski and Hubler (2013), Fischer and Fox (2012, 2011), and Fowlie (2009).[4]

2 Basic Model

We develop a simple model that captures the effects of unilateral carbon pricing in the presence of international trade in energy and manufactured goods. To do this, we assume that the world is divided into just two countries or regions, Home and Foreign. Home adopts a carbon policy while Foreign is passive in the sense that it does not react to or anticipate Home's policy (though it may have its own carbon prices).

Each country is endowed with energy deposits and with labor, L and L^* (throughout, * denotes a Foreign variable). Production takes place in two stages. First, energy is extracted from deposits using labor and is freely traded in the international market. Second, final goods are produced by combining labor and energy, and services are provided using only labor. Labor is perfectly mobile across sectors within a country.[5] Energy used in production, or consumed directly,

[4]Kruse-Andersen and Sorensen (2022) was written contemporaneously with the present paper and has some similar results in the context of multinational production.

[5]What we call "labor" can be interpreted as a combination of labor and capital used to

generates a global externality, which motivates Home's policy choices.

As in DFS, goods come in a continuum, indexed by $j \in [0,1]$, and each country specializes in the production of a range of goods based on comparative advantage. Trade in goods is subject to iceberg costs which means that some goods, in the intermediate range, will be produced in both countries. Individuals in each country consume energy, goods, and services to maximize utility, closing the model.

We solve the model using the primal method to find Home's optimal allocation and then show how this allocation can be achieved in a decentralized market using taxes and subsidies. We then compare Home's optimal policy to restricted policies that resemble those commonly considered.

We specify the model as follows:

2.1 Preferences

We assume preferences are additively separable in consumption of services, C_s, consumption of individual goods, c_j, direct consumption of energy, C_e^d, and harms from climate change:

$$U = C_s + \int_0^1 u(c_j)dj + v(C_e^d) - \varphi Q_e^W, \tag{1}$$

where φ is the marginal harm from global emissions and Q_e^W is global energy extraction. We assume u and v are increasing and concave, and furthermore:

$$u(c) = \eta^{1/\sigma}\frac{c^{1-1/\sigma}-1}{1-1/\sigma},$$

where η governs demand for goods relative to services and σ is the elasticity of substitution between goods. With this restriction, preferences can also be represented by $U = C_s + u(C_g) + v(C_e^d) - \varphi Q_e^W$, where C_g is an index of goods consumption:

$$C_g = \left(\int_0^1 c_j^{(\sigma-1)/\sigma}dj\right)^{\sigma/(\sigma-1)}.$$

extract energy, produce goods, and provide services.

Foreign preferences are the same except with u^* (η^* and σ^*), v^*, and φ^*.[6]

2.2 Technology

Energy in each country is deposited in a continuum of fields, distinguished by the labor required to extract a unit of energy, a. The quantity of energy that can be extracted from all fields with a unit labor requirement $a \leq \bar{a}$ is $E(\bar{a})$ in Home and $E^*(\bar{a})$ in Foreign.[7] The minimum amount of labor L_e required to extract a quantity $Q_e = E(\bar{a})$ in Home is:

$$L_e = \int_0^{\bar{a}} a\, E'(a) da. \tag{2}$$

Global energy extraction is $Q_e^W = Q_e + Q_e^*$. This energy is either consumed directly or used as an intermediate input by the goods sector.

Any good $j \in [0, 1]$ can be produced in Home by combining energy E_j with labor L_j under constant returns. The total unit input requirement (inverse efficiency) is a_j. In terms of energy intensity (energy per unit of labor), $k_j = E_j/L_j$, the production function can be expressed as:

$$q_j = \frac{1}{a_j} f(k_j) L_j, \tag{3}$$

where f is an increasing concave function. For example, if production is Cobb-Douglas, $q_j = (1/a_j)E_j^{\alpha} L_j^{1-\alpha}$ so that $f(k_j) = k_j^{\alpha}$. In line with our Ricardian assumptions, we treat f as common across goods and countries, which means the production functions in Foreign are the same, but with Foreign's total unit input requirement a_j^* in place of a_j.

Services, in quantitites Q_s and Q_s^*, are provided in both countries with a unit

[6]Prior to introducing multiple energy sources, including renewables, in Section 7, we equate energy with a homogeneous fossil fuel measured by its carbon content. Since all energy is combusted in producing goods, global energy extraction is equal to global emissions. Our assumption of quasi-linear preferences greatly simplifies the analysis of trade policy, as in Grossman and Helpman (1994). To ensure that the marginal utility of income is 1 we assume $C_s > 0$ and $C_s^* > 0$, a condition that is easily checked.

[7]Since energy is costly to extract, we set $E(0) = E^*(0) = 0$. For analytical tractability we assume that $E(a)$ is differentiable, with $E'(a) > 0$ and $E^{*\prime}(a) > 0$.

labor requirement of 1. We take services to be the numéraire.[8]

2.3 International Trade

Home and Foreign can costlessly trade services for energy at price p_e (services per unit of energy). This price dictates outcomes in Foreign within the planning problem that we consider below. We will also compare outcomes in the planning problem to an unfettered competitive equilibrium of the model, which we call *business as usual* (BAU). We choose units of energy so that $p_e = 1$ clears the global energy market in this BAU baseline. With this choice of units, the parameter $\varphi^W = \varphi + \varphi^*$ represents the marginal global harm from combusting a unit of fossil fuel relative to its value in BAU.

Trade in the continuum of manufactured goods follows DFS. Goods are ordered by Home comparative advantage:

$$\frac{a_j^*}{a_j} = F(j). \tag{4}$$

We assume that $F(j)$ is continuous and strictly decreasing, with $F(0)$ arbitrarily large and $F(1) = 0$.[9]

Trade in goods incurs iceberg costs $\tau \geq 1$ on Home exports and $\tau^* \geq 1$ on Home imports. Taking account of these trade costs, the overall unit input requirement for Home to supply good j to Foreign is τa_j and for Foreign to supply good j to Home is $\tau^* a_j^*$.

2.4 Labor and Energy Requirements

We allow the energy intensity for good j to depend not only on where the good is produced but also on where it will be shipped. For each good j we distinguish Home exports x_j and Home production for domestic consumption y_j. We also

[8] Throughout Section 3 the numéraire is only relevant to Home when it trades with Foreign, as Home's economy is run by a planner. In Foreign, we assume that $Q_s^* > 0$ so that, given the unit labor requirement of 1 for services, the wage in Foreign is $w^* = 1$. This outcome is guaranteed with a large enough labor endowment in Foreign.

[9] These assumptions on $F(j)$ simplify the analysis of goods trade. To simplify aggregation across goods, we assume that a_j and a_j^* are also continuous functions.

distinguish Foreign exports (Home imports) m_j and Foreign production for its own domestic consumption, y_j^*. (Here x and m represent the quantities that reach consumers, so that $c_j = y_j + m_j$ and $c_j^* = y_j^* + x_j$.) For each good j there may be four different energy intensities, k_j^i, one for each of the four lines of production, $i \in \{x, y, m, y^*\}$.[10]

Given energy intensity $k_j^i = k$, $i \in \{x, y\}$, to produce good j in Home, we can invert the production function (3) to get the unit labor and energy requirements:

$$l_j(k) = a_j/f(k); \qquad e_j(k) = kl_j(k) = \frac{a_j k}{f(k)}. \tag{5}$$

Given $k_j^i = k$, $i \in \{m, y^*\}$, to produce good j in Foreign, the unit labor and energy requirements, $l_j^*(k)$ and $e_j^*(k)$, are the same as in (5), but with a_j^* in place of a_j.[11] (We will bring back trade costs when applying these expressions for unit input requirements.)

2.5 Carbon Accounting

We take a unit of carbon to be a unit of energy. Energy can be extracted in both countries and Home may either export or import energy from Foreign. Carbon is released when the energy is directly consumed (say, heating a house) or used to produce goods. Goods, embodying carbon emissions, may be traded before being consumed indirectly by households. We can therefore trace carbon from its extraction through its release into the atmosphere and finally to its implicit consumption.

We define G_e as direct consumption of energy in Home plus intermediate demand for energy by its goods sector. Home net exports of energy, positive or negative, are thus $Q_e - G_e$, accounting for the first level of trade in carbon. Following the same logic, Foreign net exports of energy are $Q_e^* - G_e^*$.

[10]Because Foreign can set k_j^{y*} independently from how it sets k_j^m, we do not include a so-called Brussels effect, as suggested by Bradford (2020).

[11]Since emissions are proportional to energy use, our *unit energy requirement*, $e_j(k)$, is sometimes called *emissions intensity* in the environmental economics literature, e.g. Shapiro and Walker (2018). We instead use the term *energy intensity* for energy per worker, k, by analogy to *capital intensity* for capital per worker.

The second level of trade in carbon is embodied in goods. The 2 by 2 matrix in the upper left of Table 1 depicts the bilateral flows of implicit consumption, with rows indicating the location of consumption and columns the location of production. For example, implicit consumption of carbon in Home is the sum of carbon released by producers in Home serving the local market, C_e^y, and carbon released by Foreign producers in supplying Home imports, C_e^m. Each of these terms is an aggregate across the unit continuum of goods. The first entry in the middle column, for example, shows that C_e^m takes account of the quantity of each good imported by Home, the unit energy requirement to produce it (a function of energy intensity), and the trade cost. Adding direct consumption to implicit consumption gives overall consumption, C_e in Home (in the upper right) and C_e^* in Foreign.

Table 1: Carbon Accounting Matrix

	Home	Foreign	Total
Home	$C_e^y = \int_0^1 e_j(k_j^y) y_j dj$	$C_e^m = \int_0^1 \tau^* e_j^*(k_j^m) m_j dj$	$C_e = C_e^y + C_e^m + C_e^d$
Foreign	$C_e^x = \int_0^1 \tau e_j(k_j^x) x_j dj$	$C_e^{y*} = \int_0^1 e_j^*(k_j^{y*}) y_j^* dj$	$C_e^* = C_e^x + C_e^{y*} + C_e^{d*}$
Total	$G_e = C_e^y + C_e^x + C_e^d$	$G_e^* = C_e^m + C_e^{y*} + C_e^{d*}$	$G_e^W = C_e^W \leq Q_e^W$

3 The Planning Problem

A planner allocates the resources that it controls to maximize Home welfare (1), subject to three constraints: (i) use of labor in the three sectors of the economy can't exceed labor supply; (ii) the global direct consumption of energy plus its use in producing goods can't exceed global extraction of energy; and (iii) its policies can't make Foreign worse off.[12] Consumption, production, and energy extraction

[12] We introduce the constraint on Foreign welfare to focus on policies that deal with the harm from global emissions rather than on policies that manipulate the terms of trade in favor of Home. To meet the Foreign welfare constraint, the planner can adjust transfers of services from Home to Foreign, subject to $C_s + C_s^* = Q_s^W = L_s^W$. The planner is not constrained by trade balance.

in Foreign are dictated by market prices. We consider these outcomes in Foreign and set out the constraints below before stating the planning problem. Before turning to outcomes in Foreign, we state a result on efficient production that appears frequently in what follows.

3.1 Efficient Production

If p is either the market price of energy or the planner's shadow cost of energy, the cost-minimizing energy intensity for producing good j is given by:

$$k(p) = \arg\min_k \left(l_j(k) + pe_j(k)\right) = \arg\min_k \left\{ \frac{1}{f(k)} + p\frac{k}{f(k)} \right\}. \tag{6}$$

The second equality, which follows from (5), shows that this result applies to production in both Home and Foreign (although the relevant value of p may differ). Using (6), we denote the minimum cost of inputs, for a given p, as $g(p) = (1 + pk(p))/f(k(p))$. The shadow cost of producing a unit of good j would then be $a_j g(p)$ in Home or $a_j^* g(p)$ in Foreign.

The solution to this minimization problem, $k(p)$ and $g(p)$, is the same for all goods j in Home and Foreign. This result, that goods are produced using the same energy intensity, for a given value of p, arises because of our simplifying assumption that, aside from the total input requirements a_j and a_j^*, the production function f is the same.[13]

3.2 Foreign

Consumption, production, and energy extraction in Foreign are dictated by market prices. Energy extractors in Foreign can sell energy at price p_e and can hire labor at wage $w^* = 1$. They tap all energy fields with a labor requirement below p_e:

$$Q_e^* = E^*(p_e). \tag{7}$$

[13] In their analysis of trade and the environment Copeland and Taylor (1994, 1995) also build on DFS, but make a restriction opposite to ours. Expressed in the notation here, they set $a_j = a$ and $a_j^* = a^*$ while allowing $f_j(k) = k^{\alpha(j)}$ to vary across goods. A hybrid of these two approaches (allowing both efficiency and energy intensity of production to vary across goods) could lead to additional insights but appears intractable.

Foreign employment in extraction is $L_e^* = \int_0^{p_e} aE^{*\prime}(a)da$, with $\partial L_e^*/\partial p_e = p_e E^{*\prime}(p_e)$.

Consumers in Foreign can buy energy at price p_e. They choose direct consumption of energy to equate their marginal utility with that price:

$$v^{*\prime}(C_e^{d*}) = p_e. \tag{8}$$

Goods producers in Foreign can purchase energy at price p_e and can hire labor at wage $w^* = 1$. They produce for the domestic market at a common energy intensity $k_j^{y*} = k(p_e)$, where $k(p_e)$ solves (6) with $p = p_e$. They supply good j to consumers in Foreign at a price equal to unit cost:

$$p_j^* = l_j^*(k(p_e)) + p_e e_j^*(k(p_e)) = a_j^* g(p_e). \tag{9}$$

Since consumers in Foreign can purchase any good j from domestic producers at price p_j^*, this price puts an upper bound on their marginal utility, $u^{*\prime}(c_j^*) \leq p_j^*$ (hence a lower bound on c_j^*).

3.3 Constraints

3.3.1 Labor Constraint

From (2), the labor L_e required to extract a quantity of energy Q_e is:

$$L_e = \int_0^{E^{-1}(Q_e)} a\, E'(a)da. \tag{10}$$

Global employment in energy extraction is $L_e^W = L_e + L_e^*$.

Labor L_g required for goods production in Home consists of labor used for goods produced for domestic consumption and for export:

$$L_g = \int_0^1 \left(l_j(k_j^y)y_j + \tau l_j(k_j^x)x_j \right) dj.$$

Adding employment in Foreign for domestic goods production and for export

(Home imports), global employment in goods production is:

$$L_g^W = L_g + \int_0^1 \left(l_j^*(k_j^{y*})y_j^* + \tau^* l_j^*(k_j^m)m_j\right) dj.$$

Accounting for labor to provide services, $L_s^W = C_s + C_s^*$, the global labor constraint is:

$$L_e^W + L_g^W + L_s^W = L^W. \tag{11}$$

3.3.2 Energy Constraint

The global constraint on use of energy is:

$$G_e + G_e^* \leq Q_e + Q_e^* = Q_e^W, \tag{12}$$

where Q_e is chosen by the planner and Q_e^* is given by (7). Expressions for G_e and G_e^*, the quantity of energy used in production plus direct consumption, are in Table 1.

3.3.3 Foreign Welfare Constraint

We require that the planner's policy not reduce welfare in Foreign, yet Home has no obligation to raise Foreign welfare either. Hence, the policy must maintain:

$$C_s^* + u^*(C_g^*) + v^*(C_e^{d*}) - \varphi^* Q_e^W = \bar{U}^*, \tag{13}$$

where $\bar{U}^*$ is Foreign welfare in BAU. In evaluating (13) below, we will employ the Foreign analog of the labor constraint (11).

3.4 The Planner's Lagrangian

The planner wants to maximize Home welfare, $U = C_s + u(C_g) + v(C_e^d) - \varphi Q_e^W$, subject to the three constraints above: (11), (12), and (13). Substituting in the labor constraint (11) and the Foreign welfare constraint (13) in place of C_s, the planner's objective turns out to be global welfare:[14]

$$U = u(C_g) + u^*(C_g^*) + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W Q_e^W + L^W - L_e^W - L_g^W - \bar{U}^*,$$

where $\varphi^W = \varphi + \varphi^*$ is the global marginal harm from emissions and L^W is the global labor endowment (with L_e^W allocated to extraction and L_g^W to goods production).

We apply a Lagrange multiplier λ_e to the energy constraint and drop the constants L^W and $\bar{U}^*$ to form the planner's Lagrangian:

$$\begin{aligned}\mathcal{L} = & \int_0^1 u(y_j + m_j)dj + \int_0^1 u^*(y_j^* + x_j)dj + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W Q_e^W \\ & - L_e^W - \int_0^1 \left(l_j(k_j^y)y_j + \tau l_j(k_j^x)x_j + l_j^*(k(p_e))y_j^* + \tau^* l_j^*(k_j^m)m_j\right) dj \\ & - \lambda_e \left(\int_0^1 \left(e_j(k_j^y)y_j + \tau e_j(k_j^x)x_j + e_j^*(k(p_e))y_j^* + \tau^* e_j^*(k_j^m)m_j\right) dj - Q_e^W\right). \end{aligned} \tag{14}$$

The terms are, line-by-line: (i) global utility from goods consumption and direct consumption of energy less harm from emissions, (ii) the opportunity cost (in terms of lost consumption of services) from labor employed in energy extraction and goods production, and (iii) the global energy constraint weighted by the Lagrange multiplier.

Because the planner's objective is global welfare, the Lagrangian encompasses a number of different cases, which are determined by the resources that the planner is assumed to control. In our core planning problem, to derive the *unilateral* optimum, the planner can choose the quantities of each good that Home consumes and each good that it exports, $\{y_j\}$, $\{x_j\}$, $\{m_j\}$, their energy intensities, $\{k_j^y\}$, $\{k_j^x\}$, $\{k_j^m\}$, direct consumption of energy C_e^d, energy extraction Q_e, and the price of energy, p_e. To derive the *global* optimum, the planner can also choose $\{y_j^*\}$, $\{k_j^{y*}\}$, C_e^{d*}, and Q_e^*.[15] Restricting the planner's choices to narrower sets

[14]The labor constraint together with the Foreign welfare constraint implies:

$$C_s = L^W - L_e^W - L_g^W - C_s^* = L^W - L_e^W - L_g^W + u^*(C_g^*) + v^*(C_e^{d*}) - \varphi^* Q_e^W - \bar{U}^*.$$

Substituting this expression into Home welfare delivers the planner's objective.

[15]In this global case p_e is redundant. Appendix A provides a step-by-step solution.

of variables allows us to derive simpler or restricted policies to the unilateral optimum (which we explore in Section 5 and in our simulations).

We solve the maximization problem, starting with what CDVW call the *inner problem*, involving optimality conditions for an individual good given values for C_e^d, Q_e, p_e, and λ_e. We then evaluate the optimality conditions for C_e^d, Q_e and p_e in what they call the *outer problem.* The Lagrange multiplier λ_e clears the energy market.

The results that follow become more intuitive by anticipating that the solution satisfies $\lambda_e \geq p_e$, with a strict inequality in all but extreme cases. This inequality is derived in Appendix B.2. In the case of $\varphi^W = 0$ we get $\lambda_e = p_e$ and the planner's problem collapses to BAU.

3.5 Inner Problem

The inner problem is to maximize, for any arbitrary good j:

$$\begin{aligned}\mathcal{L}_j &= u\left(y_j + m_j\right) + u^*(y_j^* + x_j) \\ &\quad - \left(l_j(k_j^y) + \lambda_e e_j(k_j^y)\right) y_j - \tau^* \left(l_j^*(k_j^m) + \lambda_e e_j^*(k_j^m)\right) m_j \\ &\quad - \tau \left(l_j(k_j^x) + \lambda_e e_j(k_j^x)\right) x_j - \left(l_j^*(k(p_e)) + \lambda_e e_j^*(k(p_e))\right) y_j^*,\end{aligned} \tag{15}$$

subject to the restriction that $u^{*\prime}(y_j^* + x_j) \leq p_j^* = a_j^* g(p_e)$, since Foreign has the option to buy from local producers. We have reordered terms to make the optimality conditions more transparent: (i) optimal quantities for Home consumers, y_j and m_j; and (ii) optimal quantities for Foreign consumers, x_j (anticipating that y_j^* may, in some instances, depend on x_j); and (iii) optimal energy intensities, k_j^y, k_j^m, and k_j^x.

3.5.1 Energy Intensities and Shadow Costs

Inspecting (15), we see that energy intensities, k_j^i, for $i \in \{y, x, m\}$, enter the objective as in (6), with $p = \lambda_e$. It follows immediately that the optimal values are:

$$k_j^i = k(\lambda_e) \quad i \in \{y, x, m\}.$$

The planner chooses the same energy intensity for the production of any good consumed in Home (whether produced in Home or Foreign) and for all production in Home (whether serving consumers in Home or Foreign). (The energy intensity $k(p_e)$ of Foreign when producing for its domestic consumers is the same function, but evaluated at the energy price $p = p_e$ rather than at the planner's shadow value of energy λ_e.)

The associated shadow costs, per unit, of producing and delivering y_j, x_j, and m_j (including trade costs) are $a_j g(\lambda_e)$, $\tau a_j g(\lambda_e)$, and $\tau^* a_j^* g(\lambda_e)$. Using these results, the inner problem simplifies to:

$$\begin{aligned}\mathcal{L}_j = & \, u\left(y_j + m_j\right) + u^*(y_j^* + x_j) \\ & - a_j g(\lambda_e) y_j - \tau^* a_j^* g(\lambda_e) m_j \\ & - \tau a_j g(\lambda_e) x_j - a_j^* \left(g(p_e) + (\lambda_e - p_e) g'(p_e)\right) y_j^*.\end{aligned} \tag{16}$$

(The last line applies Shepard's lemma to replace $e_j^*(k(p_e))$ with $a_j^* g'(p_e)$.) We now turn to the problem of choosing y_j, m_j, and x_j to maximize (16).

3.5.2 Goods for Home Consumers

The first-order conditions for y_j and m_j are:

$$u'(y_j + m_j) - a_j g(\lambda_e) \leq 0,$$

with equality if $y_j > 0$, and

$$u'(y_j + m_j) - \tau^* a_j^* g(\lambda_e) \leq 0,$$

with equality if $m_j > 0$. We define $\bar{j}_m$ as the value of j at which these two conditions both hold with equality. Using (4), this threshold satisfies:

$$F(\bar{j}_m) = \frac{1}{\tau^*}. \tag{17}$$

The threshold $\bar{j}_m$ separates goods that Home produces for itself from those that it imports. If $j < \bar{j}_m$, Home has a lower shadow cost of production, the second condition holds with a strict inequality (so that $m_j = 0$), and the first holds with equality to determine $y_j > 0$. If $j > \bar{j}_m$, Foreign has a lower shadow cost of production, the first condition holds with a strict inequality (so that $y_j = 0$), and the second holds with equality to determine $m_j > 0$.

The value of $\bar{j}_m$ is the same as in BAU, which can be seen by setting $\lambda_e = p_e$. The intuition for this result is that Home sets the same energy intensity for Home production and for imports. Given the same energy intensity, there is no climate-related reason for Home to alter the import threshold.

3.5.3 Goods for Foreign Consumers

Unlike goods for Home consumers, where the planner can choose both sources of supply (y_j and m_j), for Foreign consumers the planner chooses only x_j, subject to the constraint that $u^{*\prime}(y_j^* + x_j) \leq a_j^* g(p_e)$. If this constraint is not binding then $y_j^* = 0$ and the first-order condition for x_j reduces to:

$$u^{*\prime}(x_j) = \tau a_j g(\lambda_e),$$

which determines x_j.

This solution is applicable for any $j < j_0$, where j_0 is the good that Home and Foreign producers can supply at equal cost to Foreign consumers. Using (4), his threshold good satisfies:

$$F(j_0) = \tau \frac{g(\lambda_e)}{g(p_e)}. \tag{18}$$

In BAU $\lambda_e = p_e$ so that this threshold becomes the threshold for Home exports $\bar{j}_x = j_0$, satisfying $F(\bar{j}_x) = \tau$. Our focus is on $\lambda_e > p_e$, which reduces j_0 while increasing $\bar{j}_x$.

For any good $j \geq j_0$, the constraint on Foreign marginal utility binds. Foreign consumption, $c_j^* = y_j^* + x_j$, is determined by $u^{*\prime}(c_j^*) = a_j^* g(p_e)$. Home exports crowd out y_j^* one-to-one. After taking account of this crowding-out effect, by substituting $y_j^* = c_j^* - x_j$ into (16), its derivative with respect to x_j is:

$$\frac{\partial \mathcal{L}_j}{\partial x_j} = -\tau a_j g(\lambda_e) + a_j^* g(p_e) + (\lambda_e - p_e) a_j^* g'(p_e). \tag{19}$$

If this derivative is positive, as we will show that it is for $j \in [j_0, \bar{j}_x)$, the optimal level of exports is at a corner, $x_j = c^*_j$ with $y^*_j = 0$. If it's negative, as we will show that it is for $j > \bar{j}_x$, we get $x_j = 0$ and $y^*_j = c^*_j$.

Since the derivative is positive at j_0 and decreasing in j, we can solve for the threshold good $\bar{j}_x > j_0$ that sets (19) to zero. Using (4), this threshold satisfies:

$$F(\bar{j}_x) = \tau \frac{g(\lambda_e)}{g(p_e) + (\lambda_e - p_e)g'(p_e)}. \tag{20}$$

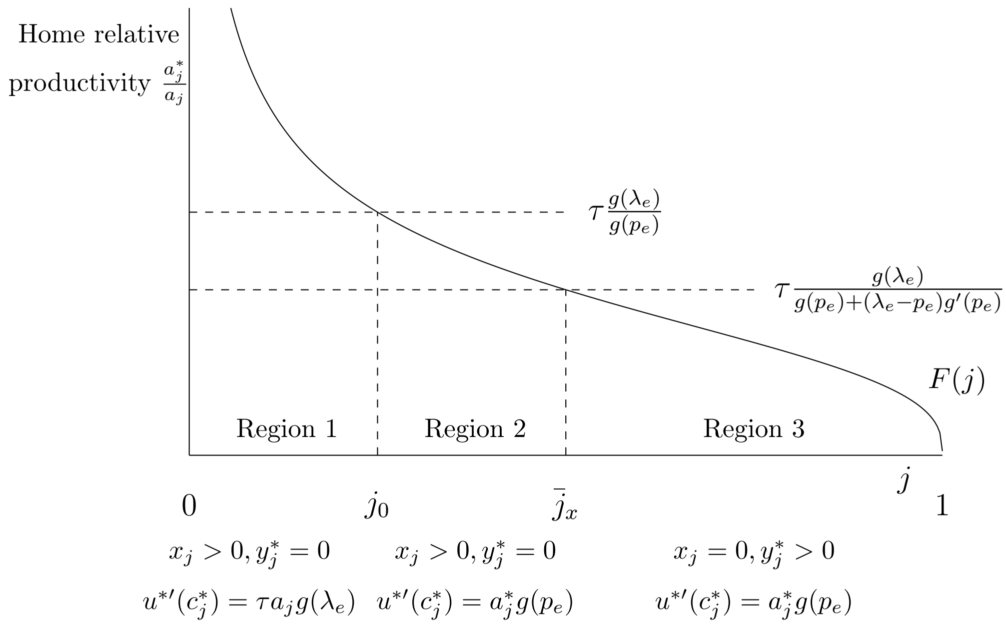
The resulting export threshold $\bar{j}_x$ is higher than it would be under BAU.[16]

Figure **??** illustrates these results. The two thresholds, j_0 and $\bar{j}_x$, divide the unit continuum of goods into three regions. Region 1 is goods $0 \leq j < j_0$ for which Home's comparative advantage is strongest. Home exports these goods in quantities that equate it's shadow cost of supply to Foreign's marginal utility. Region 2 is goods $j_0 \leq j < \bar{j}_x$ for which Home's comparative advantage is moderate. Home exports these goods at the minimum quantities sufficient to remove any incentive for Foreign to produce them for itself. Region 3 is goods $\bar{j}_x < j \leq 1$ for which Foreign's comparative advantage is strongest. Foreign produces these goods for itself.

Under BAU, Region 2 disappears, since in that case $j_0 = \bar{j}_x$. The key insight into why Region 2 emerges for $\lambda_e > p_e$ is that the planner can't specify the energy intensity $k(p_e)$ used by Foreign to produce goods for its own consumers. Foreign producers have energy costs below the planner's shadow value of energy, so their production is more energy intensive than the planner would have chosen. If Home's shadow cost is not too high, the planner prefers to have Home export goods to Foreign in place of Foreign producing them for itself. It therefore *expands* the export threshold, $\bar{j}_x$, above j_0, and even beyond what it would be under BAU. The reason is that for goods between j_0 and $\bar{j}_x$, producing those goods with lower energy intensity $k(\lambda_e)$ outweighs the cost of Home's weak comparative advantage.

[16]Recall that the cost function $g(p)$ is concave, hence, if $\lambda_e > p_e$ the denominator of (20) exceeds $g(\lambda_e)$ so that $F(\bar{j}_x) < \tau$ (which gives the result since F is strictly decreasing).

Figure 1: Goods for Foreign Consumers



3.5.4 Summary

Table 2 displays the results of the inner problem. As in Table 1, the rows indicate the location of consumption while the columns indicate the location of production. These terms are as expected from the basic structure of the model for Home consumption (top line) and for Foreign production for Foreign consumption (bottom right) and for Home exports in Region 1 (bottom left, $j \leq j_0$).

The exception is Home exports, x_j, of goods in Region 2, (bottom left, $j \in (j_0, \bar{j}_x)$): (i) exports of such goods reflect the price of energy p_e in Foreign rather than the planner's shadow price λ_e even though they are produced in Home, (ii) although produced in Home, they reflect Foreign's input requirement a_j^* rather than Home's, and (iii) they do not reflect the iceberg costs of export τ. That is, $x_j \neq \eta^* \left(\tau a_j g(\lambda_e)\right)^{-\sigma}$ in Region 2, as is the case in Region 1. The reason is that for goods in Region 2, Home crowds out Foreign production in order to produce these goods with lower energy intensity $k(\lambda_e)$. Home must export enough so that Foreign consumers are not tempted to purchase more from their own producers,

Table 2: Production and Distribution of a Good

	Home		Foreign	
Home	$y_j = \eta\left(a_j g(\lambda_e)\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta\left(\tau^* a_j^* g(\lambda_e)\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \begin{cases} \eta^*\left(\tau a_j g(\lambda_e)\right)^{-\sigma^*} \\ \eta^*\left(a_j^* g(p_e)\right)^{-\sigma^*} \end{cases}$	$j \leq j_0$ $j_0 \leq j < \bar{j}_x$	$y_j^* = \eta^*\left(a_j^* g(p_e)\right)^{-\sigma^*}$	$j > \bar{j}_x$

The thresholds, $\bar{j}_m$, j_0, and $\bar{j}_x$, are given by equations (17), (18), and (20), respectively.

at a cost of $a_j^* g(p_e)$.[17]

3.6 Outer Problem

We can rewrite the Lagrangian in terms of aggregate magnitudes as:

$$\mathcal{L} = u(C_g) + u^*(C_g^*) + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W Q_e^W - L_e^W - L_g^W - \lambda_e\left(C_e^W - Q_e^W\right), \quad (21)$$

where recall that $C_e^W = C_e^y + C_e^m + C_e^d + C_e^{y*} + C_e^x + C_e^{d*}$ and $Q_e^W = Q_e + Q_e^*$. The planner maximizes it over C_e^d, Q_e, and p_e.

3.6.1 Direct Energy Consumption

The first order condtion with respect to C_e^d is:

$$v'(C_e^d) - \lambda_e = 0.$$

This condition mimics (8), but with λ_e in place of p_e.

[17] Table 2 doesn't provide the outcomes for goods at the import and export thresholds. (By contrast, we know that good j_0 is exported by Home to Foreign.) Good $j = \bar{j}_m$ may either be imported by Home or produced domestically while good $j = \bar{j}_x$ may be either exported by Home or produced by Foreign. Aggregate results are not affected.

3.6.2 Energy Extraction

The first order condition with respect to Q_e is:

$$\frac{\partial \mathcal{L}}{\partial Q_e} = -\varphi^W - \frac{\partial L_e}{\partial Q_e} + \lambda_e \leq 0,$$

with equality if $Q_e > 0$. The extra labor to extract a bit more energy is the labor requirement for Home's marginal energy field, $E^{-1}(Q_e)$.[18] Applying this result, the first order condition simplifies to:

$$Q_e = E\left(\lambda_e - \varphi^W\right), \tag{22}$$

for $\lambda_e - \varphi^W \geq 0$ and $Q_e = 0$ otherwise. The BAU outcome replaces $\lambda_e - \varphi^W$ with p_e, analogous to (7).

3.6.3 Energy Price

Optimizing with respect to the energy price is more subtle. While C_g is already optimized via the inner problem, C_e^* depends on the energy price via the cost of producing goods in Foreign. The thresholds, j_0 and $\bar{j}_x$, depend on the energy price as well, but because these thresholds represent an interior optimum of the inner problem, by the envelope theorem we can treat them as fixed at their optimal values when we differentiate with respect to p_e.[19] Home's exports of goods in

[18] This result comes from differentiating (10) while noting that $E'(E^{-1}(Q_e)) = \partial Q_e / \partial E^{-1}(Q_e)$:

$$\frac{\partial L_e}{\partial Q_e} = E^{-1}(Q_e) E'(E^{-1}(Q_e)) \frac{\partial E^{-1}(Q_e)}{\partial Q_e} = E^{-1}(Q_e).$$

[19] In particular, we have:

$$\frac{\partial \mathcal{L}}{\partial p_e} = \left.\frac{\partial \mathcal{L}}{\partial p_e}\right|_{j_0, \bar{j}_x} + \frac{\partial \mathcal{L}}{\partial \bar{j}_x}\frac{\partial \bar{j}_x}{\partial p_e} + \frac{\partial \mathcal{L}}{\partial j_0}\frac{\partial j_0}{\partial p_e} = \left.\frac{\partial \mathcal{L}}{\partial p_e}\right|_{j_0, \bar{j}_x},$$

since, from the inner problem, $\partial \mathcal{L}/\partial \bar{j}_x = \partial \mathcal{L}/\partial j_0 = 0$. This result allows us to treat these thresholds as fixed when computing the derivatives that enter the first-order condition for the energy price. Consider, for example, energy used by Foreign producers to supply local demand:

$$C_e^{y*} = \int_{\bar{j}_x}^1 e_j^*(k(p_e)) y_j^* dj = g'(p_e) g(p_e)^{-\sigma^*} \eta^* \int_{\bar{j}_x}^1 (a_j^*)^{1-\sigma^*} dj.$$

Region 2 were optimized as well, but at a corner solution, so we must account for their dependence on p_e. Foreign direct consumption of energy, C_e^{d*}, depends on the energy price via (8) while Foreign extraction, Q_e^*, and hence L_e^*, depend on the energy price via (7).

The first order condition with respect to p_e is:

$$\frac{\partial \mathcal{L}}{\partial p_e} = \frac{\partial u^*(C_g^*)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g^W}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.$$

Appendix B.1 provides the steps to rewrite it in a more intuitive form:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e) \frac{\partial C_e^{z*}}{\partial p_e} + \int_{j0}^{\bar{j}_x} \left(\tau a_j g(\lambda_e) - a_j^* g(p_e)\right) \frac{\partial x_j}{\partial p_e} dj, \quad (23)$$

where $C_e^{z*} = C_e^{y*} + C_e^{d*}$ is Foreign's use of energy to produce goods for domestic consumption plus its direct consumption of energy. Equation (23) balances Foreign supply and demand responses to a change in p_e with the deviation between the planner's valuation and Foreign's market valuation of each response. We refer to these deviations in valuation as wedges: (i) the wedge between the planner's marginal valuation of a unit of energy extracted and the energy price (*extraction wedge*), (ii) the wedge between the planner's marginal valuation of energy used and the energy price (*consumption wedge*), and (iii) the wedges:

$$s_j = \tau a_j g(\lambda_e) - a_j^* g(p_e) = \tau a_j g(\lambda_e) - p_j^*,$$

for each good in Region 2, between the shadow cost of Home supplying exports of j and the marginal utility to consumers in Foreign (*export wedges*).

We get a compact expression for the energy-price condition by aggregating the export wedges into a single term, $S = \int_{j0}^{\bar{j}_x} s_j x_j dj$. We can then rewrite (23), since $\partial x_j / \partial p_e = -\sigma^* g'(p_e) x_j / g(p_e)$, as:

$$\lambda_e - p_e = \frac{\varphi^W \partial Q_e^* / \partial p_e - \sigma^* \frac{g'(p_e)}{g(p_e)} S}{\partial Q_e^* / \partial p_e + |\partial C_e^{z*} / \partial p_e|}. \quad (24)$$

When calculating the relevant derivative, we would treat the final integral as a constant even though $\bar{j}_x$ depends on p_e. To avoid clutter, we don't distinguish this feature in our notation for partial derivatives.

(The absolute value in the denominator makes all terms positive.)

3.7 Properties of the Solution

We can now compute the optimal policy: (i) the inner problem gives C_e and C_e^* in terms of p_e and λ_e, (ii) equations (7) and (22) give Q_e^* and Q_e as functions of p_e and λ_e, and (iii) equation (23) and the global energy constraint (12), which binds, nail down p_e and λ_e. We can also go further in characterizing the optimal wedges.

3.7.1 The Pigouvian Wedge

Adding the absolute value of the extraction wedge and the consumption wedge yields φ^W, the marginal global externality from carbon emissions. The wedge between extraction and use of energy in Home is Pigouvian. As shown in Appendix A, a global planner, that could also control outcomes in Foreign, would impose this Pigouvian wedge there as well. A unilaterally optimal policy can't harmonize this wedge internationally, yet still imposes the full Pigouvian wedge in Home.

3.7.2 Balancing Extraction and Consumption Wedges

Appendix B.2 shows that the planner picks the consumption wedge, $\lambda_e - p_e$, from the interval $[0, \varphi^W)$. It is strictly positive if $\varphi^W \partial Q_e^* / \partial p_e > 0$, i.e. if there is harm from emissions on the margin and if Foreign extraction is sensitive to the energy price.

The consumption wedge will approach the upper bound of φ^W if $\partial Q_e^* / \partial p_e$ is large relative to $|\partial C_e^{z*} / \partial p_e|$. In this case the planner chooses a low energy price to limit Foreign extraction of energy. As the consumption wedge approaches this upper bound the extraction wedge approaches 0.

The consumption wedge will approach the lower bound if $\partial Q_e^* / \partial p_e$ is small relative to $|\partial C_e^{z*} / \partial p_e|$. In this case the planner chooses a high price to limit Foreign demand for energy. With perfectly inelastic Foreign supply, the extraction wedge equals the Pigouvian wedge and the consumption wedge is 0. The unilateral policy then achieves the global optimum.[20]

[20]Following this logic, Harstad (2012) makes a case that the policy maker buy marginal energy

If iceberg costs become arbitrarily large, driving out trade in goods, the Pigouvian wedge and the consumption wedge completely characterize the optimal policy. In this case, the import and export thresholds approach the corner solutions of $\bar{j}_m = 1$ and $\bar{j}_x = 0$, and hence $C_e^{z*} = C_e^*$. Equation (24), which determines the magnitude of the consumption wedge, collapses to:

$$\lambda_e - p_e = \frac{\varphi^W \partial Q_e^* / \partial p_e}{\partial Q_e^* / \partial p_e + |\partial C_e^* / \partial p_e|}.$$

This case serves as a useful benchmark. As we bring back trade in goods, the new elements of the optimal policy are the treatment of goods imports and particularly goods exports.

3.7.3 Export Wedges and Crosshauling

The import threshold, $\bar{j}_m$, is the same under the optimal policy as in BAU. The export threshold is greater than in BAU if $\lambda_e > p_e$, as shown in footnote 16. The planner promotes exports of goods in Region 2, as dictated by the export wedges, s_j. The logic for promoting these exports follows from (19): global resources are saved by producing Region 2 goods in Home rather than Foreign.

These properties of the solution create the possibility for crosshauling. Under the optimal policy there may be a set of goods that Home simultaneously imports and exports. Such a set of goods always exists in the absence of trade costs since then $F(\bar{j}_m) = 1$ while $F(\bar{j}_x) < 1$ implying $\bar{j}_x > \bar{j}_m$.

Trade costs mute this effect. With high enough trade costs $F(\bar{j}_x) > F(\bar{j}_m)$ so that $\bar{j}_x < \bar{j}_m$. The inherent inefficiency of crosshauling overcomes its advantage in reducing the shadow value of resources used in production. Yet, even when there is no crosshauling the optimal policy broadens the range of goods that Home exports. The planner controls energy intensity not only for all production in Home but also for production in Foreign that Home imports. Goods produced in Foreign, for consumption there, escape the policy. The planner uses exports to discourage Foreign producing for itself, with the export wedge inducing Foreign

fields from Foreign to create a locally vertical Foreign supply curve. We have ruled out such an international market in Foreign energy fields in our analysis here.

consumers to buy them.

4 Optimal Taxes and Subsidies

We now describe a set of taxes and subsidies that deliver the optimal outcomes in a competitive equilibrium. In shifting from a planning problem to a market economy, recall that services are the numéraire and the unit labor requirement for services pins the wage to 1 in both countries. We treat p_e as the *global* energy price, the base to which we apply carbon taxes. The taxes and subsidies we introduce into this competitive equilibrium must generate the wedges that appear in the optimal policy.

4.1 A Simple Implementation

We focus on an implementation that is easy to describe, with three elements of intervention:

1. Impose a nominal tax on Home energy extraction, t_e^N, equal to the Pigouvian wedge:
$$t_e^N = \varphi^W.$$

2. Impose a border adjustment, t_b, on Home imports or exports of energy and on the energy content of Home imports of goods, equal to the consumption wedge:
$$t_b = \frac{\varphi^W \partial Q_e^*/\partial p_e - \sigma^* \frac{g'(p_e)}{g(p_e)} S}{\partial Q_e^*/\partial p_e + |\partial C_e^{z*}/\partial p_e|}. \tag{25}$$

3. Provide an export subsidy s_j per unit exported of any good in Region 2, $j \in (j_0, \bar{j}_x)$, equal to the export wedge:
$$s_j = \tau a_j g(p_e + t_b) - a_j^* g(p_e),$$
where $F(j_0) = \tau \frac{g(p_e+t_b)}{g(p_e)}$ and $F(\bar{j}_x) = \tau \frac{g(p_e+t_b)}{g(p_e)+t_b g'(p_e)}$.

The resulting *effective* extraction tax t_e equals the absolute value of the extraction wedge, $t_e = t_e^N - t_b$. If $\varphi^W = 0$ the optimal policy sets $t_e^N = t_b = 0$ and $s_j = 0$ for all j, resulting in the BAU competitive equilibrium.

4.2 After-Tax Prices

To eliminate ambiguity about how this policy would work, we list the net prices faced by the different agents in the global economy:

1. The global price of energy, p_e, is paid by users of energy in Foreign and is received by energy extractors in Foreign.

2. If energy is imported by Home, it is subject to a border adjustment t_b, raising the price of energy for users in Home to $p_e + t_b$.

3. Energy extractors in Home sell energy domestically at price $p_e + t_b$, so their net after paying the extraction tax is $p_e + t_b - t_e^N = p_e - t_e$.

4. They export energy at price p_e while getting a partial rebate of t_b on the nominal extraction tax, leaving them with net price $p_e - t_e$, the same as if they sell domestically.

5. Goods $j < \bar{j}_m$ are produced in Home, using energy costing $p_e + t_b$, so that local consumers pay $p_j = a_j g(p_e + t_b)$.

6. Goods $j > \bar{j}_m$ are imported by Home. Anticipating the border adjustment, Foreign produces them with energy intensity $k(p_e + t_b)$. The production cost, including the trade cost, is $\tau^* l_j^*(k(p_e + t_b)) + p_e \tau^* e_j^*(k(p_e + t_b))$. Adding in the border adjustment, $t_b \tau^* e_j^*(k(p_e + t_b))$, the price to consumers in Home is $p_j^m = \tau^* a_j^* g(p_e + t_b)$.

7. Goods in Region 1 ($j < j_0$) are produced in Home and exported. The producers use energy costing $p_e + t_b$, with no border adjustment when the goods are exported. The price in Foreign, including the trade cost, is $p_j^x = \tau a_j g(p_e + t_b)$.

8. Goods in Region 2 ($j \in (j_0, \bar{j}_x)$) are also exported by Home. The producers use energy costing $p_e + t_b$, with no border adjustment when the goods are

exported. They sell at price $p_j^* = a_j^* g(p_e)$ in Foreign. Producers get a subsidy from Home of s_j per unit so that they net $p_j^* + s_j = \tau a_j g(p_e + t_b)$, which covers their cost.

9. Goods in Region 3 ($j > \bar{j}_x$) are produced in Foreign, using energy at price p_e. They are sold to local consumers at price $p_j^* = a_j^* g(p_e)$.

4.3 Discussion

We can understand the optimal tax rates by considering how they are shaped by responses in Foreign. Extraction in Foreign and production of goods there for local consumption face no tax but respond to the equilibrium price of energy. Foreign use of energy in production has two components: the energy intensity of this production (the intensive margin) and the set of goods produced (the extensive margin). These three margins—Foreign extraction, Foreign energy intensity, and the range of goods produced in Foreign for local consumers—can be thought of as three different sources of leakage. Home sets its combination of an extraction tax, a border adjustment, and an export subsidy to indirectly affect these margins, in effect controlling all these sources of leakage.

If Foreign's extraction elasticity is large, extraction leakage is potentially high, resulting in costs to Home that go up with φ^W. Border adjustments on energy moderate this effect. Increasing the border adjustment lowers the price of energy, thereby reducing extraction leakage. Lowering p_e, however, introduces distortions on the production and consumption side. As p_e goes down, the set of goods produced in Foreign increases, and Foreign's energy intensity in producing those goods goes up. The set of goods produced in Foreign roughly corresponds to traditional (production) leakage, while the energy intensity of those goods is sometimes called the "fuel price effect."[21] The principle of optimizing over the two tax instruments, t_e and t_b, given $t_e + t_b = t_e^N = \varphi^W$, is at the heart of the seminal

[21]These terms, however, are not clearly distinguished in the literature, and our use of them is only suggestive. The fuel price effect appears to refer to any change in Foreign production or consumption due to a reduction in p_e. If true, then traditional production leakage is limited to shifts in import or export margins holding p_e fixed. Our usage does not precisely correspond to these definitions because our expressions all use the equilibrium value of p_e.

paper of Markusen (1975).[22]

The optimal policy also controls production leakage through a combination of a border adjustment on imports and a goods-specific subsidy for exports. The border tax on imports means that imports face the same effective energy price as goods produced in Home. As a result, the border tax leaves the extensive margin for imports the same as without tax and causes the energy intensity of imports to be the same as that of goods produced in Home. The policy might have controlled the export margin in a parallel fashion, by rebating taxes on export, leaving the export margin the same as it would be without tax. Doing so, however, would remove the incentive for exporters to lower their energy intensity. Rather than removing the tax on export, therefore, the policy offers good specific subsidies. Because these subsidies do not depend on energy usage, they retain incentives for exporters in Home to produce goods with low energy intensity.[23]

The subsidy goes beyond merely restoring Home's export margin: it applies to goods for which Home would not be competitive in the absence of any carbon policy. The reason follows the argument above for potential cross-hauling under the optimal policy. The policy is designed to crowd out some of Foreign's energy-intensive production for its domestic consumers. The same logic does not apply to the import margin because the border tax on imports ensures that all goods consumed in Home are produced with the same (low) energy intensity. The asymmetry between imports and exports arises because a unilateral policy can't directly control the energy intensity of goods produced in Foreign that are consumed in Foreign. The optimal export policy seeks to crowd out this activity.

[22] This connection to Markusen (1975) is disguised by differences in terminology. Our extraction tax is what he refers to as a production tax. Our border adjustment is what he refers to as a trade tax. Furthermore, his taxes are ad valorem while ours are specific. More fundamentally, he imposes trade balance, so that his trade tax incorporates terms-of-trade considerations. Finally, in his model there is no analog of our production sector, which uses energy to produce tradable goods. Hence, his analysis doesn't speak to how the border adjustment applies to the energy embodied in these goods.

[23] This basic logic comes from Fischer and Fox (2012), who point out that rebating carbon tax revenue to producers, in proportion to their production (without regard to their tax payments), retains the incentive for them to use less carbon. An optimal subsidy to production in the context of carbon pricing emerges in Fowlie et al. (2016). There it is designed to offset the output-reducing effect of market power among cement producers.

5 Constrained Optimal Policies

To assess the optimal policy, we compare it to more conventional polices: an extraction tax, a consumption tax, and a production tax. We also consider hybrids of these taxes, which are optimal combinations of the three conventional policies.[24]

We derive each policy as a variant of the planner's problem from Section 3. The Lagrangian (14) remains the same in each case, but is solved assuming that the planner can control only those variables subject to a given policy. For example, if the planner controls only Q_e and p_e, with all other variables determined in the competitive equilibrium, the resulting tax is an extraction tax. If the planner can also choose the energy intensity and quantities of both domestic production and imports, $\{k_j^y\}$, $\{k_j^m\}$, $\{y_j\}$, and $\{m_j\}$, the planner now has the flexibility to also impose a consumption tax.

Full solutions to the Lagrangian for each case are shown in Appendix C. Here we focus on the optimality condition for p_e, which conveys the essential intuition for all such policies, and then show how the solution can be implemented through taxes.

5.1 The Planner's Solution

We can write the conditions for p_e for each of the constrained policies in terms of the first two wedges seen in the optimal policy, the extraction wedge, $\lambda_e - \varphi^W - p_e$, and the consumption wedge, $\lambda_e - p_e$. In each case, the planner uses the wedge to evaluate the cost of the corresponding response outside of its control. The planner sets the size of these marginal costs equal to each other.

If the planner is constrained to choosing only Q_e and p_e, leading to an extraction tax, the planner chooses Q_e according to (22) while Foreign extraction remains outside its control. The planner does not control the demand side of the market in either region, so the global consumption response is also outside its control.

[24] We provide additional intuitions for these results in Weisbach et al. (2022). There we solve the dual problem to get conditions for optimal tax rates directly (given the taxes considered in each case). Here, in parallel to our solution of the unilateral optimal policy, we solve the primal problem to get conditions for the optimal allocation (given the appropriate constraints on the planner's choice set), and then derive conditions for the taxes that implement that allocation.

The condition for p_e balances the cost of the Foreign extraction response and the cost of the global demand response to changes in p_e:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = \left(\lambda_e - p_e\right) \frac{\partial C_e^W}{\partial p_e}.$$

The same logic holds for constraints on the planner that generate a consumption tax. Now global extraction is outside of its control while on the demand side, the planner can choose domestic consumption leaving only Foreign demand is outside of its control. The condition for p_e sets the cost of the global extraction response equal to the cost of the Foreign demand response. If the planner can control both domestic extraction and domestic consumption, leaving only Foreign extraction and consumption outside of its control, the resulting tax is a combination of an extraction tax and a consumption tax. The condition for p_e balances the costs of the responses in Foreign. Table 3 summarizes the conditions for p_e for these cases.

Table 3: Conditions for Policies Leading to Extraction and Consumption Taxes

Extraction tax	$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = \left(\lambda_e - p_e\right) \frac{\partial C_e^W}{\partial p_e}$
Consumption tax	$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^W}{\partial p_e} = \left(\lambda_e - p_e\right) \frac{\partial C_e^*}{\partial p_e}$
Extraction/Consumption	$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = \left(\lambda_e - p_e\right) \frac{\partial C_e^*}{\partial p_e}$

The optimal solution when the planner controls only domestic production (leading to a basic production tax) is more complex. The policy changes the cost of energy to producers in Home relative to the cost to producers in Foreign, which changes the trade thresholds, $\bar{j}_x$ and $\bar{j}_m$. These changes generate traditional production leakage, which the planner must take into account.

With leakage, the planner will not set the cost of energy in Home, denoted by v_e, equal to λ_e. It will choose a lower value of v_e to limit leakage. Leakage is (minus) the increase in emissions (here, energy use) in Foreign, relative to the decrease in Home, due to a change in v_e:[25]

$$\Lambda = -\frac{\partial G_e^*/\partial v_e}{\partial G_e/\partial v_e}.$$

The planner optimizes by setting the cost of energy in Home to $v_e = \Lambda p_e + (1-\Lambda)\lambda_e$, closer to p_e if leakage is higher and closer to λ_e if leakage is lower.

Using this result, we can write the condition for p_e in terms of the extraction and consumption wedges:

$$\left(\lambda_e - \varphi^W - p_e\right)\frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - p_e)\left((1-\Lambda)\frac{\partial G_e^*}{\partial p_e} + \Lambda\frac{\partial C_e^W}{\partial p_e}\right).$$

Because the planner doesn't control either domestic or Foreign extraction, the extraction wedge is multiplied by the change in Q_e^W. On the demand side, the planner cares about both the effect of the energy price on Foreign use of energy, $\partial G_e^*/\partial p_e$, and, increasingly with the extent leakage, the overall effect on global energy use.

If the planner controls all goods produced domestically (no matter where consumed) and all goods consumed domestically (no matter where produced), the resulting policy is a hybrid of a basic production tax and a basic consumption tax. In this case λ_e is the cost of energy for any producers serving consumers in Home while v_e is the cost of energy for exporters in Home serving consumers in Foreign. Only $C_e^{y^*}$ and C_e^x are affected by v_e. The expression for leakage becomes:

$$\Lambda^* = -\frac{\partial C_e^{y*}/\partial v_e}{\partial C_e^x/\partial v_e},$$

which we call "foreign leakage" to distinguish from leakage under a basic production tax. Compared to conventional leakage, foreign leakage does not include the change in Home and Foreign use of energy to serve Home consumers.

[25] Note that many analyses (e.g., Böhringer, Lange, and Rutherfod (2014)) consider two channels of leakage. The first, referred to as the fuel price effect, is the change in Foreign emissions due to the change in p_e. The second, referred to as the competitiveness channel, refers to the change in the location of production due to the change in the relative price of energy in Foreign and Home. Our definition of leakage is limited to the latter. All of the policies we consider, including the optimal policy, are concerned with the effects of changes to p_e.

The planner optimizes by setting the cost of energy in Home to $v_e = \Lambda^* p_e + (1 - \Lambda^*)\lambda_e$. Using this result, we can express the condition for p_e as:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - p_e) \left((1 - \Lambda^*) \frac{\partial C_e^{z*}}{\partial p_e} + \Lambda^* \frac{\partial C_e^*}{\partial p_e} \right).$$

Finally, if the planner can also control domestic extraction, the resulting tax is a hybrid of an extraction tax, a production tax, and a consumption tax. The above condition is still applies, except substituting Q_e^* for Q_e^W.

5.2 Taxes and Implementation

Implementing these outcomes involves imposing extraction, production, and consumption taxes, as the case may be, for each policy. If there is an extraction tax it equals the extraction wedge, $t_e = \lambda_e - \varphi^W - p_e$; if there is a consumption tax it equals the consumption wedge, $t_c = \lambda_e - p_e$; and if there is a production tax it equals the consumption wedge reduced by the extent of leakage, $t_p = (1 - \Lambda)(\lambda_e - p_e)$ (with Λ^* in place of Λ when together with a consumption tax). Table 4 summarizes the effective taxes for each case in terms of the consumption wedge. The last column gives the expressions for the consumption wedge itself.

If Home imposes one of the basic taxes—an extraction tax, a production tax, or a consumption tax—the rate is below the Pigouvian wedge, φ^W. In the case of an extraction tax, the planner will choose a lower tax rate because of concerns that a higher rate would stimulate Foreign extraction, as determined by $\partial Q_e^* / \partial p_e$. If Home imposes a consumption tax, the planner chooses a lower tax rate because of concerns that a higher rate would stimulate Foreign demand, as determined by $\partial C_e^* / \partial p_e$.

With a production tax, it is not only changes in Foreign energy use, $\partial G_e^* / \partial p_e$ that keeps the rate below φ^W, but also the degree of production leakage, Λ. The tax rate goes down linearly with leakage, and with 100% production leakage, the optimal production tax rate is 0.

When Home combines an extraction tax with a consumption tax, it can control both sides of the market and the overall tax rate $t_e + t_c$ equals the Pigouvian wedge, as with the optimal unilateral policy. This result does not carry over to

Table 4: Effective Taxes

Policy	Effective Taxes	$\lambda_e - p_e$
Extraction tax	$t_e = \varphi^W - (\lambda_e - p_e)$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e + \lvert\partial C_e^W/\partial p_e\rvert}$
Consumption tax	$t_c = \lambda_e - p_e$	$\frac{\varphi^W \, \partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e + \lvert\partial C_e^*/\partial p_e\rvert}$
Production tax	$t_p = (1 - \Lambda)(\lambda_e - p_e)$	$\frac{\varphi^W \, \partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e + (1-\Lambda)\lvert\partial G_e^*/\partial p_e\rvert + \Lambda\lvert\partial C_e^W/\partial p_e\rvert}$
Extraction/Cons	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e + \lvert\partial C_e^*/\partial p_e\rvert}$
Extraction/Prod	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_p = (1 - \Lambda)(\lambda_e - p_e) \end{cases}$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e + (1-\Lambda)\lvert\partial G_e^*/\partial p_e\rvert + \Lambda\lvert\partial C_e^W/\partial p_e\rvert}$
Production/Cons	$\begin{cases} t_p = (1 - \Lambda^*)(\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \, \partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e + (1-\Lambda^*)\lvert\partial C_e^{z*}/\partial p_e\rvert + \Lambda^*\lvert\partial C_e^*/\partial p_e\rvert}$
Extr/Prod/Cons	$\begin{cases} t_e = \varphi^W - (\lambda_e - p_e) \\ t_p = (1 - \Lambda^*)(\lambda_e - p_e) \\ t_c = \lambda_e - p_e \end{cases}$	$\frac{\varphi^W \, \partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e + (1-\Lambda^*)\lvert\partial C_e^{z*}/\partial p_e\rvert + \Lambda^*\lvert\partial C_e^*/\partial p_e\rvert}$

a combination of an extraction and production tax, however. The extraction component is set equal to the extraction wedge, but the production component is less than the consumption wedge by the factor $1 - \Lambda$. Leakage reduces the power of this hybrid compared to the extraction/consumption hybrid.

While an extraction/production hybrid tax has to contend with production leakage, it has an offsetting advantage over other policies: it can be implemented simply and accurately. To implement this tax, Home would impose a nominal extraction tax of $t_e^N = t_e + t_p = \varphi^W - \frac{\Lambda}{1-\Lambda} t_p$ and border adjustments on energy (but not on goods) at rate $t_b = t_p$. By avoiding border adjustments on goods, the tax avoids the need to estimate the marginal emissions from the production of goods in foreign countries, which is the key problem in imposing border adjustments. (Kortum and Weisbach (2016)).

Turning to the production/consumption tax hybrid, if there were no production leakage the planner would equalize the tax rates, as in the optimal unilateral policy: all production and all consumption in Home would be taxed the same way. In this case, Home could impose a production tax at a rate equal to $\lambda_e - p_e$ and a tax on imports at the same rate. With positive leakage the planner lowers the tax on Home's exports. To implement the policy, Home would again impose a production tax of $\lambda_e - p_e$, a border tax on imports at that same rate, but now would add a rebate on exports of $\Lambda^*(\lambda_e - p_e)$. That is, because of concerns about leakage, Home's policy includes a partial rebate of taxes on export, with the rebate equal to 100% only when foreign leakage is 1. The tax is lower than φ^W because this hybrid acts only on the demand side of the energy market.

Finally, when Home can impose the combination of all three taxes, the sum of the extraction and consumption rates is equal to the Pigouvian rate, as with the hybrid of just those two taxes. The production tax rate, which applies only to exports, however, is lower due to a concern about leakage. As foreign leakage goes up, the use of the production tax goes down (and the planner also shifts away from consumption taxes and toward extraction taxes).

6 Quantitative Illustration

We now turn to the quantitative implications of the optimal policy. We pursue a strategy, based on Dekle, Eaton, and Kortum (2007), calibrating the BAU competitive equilibrium to data on global carbon flows and then computing the optimal policy relative to this baseline. We also compare BAU and optimal policies to the more conventional policies derived in the previous section.[26]

6.1 Setup

We start by providing the basic elements of our procedure (with a full treatment relegated to Appendix D), and then present our key results.

[26]In principle we could incorporate a set of existing taxes into the baseline. We chose not to do so in order to keep the analysis that follows as simple as possible and because existing taxes on carbon are quite limited.

6.1.1 Functional Forms

To solve the model numerically we employ convenient functional forms for the distributions of energy fields, $E(a)$ and $E^*(a)$, for the production function, $f(k)$, and for unit labor requirements to produce goods, a_j and a_j^* (and hence also for the comparative advantage curve, $F(j)$).

Preference for Direct Consumption of Energy We parameterize the preference for direct consumption of energy as

$$v(C_e^d) = (\eta_e)^{1/\sigma_e} \frac{(C_e^d)^{1-1/\sigma_e} - 1}{1 - 1/\sigma_e}$$

where η_e is governs the demand for direct consumption of energy relative to services and σ_e is the elasticity of substitution. We define it similarly for Foreign with $\eta_e^*, \sigma_e^*, C_e^{d*}$.

Energy Supply We parameterize the distribution of energy fields by treating supply elasticities, ϵ_S and ϵ_S^*, as parameters so that for $a \geq 0$:

$$E(a) = Ea^{\epsilon_S}; \qquad E^*(a) = E^* a^{\epsilon_S^*},$$

where E and E^* are shift parameters.

Goods Production We assume a CES production function with elasticity of substitution $1/(1-\rho)$ and energy-share parameter α, so that:

$$f(k) = (1 - \alpha + \alpha k^\rho)^{1/\rho}.$$

This functional form delivers a closed-form expression for the cost function $g(p)$.[27]

[27] Energy intensity is $k(p) = (\alpha/((1-\alpha)p)^{1/(1-\rho)}$ and the cost function is:

$$g(p) = \left((1-\alpha)^{1/(1-\rho)} + \alpha^{1/(1-\rho)} p^{-\rho/(1-\rho)}\right)^{-(1-\rho)/\rho}.$$

In the Cobb-Douglas limit, $\rho \to 0$, we get $g(p) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} p^\alpha$.

Comparative Advantage We parameterize the efficiency of the goods sector in each country by:

$$a_j = \left(\frac{j}{A}\right)^{1/\theta}; \qquad a_j^* = \left(\frac{1-j}{A^*}\right)^{1/\theta},$$

where A and A^* determine absolute advantage in either country, and θ determines (inversely) the scope of comparative advantage. Taking the ratio of these two gives the comparative advantage curve:

$$F(j) = \frac{a_j^*}{a_j} = \left(\frac{A}{A^*}\frac{1-j}{j}\right)^{1/\theta}.$$

This functional form allows us to solve for the BAU import and export thresholds. From (17), the BAU import threshold is $\bar{j}_m = A/\left(A + (\tau^*)^{-\theta} A^*\right)$, while setting $\lambda_e = p_e$ in (20), the BAU export threshold is $\bar{j}_x = \tau^{-\theta} A/\left(\tau^{-\theta} A + A^*\right)$.

6.1.2 Calibration of BAU Scenario.

We calibrate BAU to carbon accounting data for 2018 from the Trade Embodied in CO_2 ($TECO_2$) database made available by the OECD.[28] Units are gigatonnes of CO_2. Energy extraction data for 2018 is from the International Energy Agency World Energy Statistics Database. We use emissions factors to convert units of energy to units of CO_2.

For most of our results, members of the OECD form the taxing region, or Home, and non-OECD countries are Foreign. Table 5 provides the data that we calibrate to. By this CO_2 metric the OECD represents about one-third of the world. It represents a smaller share of extraction and a larger share of implicit consumption, nearly twenty percent of which is imported.

Two examples provide the basic logic for how we can calibrate the model to the data in Table 5. As noted in Section 2.3, we choose units of energy so that in BAU the global energy price is 1. Hence baseline extraction is $E = Q_e$ and $E^* = Q_e^*$. In BAU a country's average spending per good doesn't depend on the

[28]The values that we take from $TECO_2$ are broadly consistent with those available from the Global Carbon Project.

Table 5: Baseline Calibration for Home as the OECD

	Home	Foreign	Direct	Total
Home	$C_e^y = 8.7$	$C_e^m = 2.5$	$C_e^d = 2.5$	$C_e = 13.7$
Foreign	$C_e^x = 1.0$	$C_e^{y*} = 16.7$	$C_e^{d*} = 2.2$	$C_e^* = 19.9$
Direct	$C_e^d = 2.5$	$C_e^{d*} = 2.2$		
Total	$G_e = 12.2$	$G_e^* = 21.4$		$C_e^W = 33.6$
Extraction	$Q_e = 9.3$	$Q_e^* =$24.3		$Q_e^W = 33.6$

source of the good. Since the share of energy in the cost of any good is the same, in the baseline $\bar{j}_m = C_e^y/C_e$ and $\bar{j}_x = C_e^x/C_e^*$.

In addition to the carbon accounting data, we need values for seven parameters: θ, ϵ_S, ϵ_S^*, σ, σ^*, α, and ρ.[29] Table 6 lists our central values for these parameters, which we have determined using a variety of sources.[30] Appendix E provides additional details on our calibration procedure.

Prior studies, such as Elliott et. al. (2009) show that the foreign elasticity of energy supply, ϵ_s^*, is the key parameter affecting leakage and the effectiveness of a production tax. We estimate that $\epsilon_S = \epsilon_S^* = 0.5$ using data in Asker, Collard-Wexler, and De Loecker (2018), by fitting the slope of $E(a)$ and $E^*(a)$ among oil fields with costs above the median. Based on a literature review, Kotchen (2021) uses much higher values for the United States, with a point estimate for coal of $\epsilon_s^{coal} = 1.9$, for natural gas of $\epsilon_s^{NG} = 1.6$, and for gasoline of $\epsilon_s^{gas} = 2.0$. To account for the uncertainty in these values, we show most of our results using both our baseline calibration and also setting $\epsilon_s^* = 2.0$.

[29]The eight other parameters: A, A^*, E, E^*, η, η^*, τ, and τ^* are all subsumed by calibrating to the carbon accounts.

[30]We choose $\alpha = 0.15$ based on the ratio of the value of energy used in production to value added. (In our model that ratio is $\alpha/(1-\alpha)$.) We take $\theta = 4$ based on the preferred estimate in Simonovska and Waugh (2014). The values for $\sigma = \sigma^* = 1$ are chosen as a compromise between a likely higher elasticity of substitution between individual goods and a lower elasticity of demand for the goods aggregate.

Table 6: Parameter Values

α	ρ	ϵ_S	ϵ_S^*	σ	σ^*	θ
0.15	0	0.5	0.5	1	1	4

6.1.3 From BAU to Optimal

For any endogenous variable x we denote its value, given p_e and t_b, as $x(p_e, t_b)$. We will need to solve numerically for the optimal values of p_e and t_b, while we know that $t_e^N = \varphi^W$. In BAU $p_e = 1$ and $t_b = t_e^N = 0$. To simplify notation we let $x(1, 0) = x$.

Under the optimal policy, Home energy extraction is simply:

$$Q_e(p_e, t_b) = \left(p_e + t_b - \varphi^W\right)^{\epsilon_S} Q_e,$$

for $p_e + t_b - \varphi^W \geq 0$ and $Q_e(p_e, t_b) = 0$ otherwise. Foreign extraction is simply:

$$Q_e^*(p_e) = p_e^{\epsilon_S^*} Q_e^*,$$

where Q_e and Q_e^* are BAU baseline values.

The import margin for the optimal policy is unchanged from BAU, so that $\bar{j}_m(p_e, t_b) = \bar{j}_m = C_e^y / C_e$. Under the unilaterally optimal policy, the export margin changes from $\bar{j}_x = C_e^x / C_e^*$ to:

$$\bar{j}_x(p_e, t_b) = \frac{g(p_e + t_b)^{-\theta} C_e^x}{g(p_e + t_b)^{-\theta} C_e^x + \left(g(p_e) + t_b g'(p_e)\right)^{-\theta} C_e^{y*}}.$$

Consumption of energy in Foreign from Foreign production is:

$$C_e^{y*}(p_e, t_b) = D^*(p_e) \frac{\left(A^*\right)^{(\sigma^* - 1)/\theta}}{1 + (1 - \sigma^*)/\theta} \left(1 - \bar{j}_x(p_e, t_b)\right)^{1 + (1 - \sigma^*)/\theta},$$

where $D^*(p) = \eta^* g'(p) g(p)^{-\sigma^*}$ captures how demand for energy depends on the energy price, evaluated at $p_e = p$. Expressed in terms of BAU:

$$C_e^{y*}(p_e, t_b) = \frac{D^*(p_e)}{D^*(1)} \left(\frac{1 - \bar{j}_x(p_e, t_b)}{1 - \bar{j}_x} \right)^{1+(1-\sigma^*)/\theta} C_e^{y*}.$$

Appendix D provides a step-by-step derivation of all such terms.

To compute the optimal border adjustment t_b along with the equilibrium energy price p_e, we require that they clear the global energy market and satisfy (25):

$$C_e^W(p_e, t_b) = Q_e^W(p_e, t_b),$$

$$t_b = \frac{\varphi^W \epsilon_S^* Q_e^*(p_e) - \sigma^* \epsilon_g(p_e) S(p_e, t_b)}{\epsilon_S^* Q_e^*(p_e) + \epsilon_D^*(p_e) C_e^{z*}(p_e, t_b)}, \quad (26)$$

where ϵ_D^* (the Foreign demand elasticity) is the price elasticity of $D^*(p)$ while ϵ_g is the price elasticity of $g(p)$, which turns out to be the energy share in goods production.[31] Our algorithm simply iterates between the first two equations until we find the vector (p_e, t_b) that satisfies them both. We follow similar procedures for the optimal constrained policies.

We can evaluate any outcome of the model at the equilibrium (p_e, t_b) to explore the implications of the optimal policy. A key implication is the welfare benefit of the policy to Home. Our measure of welfare gain starts with the change in the planner's objective, $U(p_e, t_b) - U$. This term is equivalent to increased spending on services by Home, since consumption of services enters preferences linearly with price 1. To interpret the magnitude, and to make it scale free, we normalize it by Home's baseline spending on goods (baseline spending on energy divided by the energy share), $V_g = p_e C_e / \epsilon_g(p_e)$. The measure we present is thus, $W = (U(p_e, t_b) - U)/V_g$.

Our script is in Matlab. We use the solving procedure described above rather

[31]The connection to the energy share is as follows:

$$\epsilon_g(p_e) = \frac{p_e g'(p_e)}{g(p_e)} = \frac{p_e e_j^*(k(p_e))}{l_j^*(k(p_e)) + p_e e_j^*(k(p_e))},$$

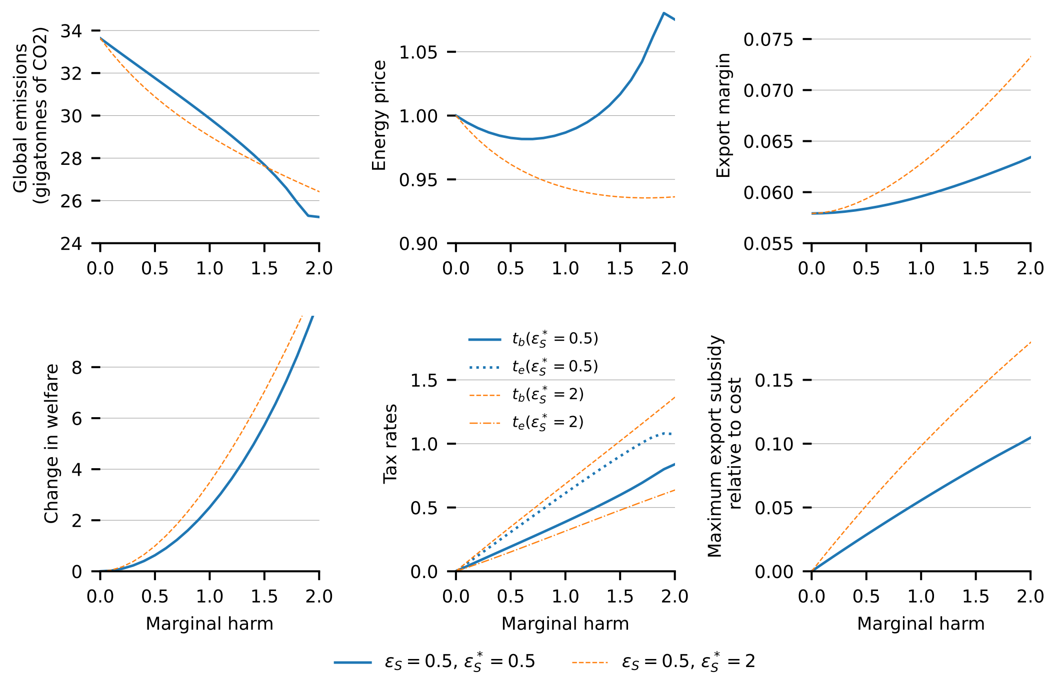
for any good j produced in Foreign for consumers there (or universally in BAU). Similarly, $\epsilon_g(p_e + t_b)$ would be the energy share for any good produced in Home. Both ϵ_g and ϵ_D^* typically depend on the cost of energy, although in the special case of $\rho = 0$ they become constants, $\epsilon_g = \alpha$ and $\epsilon_D^* = 1 - \alpha + \alpha\sigma^*$.

than a built-in solver. Our code is available at `https://github.com/dweisbach/Optimal-Unilateral-Carbon-Policy`.

6.2 Results

6.2.1 Optimal Policy

Figure 2: Optimal Policy in the OECD



We begin with a simulation of the optimal policy in the OECD (Figure 2). We illustrate the policy for marginal harm ranging from $\varphi^W = 0$ to $\varphi^W = 2$, showing the result for our baseline calibration of $\epsilon_S^* = 0.5$ and for $\epsilon_S^* = 2.0$. We show (i) the emissions reductions, (ii) the change in welfare (W), (iii) the change in p_e, (iv) the tax rates under the optimal policy, (v) the change in Home's export margin, $\bar{j}_x$, and (vi) the maximum export subsidy.[32]

[32]The maximum export subsidy is applied to Home's marginal export good, $\bar{j}_x$. The figure expresses this maximum subsidy relative to Home's cost of producing and delivering the good

Focusing on our baseline calibration, global emissions go down by about ¼ when $\varphi^W = 2$, a substantial reduction given that emissions in the OECD are only about ⅓ of global emissions (as reflected in the value of G_e in Table 5). Note that the substantial reduction from the OECD policy does not mean that the OECD's emissions are near zero. Some of the reductions arise in other parts of the world because of how the optimal policy expands the carbon price to trading partners. Notably, the OECD would choose to impose a significant carbon policy even when the rest of the world does not.

With $\epsilon_s^* = 0.5$, Home relies substantially on the extraction tax. The value of t_e is always higher than t_b, and increasingly so as φ^W goes up. The optimal tax rates range from 0 to up to about 1.5 times the initial (BAU) price of energy. The OECD's policy, however, still pushes the energy price (top middle) below 1 until φ^W approaches 1.5. For even higher values of φ^W, the net price received by energy extractors in the OECD, $p_e - t_e$, approaches zero. As a result, extraction in the OECD hits zero as φ^W approaches 2, which can be seen in the kink in the lines for high values of φ^W.

Examining the two graphs on the right-hand column of Figure 2, we can see that Home expands its export margin as marginal damages increase. By expanding its export margin, Home is able to broaden the application of its carbon policy, which becomes more important as the marginal harm from emissions increases. This feature of the policy comes at a cost that rises with φ^W.

Our alternative calibration sets $\epsilon_s^* = 2.0$. With a higher foreign elasticity of energy supply, Home makes less use of an extraction tax, because the tax would induce a significant response in Foreign. Instead, Home shifts most of the tax to the demand side: in the bottom middle panel, t_b now exceeds t_e. The value of p_e, correspondingly, goes down. Because Home relies more on demand-side taxes, it adjusts the trade margins more aggressively, as seen in the two right hand panels. Notably, emissions reductions (top left panel) are similar in the two cases. By shifting the mix of taxes and subsidies, the optimal policy is able to

to Foreign. This ratio turns out to reflect only the energy share, ϵ_g, and the ad-valorem border adjustment, t_b/p_e:

$$\frac{s_{\bar{j}_x}}{\tau a_{\bar{j}_x} g(p_e + t_b)} = \frac{(t_b/p_e)\epsilon_g}{1 + (t_b/p_e)\epsilon_g}.$$

achieve roughly the same outcome regardless of the value of ϵ_s^*.

To further examine the features of the optimal policy, we present four simulations that vary different elements of Home's policy.

6.2.2 Coalition Size

A key factor in global climate negotiations is the set of countries that will agree to emissions reductions. To examine the effects of coalition size, Figure 3 shows global emissions under optimal policies with five increasingly large coalitions, starting with just the EU and moving up to a global coalition.[33] Tables 7, 8 and 9 provide the calibrations for the three new scenarios. We show effects for our baseline calibration of ϵ_s^* (left panel) and our alternative calibration (right panel). All other parameters remain the same across each case.

Figure 3 can be thought of as a production possibility frontier showing the trade-offs between emissions reductions and cost for a given pricing coalition. Cost is measured as the reduced consumption needed to achieve a given percentage reduction in emissions from the 2018 level (33.6 Gt CO_2).[34] The x's in each line show the optimal emissions reduction when $\varphi^W = 2$.

Both panels show a consistent story, which is that there are substantial gains from expanding the taxing coalition. The EU alone has almost no power to reduce emissions. Adding the United States or the rest of the OECD countries helps significantly and increases the willingness of the coalition to incur costs to reduce emissions. Adding China to the taxing coalition leads to even greater emissions reductions for any given consumption cost.

Looking at the calibration tables, we can see that the size of the extraction

[33] We treat the global case as the limit of our two-region model as Foreign becomes infinitesimally small. For the EU-only case, we treat the EU as having 28 members as it had, prior to Brexit, in 2015.

[34] Our measure of economic cost of the policy to Home starts with the welfare measure W given above, but adds $\varphi^W \left(Q_e^W(p_e, t_b, t_e) - Q_e^W\right)$ (which is negative) to the numerator. The result is necessarily a negative number, becoming more negative as a larger φ^W leads to greater emissions reductions. This measure is convenient to compute, but implicitly assumes $\varphi^* = 0$. If $\varphi^* > 0$ then we overstate the economic cost to Home by ignoring transfers from Foreign to Home that offset gains to Foreign from reduced global emissions. Given a non-zero value for φ^* it is straightforward to make the necessary adjustment, which would push our measure of economic cost toward zero.

Figure 3: Choice of Pricing Coalition

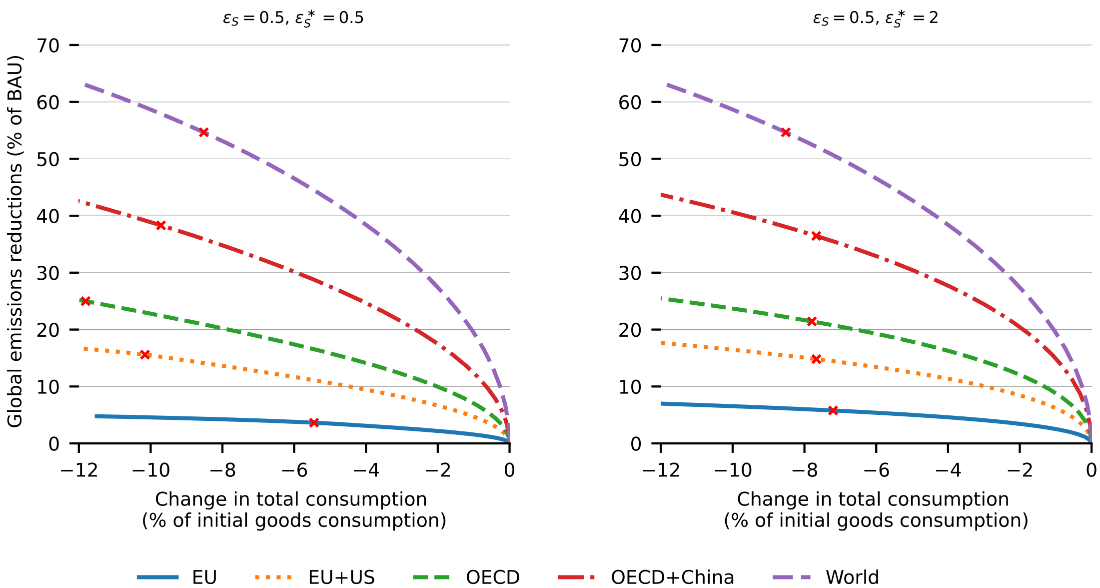


Table 7: Calibration for the European Union

	Home	Foreign	Direct	Total
Home	$C_e^y = 3.4$	$C_e^m = 1.2$	$C_e^d = 1.1$	$C_e = 5.7$
Foreign	$C_e^x = 0.5$	$C_e^{y*} = 23.8$	$C_e^{d*} = 3.6$	$C_e^* = 27.9$
Direct	$C_e^d = 1.1$	$C_e^{d*} = 3.6$		
Total	$G_e = 5.0$	$G_e^* = 28.6$		$C_e^W = 33.6$
Extraction	$Q_e = 4.7$	$Q_e^* = 28.9$		$Q_e^W = 33.6$

base is the key difference between the EU and the coalition of the EU and the United States. Production and consumption roughly double, reflecting the relative size of the two economies, but extraction goes up by a factor of more than 5. With almost no extraction, the EU on its own gets little advantage from the extraction tax portion of the optimal policy, which means that acting alone, it is ineffective at reducing global emissions. Adding the United States expands the extraction base and makes the policy more effective.

Table 8: Calibration for the EU and the United States

	Home	Foreign	Direct	Total
Home	$C_e^y = 5.6$	$C_e^m = 2.1$	$C_e^d = 2.0$	$C_e = 9.7$
Foreign	$C_e^x = 0.8$	$C_e^{y*} = 20.4$	$C_e^{d*} = 2.7$	$C_e^* = 23.9$
Direct	$C_e^d = 2.0$	$C_e^{d*} = 2.7$		
Total	$G_e = 8.4$	$G_e^* = 25.2$		$C_e^W = 33.6$
Extraction	$Q_e = 5.6$	$Q_e^* = 28.0$		$Q_e^W = 33.6$

Table 9: Calibration for the OECD plus China

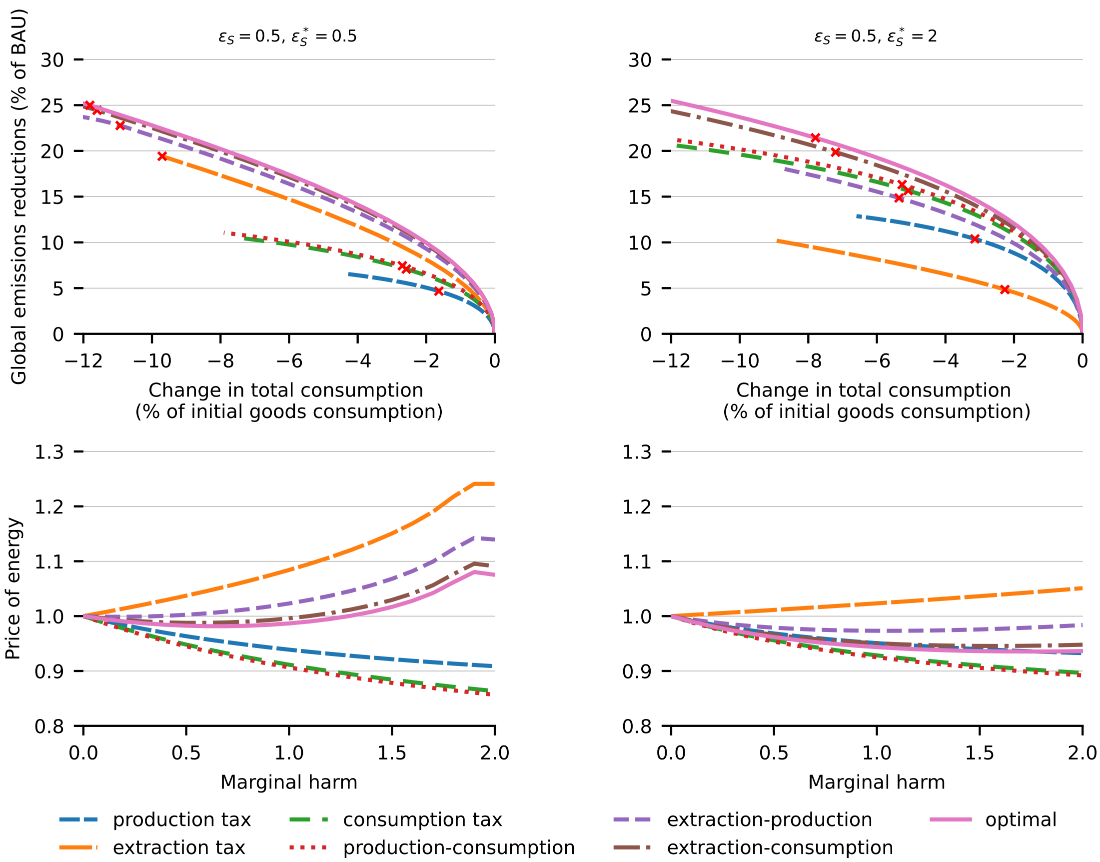
	Home	Foreign	Direct	Total
Home	$C_e^y = 17.8$	$C_e^m = 1.9$	$C_e^d = 3.0$	$C_e = 22.7$
Foreign	$C_e^x = 1.4$	$C_e^{y*} = 7.8$	$C_e^{d*} = 1.7$	$C_e^* = 10.9$
Direct	$C_e^d = 3.0$	$C_e^{d*} = 1.7$		
Total	$G_e = 22.2$	$G_e^* = 11.4$		$C_e^W = 33.6$
Extraction	$Q_e = 16.9$	$Q_e^* = 16.7$		$Q_e^W = 33.6$

Comparing the left and right panels, we can see that regardless of the value of ϵ_s^*, the taxing coalition is able to achieve about the same emissions reductions for a given cost. With the exception of the EU-only tax, however, the taxing coalition is willing to incur a higher cost when ϵ_s^* is low than when it is high. For example, the OECD would choose to reduce emissions by 25% at a cost of 12% when $\epsilon_s^* = 0.5$, but would only be willing to spend 7.8% to reduce emissions by 21% when $\epsilon_s^* = 2.0$.

6.2.3 Choice of Tax

The top panels of Figure 4 compares the optimal tax to the six constrained optimal taxes, under our baseline calibration and for $\epsilon_S^* = 2.0$.[35] The bottom panels show the effects on p_e for each tax.

Figure 4: Effects of different taxes on emissions and p_e



With a low value of ϵ_S^*, extraction taxes perform much better than the demand-side taxes (production or consumption taxes, or a hybrid of the two). The bottom panel illustrates why: the extraction tax raises p_e while the demand-side taxes

[35]We leave out the extraction/production/consumption hybrid as it turns out to be indistinguishable from the unilaterally optimal policy in this figure. Some of the lines in Figure 4 stop short of a cost of 10%. This is for two reasons. First, we only ran our simulation up to values of $\varphi^W = 20$. Second, Home extraction goes to zero for sufficiently high values of φ^W, so an extraction tax become ineffective beyond that point.

lower it. Increasing p_e in this case induces a demand-side response in Foreign without generating a large supply-side response. In fact, when ϵ_s^* is low, both hybrids involving an extraction tax perform almost as well as the optimal tax.

When $\epsilon_S^* = 2.0$ (the right hand panel) extraction taxes are no longer as desirable. Increasing p_e would cause a substantial increase in Foreign extraction, offsetting the effectiveness of the tax. Demand-side taxes are correspondingly more effective because lowering p_e causes a significant reduction in Foreign extraction. For example, the basic production tax goes from an optimal emissions reduction of 4.7% when $\epsilon_s^* = 0.5$ to reductions of 10.4% when $\epsilon_s^* = 2.0$. Looking at the bottom right panel, we can see that Home is less willing to allow p_e to change when ϵ_s^* is high.

6.2.4 Location

Figure 5: Effects on Foreign Activities

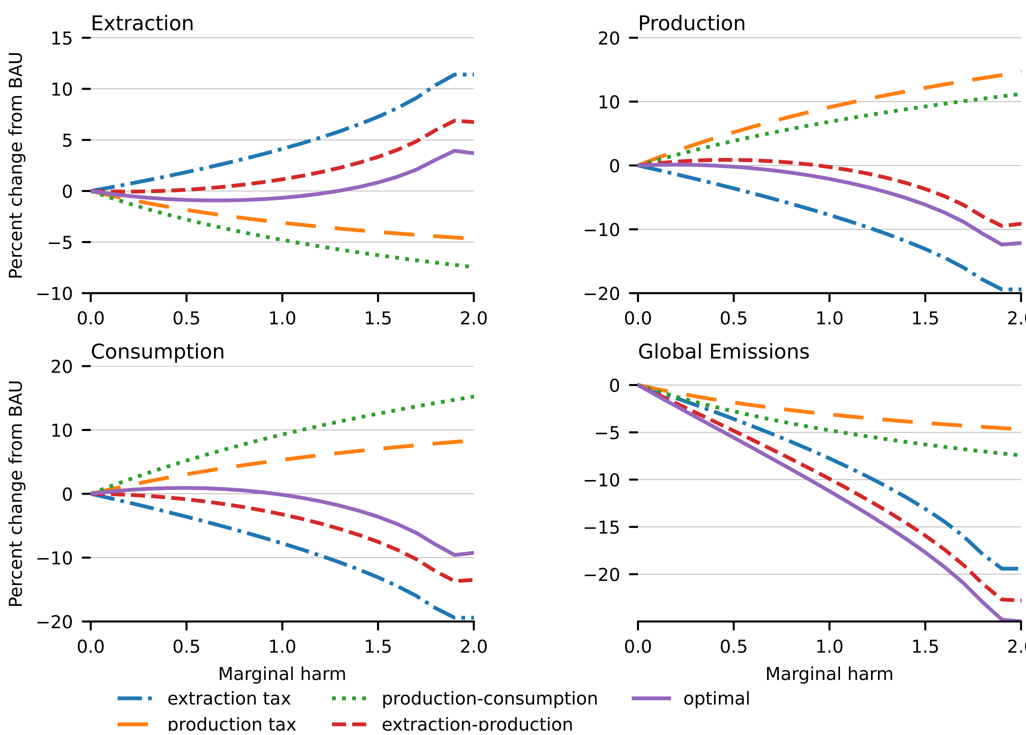


Figure 5 explores the effects of taxes on leakage and other shifts in location, focusing on how activities in Foreign change in response to Home's taxes. It illustrates the optimal tax and four of the constrained taxes (dropping the basic consumption and extraction/consumption hybrid to reduce clutter). It shows the percent changes in Q_e^*, G_e^*, and C_e^* relative to their values with no tax. The bottom right panel shows the change in global emissions that the OECD would choose if it were constrained to using each of these taxes.

Changes to extraction (top left) are consistent with the changes to p_e seen in Figure 4. Extraction taxes drive up p_e and as a result, cause Foreign to increase its extraction. Production and consumption taxes drive p_e down, causing Foreign to reduce its extraction. The optimal tax and the extraction/production hybrid moderate the effects on Foreign extraction.

The opposite occurs for G_e^* and C_e^* (top right, bottom left). Because production and consumption taxes drive p_e down, G_e^* and C_e^* both go up when Home imposes those taxes. Correspondingly, Foreign production and consumption both go down when Home imposes an extraction tax. And once again, the optimal and the extraction/production hybrid operate in the middle.

7 Multiple Energy Sources

Up to this point we have assumed that all energy is from fossil fuel with a fixed carbon content. We could therefore normalize a unit of CO_2 to be a unit of energy, treating energy and CO_2 interchangeably. We also assumed that energy is costlessly traded, crude oil being the closest example. Here we explore how our analysis can accommodate a variety of energy sources.

We introduce $R \geq 1$ sources, indexed by r, such as coal, natural gas, and solar. We assume that these sources are perfect substitutes in providing energy, but may differ in dirtiness, d_r, measured as CO_2 emissions per unit of energy. We take $r = 1$ to be crude oil, and normalize $d_1 = 1$. If r is a renewable source, $d_r = 0$. Each source has a corresponding distribution of energy fields, $E_r(a)$ in Home and $E_r^*(a)$ in Foreign.[36] This formulation, in terms of energy fields, can describe renewable sources as well since costs of generating solar, wind, and water

[36] We assume these functions satisfy the conditions described in footnote 7.

power are also dictated by scarce geographic factors.

We assume that the world energy market is integrated through trade in oil, while other sources of energy can't be shipped internationally. This assumption rules out potential policy interventions by Home to shift Foreign supply toward sources with lower CO_2 content.[37] These assumptions lead to a simple generalization of our analysis above. The quantity of energy supplied by Home becomes $Q_e = \sum_{r=1}^{R} Q_{e,r}$, while at an energy price p_e Foreign supplies $Q_e^* = \sum_{r=1}^{R} Q_{e,r}^* = \sum_{r=1}^{R} E_r^*(p_e)$. Setting $R = 1$ we return to the model used in Sections 2-6.

7.1 Amendments to the Planning Problem

This extension requires only modest changes to the unilaterally optimal solution presented in Section 3. The inner problem is unchanged since different sources of energy are perfect substitutes in production. The outer problem must be modified to accommodate the planner's choice of extraction from each source, $\{Q_{e,r}\}_{r=1}^{R}$, and to determine how its choice of p_e depends on Foreign's energy sources.

The first order condition of direct energy consumption is unchanged, $v'(C_e^d) = \lambda_e$.

The first order condition for energy supplied by source r is:

$$\frac{\partial \mathcal{L}}{\partial Q_{e,r}} = -d_r \varphi^W - \frac{\partial L_e}{\partial Q_{e,r}} + \lambda_e \leq 0,$$

with equality if $Q_{e,r} > 0$. The extra labor in Home to supply a bit more energy from source r is the labor requirement on its marginal energy field for this source, $E_r^{-1}(Q_{e,r})$, implying:

$$Q_{e,r} = E_r\left(\lambda_e - d_r \varphi^W\right),$$

for $\lambda_e - d_r \varphi^W \geq 0$ and $Q_{e,r} = 0$ otherwise.

[37] For example, if renewables were tradable at a low cost, Home might import at a high price while also exporting them at a low price in order to limit Foreign's extraction of fossil fuels. While intriguing, a careful analysis of such policies is beyond the scope of this paper.

The first order condition for the energy price becomes:

$$\frac{\partial u^*(C_g^*)}{\partial p_e}+\frac{v^*(C_e^d)}{\partial p_e}-\varphi^W\sum_{r=1}^{R}d_r\frac{\partial Q_{e,r}^*}{\partial p_e}-\sum_{r=1}^{R}\frac{\partial L_{e,r}^*}{\partial p_e}-\frac{\partial L_g^W}{\partial p_e}-\lambda_e\left(\frac{\partial C_e^*}{\partial p_e}-\sum_{r=1}^{R}\frac{\partial Q_{e,r}^*}{\partial p_e}\right)=0,$$

where $\partial L_{e,r}^*/\partial p_e = p_e\partial Q_{e,r}^*/\partial p_e$. We can simplify this condition to rewrite it as:

$$\sum_{r=1}^{R}\left(\lambda_e - d_r\varphi^W - p_e\right)\frac{\partial Q_{e,r}^*}{\partial p_e}=(\lambda_e-p_e)\frac{\partial C_e^{z*}}{\partial p_e}+\int_{j_0}^{\bar{j}_x}\left(\tau a_j g(\lambda_e)-a_j^* g(p_e)\right)\frac{\partial x_j}{\partial p_e}dj.$$

This condition is identical to (23) except that there is now a separate extraction wedge, $\lambda_e - d_r\varphi^W - p_e$, for each energy source. The wedge is equal to the difference between the planner's marginal valuation of supplying energy from a given source and the price of energy.

In its simplified form the condition is a straightforward generalization of (24):

$$\lambda_e - p_e = \frac{\varphi^W\sum_{r=1}^{R}d_r\partial Q_{e,r}^*/\partial p_e - \sigma^*\epsilon_g(p_e)S}{\sum_{r=1}^{R}d_r\partial Q_{e,r}^*/\partial p_e + |\partial C_e^{z*}/\partial p_e|}. \quad (27)$$

The key insight is that only the marginal response of Foreign energy supply to the energy price enters into the optimal unilateral policy. Even if much of Foreign's energy comes from renewables, if the marginal source of energy is coal the planner will tilt away from policies that raise the price of energy.

7.2 Amendments to Optimal Taxes

The optimal policy can still be implemented with an extraction tax, a border adjustment, and a subsidy to Home's marginal exporters. We can no longer treat energy and CO_2 as functionally the same, however. If the nominal extraction tax is applied to the carbon content of each type of energy, the rate per unit of CO_2 would still be equal to the Pigouvian wedge, φ^W. But, the nominal tax per unit of energy supplied by source r is $t_{e,r}^N = d_r\varphi^W$.

The level of the border adjustment is $t_b = \lambda_e - p_e$, per unit of energy, given by equation (27). This result formalizes the argument made in Kortum and Weisbach (2017) that the border adjustment should not be based on the carbon content of the energy source used to produce a particular good. Instead, what matters is the carbon content of the marginal energy source for the country exporting the good. The price faced by users of energy is $p_e + t_b$, without regard to the source.

The final element of Home's carbon policy, the subsidy to Home's marginal exporters, is unchanged by the addition of multiple energy sources.

Putting the nominal extraction tax and border adjustment together, the effective tax on energy supplied by source r is $t_{e,r} = t^N_{e,r} - t_b$, equal to (minus) the associated extraction wedge. Energy supplied by source r in Home is thus:

$$Q_{e,r} = E_r \left(p_e - t_{e,r}\right) = E_r \left(p_e + t_b - d_r \varphi^W\right),$$

for $p_e - t_{e,r} > 0$ and $Q_{e,r} = 0$ otherwise. Extraction from a high-carbon source r may be shut down under the optimal policy. Supply from low-carbon sources will be stimulated relative to high-carbon sources.

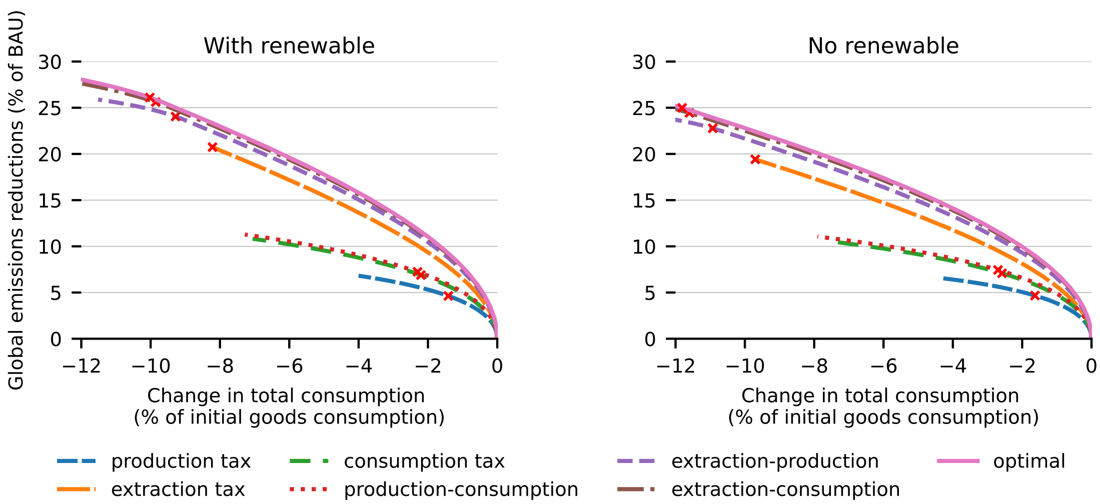
7.3 Simulations

To understand these effects, we modify our calibration to include renewable energy. To do this, we simply scale up total energy by the global fraction of renewables, which is 13% using the same IEA source. All functional forms from the prior simulation remain the same, and we assume for simplicity that the elasticity of renewable energy supply is the same as for fossil fuels.

Figure 6 compares the emissions reductions achievable with renewables (left hand panel) to those without (right hand panel, which is identical to that in Figure 4). Emissions reductions are slightly greater with renewables for any given cost, but not by a large margin. In addition, the ranking of the policies stays the same.

Figure 7 shows how various policies affect the use of renewables. The top left panel shows the global use of renewables under the unilateral optimal tax and the three pure taxes. All four policies increase the global use of renewables, with the optimal policy inducing the highest use followed by an extraction tax. (The kinks in the optimal and extraction tax lines are where Home ceases to extract fossil fuels and, as a result, stops increasing the extraction tax even when marginal

Figure 6: Renewable Comparison

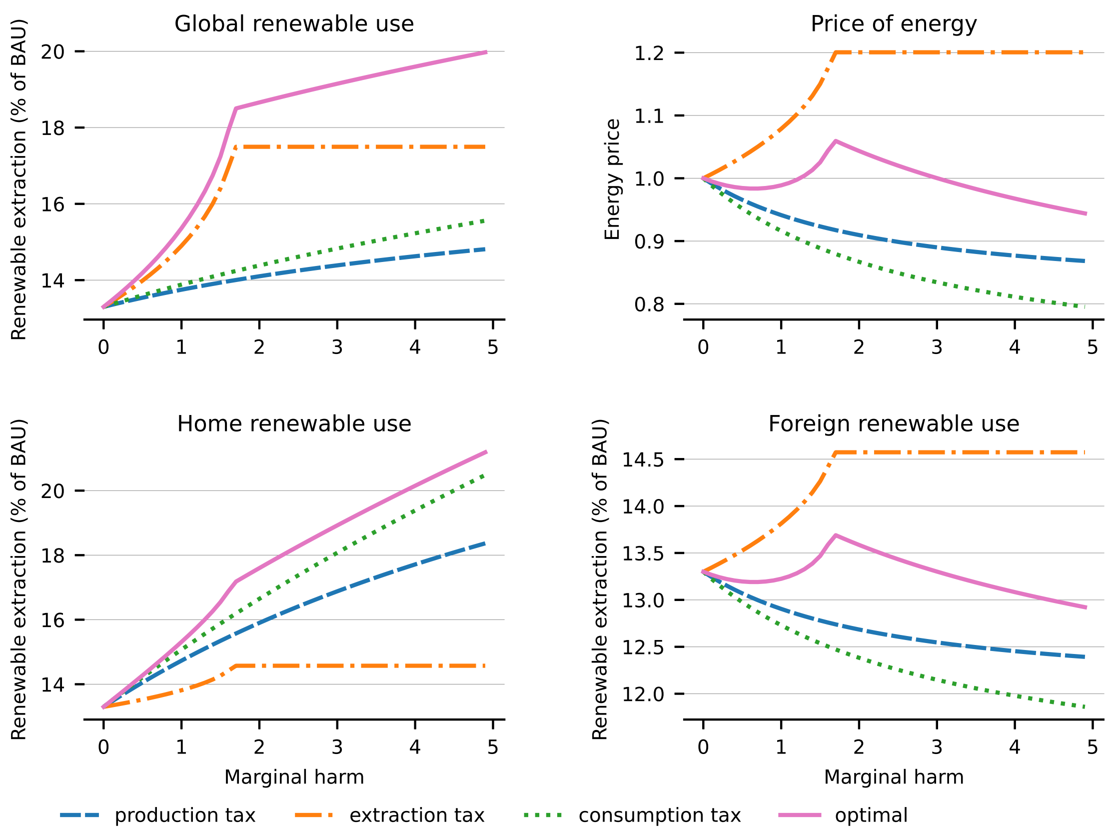


harm goes up.)

The reason the extraction tax increases renewables use so strongly is that it raises the global price of energy (top right panel). Both Home and Foreign renewables extractors can sell their energy at this higher price, inducing greater use of renewables in both locations (while Home fossil fuel extractors only receive $p_e - t_e$ for their energy).

Production and consumption taxes are much less successful at inducing the use of renewables. The reason is that they lower the global price of energy. As a result, those policies reduce the use of renewables in Foreign. In Home, however, sellers of renewable energy receive $p_e + t_b$ for their energy, which is higher than in the BAU. As a result, they increase their sales. The net is a modest increase in renewable use. To the extent that renewables use is independently valuable (for example, it might allow learning by doing, lowering the future price of renewables), the differing effects of supply-side and demand-side taxes might be important. (Although we omit the lines for the hybrid policies to keep the figures legible, the results are similar in those cases.)

Figure 7: Effects of Renewables



8 Conclusion

While the model in this paper is highly stylized, its simplicity yields analytical insights into the features of an optimal unilateral carbon policy. To go deeper requires pushing the analysis in a more quantitative direction, extending it to multiple countries and perhaps to multiple periods of time as well. Barresi (2022) shows how our unilateral carbon policy fits into the multi-country trade model of Eaton and Kortum (2002), with coalitions of countries representing Home and Foreign. Larch and Wanner (2019) provide a natural multi-country analysis of the energy sector. On the second extension, Golosov, Hassler, Krusell, and Tsyvinski (2014) and Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2021) are roadmaps for introducing dynamics.

Another important extension, in a multi-country world, is to consider endogenizing the region we call Home. Our current approach follows Markusen (1975) and CDVW in assuming that Foreign is intransigent. Home's optimal policy will likely be different if it can entice (or coerce) Foreign countries to join its coalition. Promising steps in this direction have been taken by Nordhaus (2015), Farrokhi and Lashkaripour (2020), and Barresi (2022).

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A Global Planner's Problem

Suppose the planner controls all decisions in Foreign as well as in Home. We can solve this problem by maximizing the Lagrangian (14) while enlarging the set of choice variables to C_e^d, C_e^{d*}, Q_e, Q_e^*, $\{y_j\}$, $\{y_j^*\}$, $\{x_j\}$, $\{m_j\}$, $\{k_j^y\}$, $\{k_j^{y*}\}$, $\{k_j^x\}$, and $\{k_j^m\}$. (The variable p_e is no longer relevant.)

A.1 Solution

Following CDVW, we first solve the inner problem, involving conditions for an individual good j, given λ_e. We then turn to the outer problem, optimizing over C_e^d, C_e^{d*}, Q_e and Q_e^* while solving for λ_e.

A.1.1 Inner Problem

The inner problem, much like (15), is to choose y_j, y_j^*, x_j, m_j, k_j^y, k_j^{y*}, k_j^x, and k_j^m to maximize:

$$\begin{aligned}\mathcal{L}_j &= u\left(y_j + m_j\right) + u^*(y_j^* + x_j) \\ &\quad - a_j\left(\frac{1}{f(k_j^y)} + \lambda_e\frac{k_j^y}{f(k_j^y)}\right)y_j - a_j^*\left(\frac{1}{f(k_j^{y*})} + \lambda_e\frac{k_j^{y*}}{f(k_j^{y*})}\right)y_j^* \\ &\quad - \tau a_j\left(\frac{1}{f(k_j^x)} + \lambda_e\frac{k_j^x}{f(k_j^x)}\right)x_j - \tau^* a_j^*\left(\frac{1}{f(k_j^m)} + \lambda_e\frac{k_j^m}{f(k_j^m)}\right)m_j.\end{aligned}$$

Energy intensities k_j^i, for $i \in \{y, y^*, x, m\}$, enter this objective as in (6), which implies $k_j^i = k(\lambda_e)$. We also get that the shadow cost of producing good j is $a_j g(\lambda_e)$ in Home and $a_j^* g(\lambda_e)$ in Foreign. The unit energy requirement is $e_j(k(\lambda_e)) = a_j g'(\lambda_e)$ in Home and $e_j^*(k(\lambda_e)) = a_j^* g'(\lambda_e)$ in Foreign.

The FOC for y_j implies $u'(y_j + m_j) \leq a_j g(\lambda_e)$, with equality if $y_j > 0$. The FOC for m_j implies $u'(y_j + m_j) \leq a_j^* \tau^* g(\lambda_e)$, with equality if $m_j > 0$. Both FOC's hold with equality for good $j = \bar{j}_m$ satisfying $F(\bar{j}_m) = 1/\tau^*$. For $j < \bar{j}_m$ we have $y_j = \eta\left(a_j g(\lambda_e)\right)^{-\sigma}$ and $m_j = 0$ while for $j > \bar{j}_m$ we have $m_j = \eta\left(a_j^* \tau^* g(\lambda_e)\right)^{-\sigma}$ and $y_j = 0$.

The FOC for y_j^* implies $u^{*\prime}(y_j^* + x_j) \leq a_j^* g(\lambda_e)$, with equality if $y_j^* > 0$. The FOC for x_j implies $u^{*\prime}(y_j^* + x_j) \leq a_j \tau g(\lambda_e)$, with equality if $x_j > 0$. Both FOC's hold with equality for good $j = \bar{j}_x$ satisfying $F(\bar{j}_x) = \tau$. (Since F is monotonically decreasing, it follows that $\bar{j}_x < \bar{j}_m$.) For $j < \bar{j}_x$ we have $x_j = \eta^* \left(a_j \tau g(\lambda_e)\right)^{-\sigma^*}$ and $y_j^* = 0$ while for $j > \bar{j}_x$ we have $y_j^* = \eta^* \left(a_j^* g(\lambda_e)\right)^{-\sigma^*}$ and $x_j = 0$.

Aggregating over goods, taking account of demand and unit energy requirements, the implicit consumption of energy in Home is:

$$C_e^y(\lambda_e) + C_e^m(\lambda_e) = \eta g'(\lambda_e) g(\lambda_e)^{-\sigma} \left(\int_0^{\bar{j}_m} a_j^{1-\sigma} dj + (\tau^*)^{1-\sigma} \int_{\bar{j}_m}^1 \left(a_j^*\right)^{1-\sigma} dj \right),$$

while in Foreign:

$$C_e^{y*}(\lambda_e) + C_e^x(\lambda_e) = \eta^* g'(\lambda_e) g(\lambda_e)^{-\sigma} \left(\tau^{1-\sigma^*} \int_0^{\bar{j}_x} a_j^{1-\sigma^*} dj + \int_{\bar{j}_x}^1 \left(a_j^*\right)^{1-\sigma^*} dj \right).$$

Both are functions of the Lagrange multiplier λ_e.

A.1.2 Outer Problem

The outer problem is to choose C_e^d, C_e^{d*}, Q_e and Q_e^* while solving for λ_e that clears the global energy market. The Lagrangian remains as in (21). The first order condition with respect to direct consumption gives $v(C_e^d) = v^*(C_e^{d*}) = \lambda_e$. The first order condition with respect to Home energy extraction implies $Q_e = E(\lambda_e - \varphi^W)$, for $\lambda_e - \varphi^W \geq 0$, or else $Q_e = 0$. Likewise for Foreign energy extraction, $Q_e^* = E^*(\lambda_e - \varphi^W)$, for $\lambda_e - \varphi^W \geq 0$, or else $Q_e^* = 0$. The Lagrange multiplier solves:

$$C_e(\lambda_e) + C_e^*(\lambda_e) = E\left(\lambda_e - \varphi^W\right) + E^*\left(\lambda_e - \varphi^W\right).$$

A.2 Decentralized Global Optimum

We can interpret the planner's solution in terms of a decentralized economy with a price of energy $p_e = \lambda_e$. An extraction tax in both Home and Foreign, equal to global marginal damages from emissions, $t_e = t_e^* = \varphi^W$, solves the global externality. Energy extractors in Home and Foreign receive an after-tax price of $p_e - \varphi^W$ (nominal and effective extraction tax rates are the same). With a globally harmonized policy, a consumption tax at rate φ^W (with appropriate transfers of

Table 10: BAU Competitive Equilibrium (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta\left(a_j g(p_e)\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta\left(\tau^* a_j^* g(p_e)\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^*\left(\tau a_j g(p_e)\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^*\left(a_j^* g(p_e)\right)^{-\sigma^*}$	$j > \bar{j}_x$

Thresholds: $F(\bar{j}_m) = 1/\tau^*$ and $F(\bar{j}_x) = \tau$

services) results in the same outcomes.[38]

A.3 Competitive Equilibrium

In a competitive equilibrium all outcomes are the same as in the decentralized global optimum above, except with no tax on energy extractors. For later reference, we list the outcomes for any good j in Table 10. We treat this case as our BAU baseline.

B Unilateral Planner's Problem

We fill in missing derivations concerning the optimal unilateral policy.

B.1 Optimality Condition for the Energy Price

We start with the first-order condition with respect to p_e, as in the paper:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^*}{\partial p_e} - \frac{\partial L_e^*}{\partial p_e} - \frac{\partial L_g^W}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^*}{\partial p_e} \right) = 0.$$

[38]Inspection of the global market clearing condition for energy shows that extraction and consumption of energy remain the same if we instead set $p_e = \lambda_e + \varphi^W$. This change corresponds to adding full border adjustments, $t_b = t_b^* = \varphi^W$, to a nominal extraction tax, $t_e^N = t_e^{N*} = \varphi^W$, turning it into a consumption tax. Any differences in the distribution of services consumption between these two policies (a global extraction tax versus a global consumption tax) can be undone with transfers.

We distinguish energy C_e^x and labor L_g^x used by Home to supply goods to Foreign consumers from energy C_e^{y*} and labor L_g^{y*} used by Foreign to supply goods to Foreign consumers. (Energy $C_e^y + C_e^m$ and labor $L_g^y + L_g^m$ used to supply goods to Home consumers don't depend on p_e since C_g is determined by the inner problem in Section 3.5.2.) Substituting in $\partial L_e^*/\partial p_e = p_e \partial Q_e^*/\partial p_e$, we can rearrange the first-order condition as:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = \lambda_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} - \frac{\partial u^*(C_g^*)}{\partial p_e} - \frac{v^*(C_e^{d*})}{\partial p_e} + \lambda_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e}. \tag{28}$$

First note that by chain rule

$$\frac{\partial v^*(C_e^{d*})}{\partial p_e} = \frac{\partial v^*(C_e^{d*})}{\partial C_e^{d*}} \frac{\partial C_e^{d*}}{\partial p_e} = p_e \frac{\partial C_e^{d*}}{\partial p_e}$$

We now derive an expression for the term $\partial u^*(C_g^*)/\partial p_e$ that appears in (28). Recall that we can write Foreign utility from goods consumption as:

$$u^*(C_g^*) = \int_0^1 u(c_j^*) dj,$$

so that its derivative with respect to the energy price is:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} = \int_0^1 u^{*\prime}(c_j^*) \frac{\partial c_j^*}{\partial p_e} dj.$$

From the inner problem in Section 3.5.3, the c_j^* in Region 1 don't depend on the energy price, the c_j^* in Region 2 are exported by Home in quantities x_j that equates Foreign marginal utility to what it would cost Foreign to produce them itself, and c_j^* in Region 3 are produced by Foreign in quantities y_j^* and consumed there. Hence:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} = \int_{j_0}^{\bar{j}_x} a_j^* g(p_e) \frac{\partial x_j}{\partial p_e} dj + \int_{\bar{j}_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj. \tag{29}$$

We can go a step further by aggregating the implicit cost functions for supplying

Foreign consumption:

$$\lambda_e C_e^x + L_g^x + p_e C_e^{y*} + L_g^{y*} = \int_0^{\bar{j}_x} \tau a_j g(\lambda_e) x_j dj + \int_{\bar{j}_x}^1 a_j^* g(p_e) y_j^* dj.$$

Differentiating both sides (noting that x_j depends on p_e only in Region 2) and canceling out C_e^{y*} yields:

$$\lambda_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} = \int_{j_0}^{\bar{j}_x} \tau a_j g(\lambda_e) \frac{\partial x_j}{\partial p_e} dj + \int_{\bar{j}_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj.$$

Combined with (29):

$$\begin{aligned} \frac{\partial u^*(C_g^*)}{\partial p_e} &= \lambda_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} \\ &\quad - \int_{j_0}^{\bar{j}_x} \left(\tau a_j g(\lambda_e) - a_j^* g(p_e)\right) \frac{\partial x_j}{\partial p_e} dj. \end{aligned} \tag{30}$$

Substituting (30) into (28) yields (23) from the paper:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^*}{\partial p_e} = (\lambda_e - p_e) \frac{\partial C_e^{z*}}{\partial p_e} + \int_{j_0}^{\bar{j}_x} \left(\tau a_j g(\lambda_e) - a_j^* g(p_e)\right) \frac{\partial x_j}{\partial p_e} dj,$$

B.2 Bounds on the Consumption Wedge

We establish a lower bound on $\lambda_e - p_e$ by decomposing the wedges (or subsidies) s_j, for $j \in (j_0, \bar{j}_x)$, which enter (24) through S. Adding and subtracting $\lambda_e e_j^*(k(p_e))$ from each wedge:

$$s_j = \tau a_j g(\lambda_e) - l_j^*(k(p_e)) - p_e e_j^*(k(p_e)) = (\lambda_e - p_e) e_j^*(k(p_e)) - \pi_j,$$

where $\pi_j = l_j^*(k(p_e)) + \lambda_e e_j^*(k(p_e)) - \tau a_j g(\lambda_e)$ is the planner's value of global resources saved when a unit of good j is produced in Home and exported rather than being produced in Foreign. Equation (19) shows that π_j is also the derivative of the inner problem with respect to x_j, so that $\pi_j > 0$ for $j < \bar{j}_x$ and zero at $j = \bar{j}_x$.

Substituting this expression for s_j into the overall implicit subsidy S, we can rewrite (24) as:

$$\lambda_e - p_e = \frac{\varphi^W \partial Q_e^*/\partial p_e + \sigma^* \frac{g'(p_e)}{g(p_e)} \int_{j_0}^{\bar{j}_x} \pi_j x_j dj}{\partial Q_e^*/\partial p_e + |\partial C_e^{z*}/\partial p_e| + \sigma^* \frac{g'(p_e)}{g(p_e)} \int_{j_0}^{\bar{j}_x} e_j^*(k(p_e)) x_j dj}.$$

The denominator is strictly positive while the numerator is weakly positive, establishing the result that $\lambda_e - p_e \geq 0$. If $\varphi^W \partial Q_e^*/\partial p_e = 0$ then $\lambda_e - p_e = 0$, with $j_0 = \bar{j}_x$.

Having shown that $\lambda_e \geq p_e$, it follows that $j_0 \leq \bar{j}_x$ and hence $S > 0$. We get an upper bound on λ_e by using (24) to write:

$$\varphi^W - (\lambda_e - p_e) = \frac{|\partial C_e^{z*}/\partial p_e| \varphi^W + \sigma^* \frac{g'(p_e)}{g(p_e)} S}{\partial Q_e^*/\partial p_e + |\partial C_e^{z*}/\partial p_e|}.$$

The right-hand side is positive, which implies $\lambda_e - p_e \leq \varphi^W$, with a strict inequality if $\varphi^W > 0$.

C Constrained-Optimal Policies

We derive the formulas for the constrained-optimal policies that appear in Tables 3-4 of the paper. Each maximizes the Lagrangian (14), but with with different constraints on the planner's choice variables. Similarly, the Lagrangian of the outer problem (21), which we repeat here for convenience, is common to all seven policies:

$$\mathcal{L} = u(C_g) + u^*(C_g^*) + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W Q_e^W - L_e^W - L_g^W - \lambda_e \left(C_e^W - Q_e^W \right),$$

Expressions for the terms that enter this outer Lagrangian will be different for each policy we consider below.

As in the paper, we first consider policies in which the planner chooses extraction and/or domestic consumption before turning to policies in which the planner controls domestic production, or some combination of extraction, consumption, and production.

In the derivations that follow we will often need to distinguish labor used in each of the four lines of goods production, in parallel to how we distinguish the use of energy in these four lines. Thus, we introduce L_g^i for $i \in \{y, x, m, y^*\}$ with $L_g^y + L_g^x = L_g$ and $L_g^m + L_g^{y*} = L_g^*$. (These terms appeared briefly in Appendix B.1 as well.)

The primary challenge is to derive, for each policy, the optimal energy price (or prices in the case of a production tax) that maximizes the outer Lagrangian (21). A key step is to use a simpler version of (30) from Appendix B.1, which we derive here. Recall that:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} = \int_0^1 u^{*\prime}(c_j^*) \frac{\partial c_j^*}{\partial p_e} dj.$$

Assuming that producers serving Foreign consumers face an energy price p_e (we will amend this assumption for the case of a production tax), we have:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} = \int_0^{\bar{j}_x} \tau a_j g(p_e) \frac{\partial x_j}{\partial p_e} dj + \int_{\bar{j}_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj.$$

Aggregating the implicit cost functions for supplying Foreign consumption, differentiating with respect to the energy price, and canceling out C_e^x and C_e^{y*}:

$$p_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} = \int_0^{\bar{j}_x} \tau a_j g(p_e) \frac{\partial x_j}{\partial p_e} dj + \int_{\bar{j}_x}^1 a_j^* g(p_e) \frac{\partial y_j^*}{\partial p_e} dj.$$

Combining the two equations above:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} = p_e \frac{\partial C_e^x}{\partial p_e} + \frac{\partial L_g^x}{\partial p_e} + p_e \frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e}. \tag{31}$$

We will refer back to (31) in the derivations that follow.

C.1 Basic Extraction Policy

For an extraction policy, we constrain the planner to choose only Q_e and p_e. Energy intensities, quantities produced, and quantities consumed of each good j are as in the BAU competitive equilibrium, given p_e. Hence we can skip the inner problem. and go directly to the outer problem.

C.1.1 Outer Problem

We want to maximize (21) over Q_e and p_e. The first order condition for Q_e is identical to that for the unilaterally optimal policy. For $\lambda_e - \varphi^W \geq 0$ we have $Q_e = E(\lambda_e - \varphi^W)$, and otherwise $Q_e = 0$.

The first order condition for p_e is:

$$\frac{\partial u(C_g)}{\partial p_e}+\frac{\partial u^*(C_g^*)}{\partial p_e}+\frac{\partial v(C_e^d)}{\partial p_e}+\frac{\partial v^*(C_e^{d*})}{\partial p_e}-\varphi^W\frac{\partial Q_e^*}{\partial p_e}-\frac{\partial L_e^*}{\partial p_e}-\frac{\partial L_g^W}{\partial p_e}-\lambda_e\left(\frac{\partial C_e^W}{\partial p_e}-\frac{\partial Q_e^*}{\partial p_e}\right)=0.$$

Since all goods producers in both countries face price p_e for energy, equation (31) becomes:

$$\frac{\partial u(C_g)}{\partial p_e}+\frac{\partial u^*(C_g^*)}{\partial p_e}+\frac{\partial v(C_e^d)}{\partial p_e}+\frac{\partial v^*(C_e^{d*})}{\partial p_e}=\frac{\partial L_g^W}{\partial p_e}+p_e\frac{\partial C_e^W}{\partial p_e}.$$

Using this result and $\partial L_e^*/\partial p_e = p_e \partial Q_e^*/\partial p_e$, the first-order condition collapses to:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e}-\frac{\partial C_e^W}{\partial p_e}\right)=\varphi^W\frac{\partial Q_e^*}{\partial p_e}.$$

C.1.2 Decentralization

In a market economy we can impose an extraction tax of $t_e = \varphi^W - (\lambda_e - p_e)$ so that the after-tax price, $p_e - t_e = \lambda_e - \varphi^W$, induces the optimal level of extraction in Home. The extraction tax rate is thus:

$$t_e = \varphi^W \frac{\left|\partial C_e^W/\partial p_e\right|}{\partial Q_e^*/\partial p_e + \left|\partial C_e^W/\partial p_e\right|}.$$

C.2 Basic Consumption Policy

For a consumption policy we constrain the planner to choose only: $\left\{k_j^y\right\}$, $\left\{k_j^m\right\}$, $\{y_j\}$, $\{m_j\}$, C_e^d, and p_e. These choices involve both the inner problem and the outer problem.

Table 11: Basic Consumption Policy (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta \left(a_j g(\lambda_e)\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta \left(\tau^* a_j^* g(\lambda_e)\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^* \left(\tau a_j g(p_e)\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^* \left(a_j^* g(p_e)\right)^{-\sigma^*}$	$j > \bar{j}_x$

Thresholds: $F(\bar{j}_m) = 1/\tau^*$ and $F(\bar{j}_x) = \tau$

C.2.1 Inner Problem

We first consider the inner problem (conditions for an individual good j given values for p_e and λ_e). The terms involving Foreign consumption drop out of the inner problem, as they are determined by p_e, leaving:

$$\mathcal{L}_j = u(y_j + m_j) - a_j \left(\frac{1}{f(k_j^y)} + \lambda_e \frac{k_j^y}{f(k_j^y)} \right) y_j - \tau^* a_j^* \left(\frac{1}{f(k_j^m)} + \lambda_e \frac{k_j^m}{f(k_j^m)} \right) m_j.$$

The first order conditions for k_j^y, k_j^m, y_j, and y_j^m will clearly be identical to those for the unilaterally optimal policy. Results from the inner problem, together with market-determined outcomes, are summarized in Table 11.

All producers serving consumers in Home, whether domestic or foreign, use the same energy intensity, but Home uses a different energy intensity for serving Foreign consumers (unlike in the unilaterally optimal case). The import and export thresholds are the same as in the BAU competitive equilibrium.

C.2.2 Outer Problem

The outer problem is to maximize (21) over C_e^d and p_e. Taking into account that C_g (and hence the labor and energy to produce these goods) is determined by the inner problem, while Q_e (like Q_e^*) is now left to depend on the energy price. The first order condition for C_e^d is $v'(C_e^d) = \lambda_e$ which implies C_e^d is characterized by λ_e only and hence $\partial C_e^d / \partial p_e = 0$. The first order condition for p_e is:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} + \frac{v^*(C_e^{d*})}{\partial p_e} - \varphi^W \frac{\partial Q_e^W}{\partial p_e} - \frac{\partial L_e^W}{\partial p_e} - \frac{\partial L_g^x}{\partial p_e} - \frac{\partial L_g^{y*}}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.$$

Since all goods producers serving consumers in Foreign face price p_e for energy, we can exploit (31). Together with $\partial L_e^W/\partial p_e = p_e \partial Q_e^W/\partial p_e$, the first-order condition collapses to:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^W}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e}\right) = \varphi^W \frac{\partial Q_e^W}{\partial p_e}.$$

C.2.3 Decentralization

In a market economy we can impose a consumption tax of $t_c = \lambda_e - p_e$ so that the after-tax price of energy embodied in goods consumed in Home, $p_e + t_c = \lambda_e$, induces the optimal level of demand. The consumption tax rate is thus:

$$t_c = \varphi^W \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e + |\partial C_e^*/\partial p_e|}.$$

C.3 Extraction-Consumption Hybrid Policy

We now augment the basic consumption policy by allowing the planner to choose the amount of energy extraction in Home. To solve this problem we need only tweak the basic consumption case by replacing the competitively determined Q_e with the optimally chosen value. The inner problem is unchanged.

C.3.1 Outer Problem

The outer problem is to maximize (21) over C_e^d, Q_e and p_e. The first-order condition for Q_e is identical to that for the basic extraction policy. The first-order conditions for C_e^d and p_e are identical to that for the basic consumption policy except that $\partial Q_e^*/\partial p_e$ replaces $\partial Q_e^W/\partial p_e$:

$$(\lambda_e - p_e)\left(\frac{\partial Q_e^*}{\partial p_e} - \frac{\partial C_e^*}{\partial p_e}\right) = \varphi^W \frac{\partial Q_e^*}{\partial p_e}.$$

C.3.2 Decentralization

In a market economy, the optimal consumption tax is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e + |\partial C_e^*/\partial p_e|}.$$

Since the optimal nominal extraction tax is $t_e^N = \varphi^W$ (as in the unilaterally optimal policy) the corresponding effective extraction tax is $t_e = \varphi^W - (\lambda_e - p_e)$.

C.4 Basic Production Policy

To capture the essence of a production policy, we constrain the planner to choose only p_e together with the price of energy for Home producers, v_e. Importantly, we do not constrain v_e to equal the shadow value of energy, λ_e.

C.4.1 Inner Problem

The inner problem reduces to competitive behavior conditional on energy costs, p_e and v_e. Goods prices are: $p_j^* = a_j^* g(p_e)$ and $p_j = a_j g(v_e)$ with $p_j^m = \tau^* p_j^*$ and $p_j^x = \tau p_j$.

We get the export margin by equating p_j^x with p_j^* at $j = \bar{j}_x$:

$$F(\bar{j}_x) = \tau \frac{g(v_e)}{g(p_e)}.$$

For any good $j < \bar{j}_x$ the quantity of Home exports demanded by Foreign is:

$$x_j = \eta^* \left(\tau a_j g(v_e)\right)^{-\sigma^*},$$

while $y_j^* = 0$. For any good $j > \bar{j}_x$ the quantity demanded by Foreign from its local producers is:

$$y_j^* = \eta^* \left(a_j^* g(p_e)\right)^{-\sigma^*},$$

while $x_j = 0$.

We get the import margin by equating p_j^m with p_j at $j = \bar{j}_m$:

$$F(\bar{j}_m) = \frac{1}{\tau^*} \frac{g(v_e)}{g(p_e)}.$$

For any good $j > \bar{j}_m$, Home imports:

$$m_j = \eta(\tau^* a_j^* g(p_e))^{-\sigma},$$

Table 12: Basic Production Policy (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta\left(a_j g(v_e)\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta\left(\tau^* a_j^* g(p_e)\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^*\left(\tau a_j g(v_e)\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^*\left(a_j^* g(p_e)\right)^{-\sigma^*}$	$j > \bar{j}_x$

Thresholds: $F(\bar{j}_m) = (1/\tau^*)g(v_e)/g(p_e)$ and $F(\bar{j}_x) = \tau g(v_e)/g(p_e)$.

while $y_j = 0$. For any good $j < \bar{j}_m$ Home purchases:

$$y_j = \eta(a_j g(v_e))^{-\sigma}$$

from local producers, while $m_j = 0$.

Table 12 summarizes these results. The intensive margin of demand for goods produced in Home depends on v_e, the intensive margin for goods produced in Foreign depends on p_e, and the extensive margins of trade depend separately on v_e and p_e.

C.4.2 Outer Problem

The outer problem is to maximize (21) over C_e^d, p_e and v_e. First order condition with respect to C_e^d is $v'(C_e^d) = v_e$ which implies $\partial C_e^d/\partial p_e = 0$ but note here that $\partial C_e^d/\partial v_e \neq 0$. The key results that simplify the first order conditions for p_e and v_e, which follow the same derivation as equations (30) and (31), are:

$$\frac{\partial u(C_g)}{\partial p_e} + \frac{\partial u^*(C_g^*)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial p_e} = \frac{\partial L_g^W}{\partial p_e} + v_e \frac{\partial (C_e^y + C_e^x)}{\partial p_e} + p_e \frac{\partial G_e^*}{\partial p_e} \tag{32}$$

and

$$\frac{\partial u(C_g)}{\partial v_e} + \frac{\partial u^*(C_g^*)}{\partial v_e} + \frac{\partial v(C_e^d)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial v_e} = \frac{\partial L_g^W}{\partial v_e} + v_e \frac{\partial G_e}{\partial v_e} + p_e \frac{\partial G_e^*}{\partial v_e}. \tag{33}$$

The first order condition for p_e is:

$$\frac{\partial u(C_g)}{\partial p_e}+\frac{\partial u^*(C_g^*)}{\partial p_e}+\frac{v^*(C_e^{d*})}{\partial p_e}-\varphi^W\frac{\partial Q_e^W}{\partial p_e}-\frac{\partial L_e^W}{\partial p_e}-\frac{\partial L_g^W}{\partial p_e}-\lambda_e\left(\frac{\partial C_e^W}{\partial p_e}-\frac{\partial Q_e^W}{\partial p_e}\right)=0,$$

or after substituting in $\partial L_e^W/\partial p_e = p_e \partial Q_e^W/\partial p_e$ and rearranging:

$$\left(\lambda_e-\varphi^W-p_e\right)\frac{\partial Q_e^W}{\partial p_e}=-\frac{\partial u(C_g)}{\partial p_e}-\frac{\partial u^*(C_g^*)}{\partial p_e}-\frac{v^*(C_e^{d*})}{\partial p_e}+\frac{\partial L_g^W}{\partial p_e}+\lambda_e\frac{\partial C_e^W}{\partial p_e}.$$

Substituting in (32) gives:

$$\left(\lambda_e-\varphi^W-p_e\right)\frac{\partial Q_e^W}{\partial p_e}=(\lambda_e-v_e)\frac{\partial(C_e^y+C_e^x)}{\partial p_e}+(\lambda_e-p_e)\frac{\partial G_e^*}{\partial p_e}. \tag{34}$$

The first order condition for v_e is:

$$\frac{\partial u(C_g)}{\partial v_e}+\frac{\partial u^*(C_g^*)}{\partial v_e}+\frac{\partial v(C_e^d)}{\partial v_e}+\frac{\partial v^*(C_e^{d*})}{\partial v_e}-\frac{\partial L_g^W}{\partial v_e}-\lambda_e\frac{\partial C_e^W}{\partial v_e}=0.$$

Substituting in (33) we have:

$$(\lambda_e-v_e)\frac{\partial G_e}{\partial v_e}=(p_e-\lambda_e)\frac{\partial G_e^*}{\partial v_e}.$$

The optimal v_e balances the two wedges, $\lambda_e - v_e$ and $\lambda_e - p_e$, based on the extent of leakage.

We define production leakage by:

$$\Lambda=\frac{-\partial G_e^*/\partial v_e}{\partial G_e/\partial v_e}.$$

Due to a rise in v_e, production leakage is the ratio of the increase in Foreign use of energy in goods production relative to the decline in Home use of energy. We say Foreign use of energy in goods production because $\partial v^*(C_e^{d*})/\partial v_e = p_e \implies \partial C_e^{d*}/\partial v_e = 0$.

In terms of production leakage, the first-order condition for v_e implies:

$$\frac{\lambda_e - v_e}{\lambda_e - p_e} = \Lambda.$$

Substituting into (34), noting that $\partial(C_e^y + C_e^x)/\partial p_e = \partial C_e^W/\partial p_e - \partial G_e^*/\partial p_e$, yields:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - (1-\Lambda)\partial G_e^*/\partial p_e - \Lambda \partial C_e^W/\partial p_e}.$$

Since $\Lambda \geq 0$ it's clear that $\lambda_e \geq p_e$ and hence $\lambda_e \geq v_e$ as well.

C.4.3 Decentralization

In a market economy we can impose a production tax of $t_p = v_e - p_e$ so that the after-tax price of energy used to produce goods in Home, $p_e + t_p = v_e$, induces the optimal energy intensity. The production tax rate is thus:

$$t_p \quad = \quad v_e - p_e = \varphi^W \frac{(1-\Lambda)\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e + (1-\Lambda)\left|\partial G_e^*/\partial p_e\right| + \Lambda \left|\partial C_e^W/\partial p_e\right|}.$$

In the case of no trade in goods there is no leakage and the basic production tax becomes the same as the basic consumption tax.

C.5 Extraction-Production Hybrid Policy

Suppose we augment the basic production policy by allowing the planner to also choose Q_e. The inner problem is identical to the basic production policy.

C.5.1 Outer Problem

The outer problem is to maximize (21) over Q_e, C_e^d, p_e, and v_e. The first-order condition for Q_e is identical to that for the basic extraction policy. The first-order conditions for C_e^d and v_e are identical to the basic production policy.

The first-order conditions for p_e is identical to that for the basic consumption policy except that $\partial Q_e^*/\partial p_e$ replaces $\partial Q_e^W/\partial p_e$. Hence:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - (1-\Lambda)\partial G_e^*/\partial p_e - \Lambda \partial C_e^W/\partial p_e}.$$

C.5.2 Decentralization

In a market economy, the optimal production tax rate is:

$$t_p = v_e - p_e = \varphi^W \frac{(1-\Lambda)\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e + (1-\Lambda)\left|\partial G_e^*/\partial p_e\right| + \Lambda \left|\partial C_e^W/\partial p_e\right|}.$$

From the first order condition for Q_e we know that the after-tax price received by extractors must satisfy:

$$v_e - t_e^N = p_e - t_e = \lambda_e - \varphi^W.$$

The optimal nominal extraction tax is thus:

$$t_e^N = \varphi^W - (\lambda_e - v_e) = \varphi^W - \frac{\Lambda}{1-\Lambda} t_p,$$

while the corresponding effective extraction tax is:

$$t_e = t_e^N - t_p = \varphi^W - \frac{t_p}{1-\Lambda}.$$

C.6 Production-Consumption Hybrid Policy

We now augment the basic consumption policy by allowing the planner to choose v_e, which becomes the cost of energy for producing Home's exports. (The consumption policy determines the cost of producing in Home for Home consumers.) The choice of v_e therefore only alters the set of goods Home exports and the quantity demanded of those goods. We summarize the results of the inner problem for a particular good j in Table 13.

Table 13: Production-Consumption Hybrid Policy (Good-j Outcomes)

	Home		Foreign	
Home	$y_j = \eta\left(a_j g(\lambda_e)\right)^{-\sigma}$	$j < \bar{j}_m$	$m_j = \eta\left(\tau^* a_j^* g(\lambda_e)\right)^{-\sigma}$	$j > \bar{j}_m$
Foreign	$x_j = \eta^*\left(\tau a_j g(v_e)\right)^{-\sigma^*}$	$j < \bar{j}_x$	$y_j^* = \eta^*\left(a_j^* g(p_e)\right)^{-\sigma^*}$	$j > \bar{j}_x$

Thresholds: $F(\bar{j}_m) = 1/\tau^*$ and $F(\bar{j}_x) = \tau g(v_e)/g(p_e)$

C.6.1 Outer Problem

The outer problem is to maximize (21) over C_e^d, p_e and v_e. The first order condition for C_e^d is $\partial v(C_e^d)/\partial C_e^d = \lambda_e \implies \partial C_e^d/\partial p_e = \partial C_e^d/\partial v_e = 0$, since C_e^d is completely characterized by λ_e. The first order condition for p_e is:

$$\frac{\partial u^*(C_g^*)}{\partial p_e} + \frac{v^*(C_e^d)}{\partial p_e} - \varphi^W \frac{\partial Q_e^W}{\partial p_e} - \frac{\partial L_e^W}{\partial p_e} - \frac{\partial L_g^x}{\partial p_e} - \frac{\partial L_g^{y*}}{\partial p_e} - \lambda_e \left(\frac{\partial C_e^*}{\partial p_e} - \frac{\partial Q_e^W}{\partial p_e} \right) = 0.$$

We can simplify the first-order condition using an analog of equation (31):

$$\frac{\partial u^*(C_g^*)}{\partial p_e} + \frac{\partial v^*(C_e^{d*})}{\partial p_e} = \frac{\partial L_g^x}{\partial p_e} + \frac{\partial L_g^{y*}}{\partial p_e} + v_e \frac{\partial C_e^x}{\partial p_e} + p_e \left(\frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial C_e^{d*}}{\partial p_e} \right).$$

Substituting in this result, together with $\partial L_e^W/\partial p_e = p_e \partial Q_e^W/\partial p_e$, we get:

$$\left(\lambda_e - \varphi^W - p_e\right) \frac{\partial Q_e^W}{\partial p_e} = (\lambda_e - v_e) \frac{\partial C_e^x}{\partial p_e} + (\lambda_e - p_e) \left(\frac{\partial C_e^{y*}}{\partial p_e} + \frac{\partial C_e^{d*}}{\partial p_e} \right).$$

The first order condition for v_e is:

$$\frac{\partial u^*(C_g^*)}{\partial v_e} - \frac{\partial L_g^x}{\partial v_e} - \frac{\partial L_g^{y*}}{\partial v_e} - \lambda_e \frac{\partial C_e^*}{\partial v_e} = 0.$$

We can simplify it by substituting in the analog of equation (33):

$$\frac{\partial u^*(C_g^*)}{\partial v_e} + \frac{\partial v^*(C_e^{d*})}{\partial v_e} = \frac{\partial L_g^x}{\partial v_e} + \frac{\partial L_g^{y*}}{\partial v_e} + v_e \frac{\partial C_e^x}{\partial v_e} + p_e \left(\frac{\partial C_e^{y*}}{\partial v_e} + \frac{\partial C_e^{d*}}{\partial v_e} \right).$$

The result is:

$$(\lambda_e - v_e)\frac{\partial C_e^x}{\partial v_e} = (p_e - \lambda_e)\frac{\partial C_e^{y*}}{\partial v_e}.$$

because $\partial v^*(C_e^{d*})/\partial p_e = p_e \implies \partial C_e^{d*}/\partial v_e = 0$. The optimal v_e balances the two wedges, $\lambda_e - v_e$ and $\lambda_e - p_e$, based on a measure of leakage that only involves production for Foreign consumers.

We define foreign leakage by:

$$\Lambda^* = \frac{-\partial C_e^{y*}/\partial v_e}{\partial C_e^x/\partial v_e}.$$

Due to a rise in v_e, foreign leakage is the ratio of the increase in Foreign use of energy to serve its own customers relative to the decline in Home use of energy to serve Foreign customers.

Using our expression for foreign leakage, the first-order condition for v_e becomes:

$$\frac{\lambda_e - v_e}{\lambda_e - p_e} = \Lambda^*.$$

Substituting into the first-order condition for p_e, noting that $\partial C_e^x/\partial p_e = \partial C_e^*/\partial p_e - \partial C_e^{z*}/\partial p_e$, yields:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e - (1-\Lambda^*)\partial C_e^{z*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}.$$

Since $\Lambda^* \geq 0$ it's clear that $\lambda_e \geq p_e$ and hence $\lambda_e \geq v_e$ as well.

C.6.2 Decentralization

In a market economy the optimal consumption tax is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\partial Q_e^W/\partial p_e}{\partial Q_e^W/\partial p_e + (1-\Lambda^*)\left|\partial C_e^{z*}/\partial p_e\right| + \Lambda^* \left|\partial C_e^*/\partial p_e\right|}.$$

The optimal production tax on Home's exports is:

$$t_p = v_e - p_e = (1-\Lambda^*)t_c.$$

C.7 Extraction-Production-Consumption Hybrid Policy

The final case augments the production-consumption policy to allow the planner to choose Q_e. Many of the results for the production-consumption case carry over, including those for individual goods shown in Table 13.

C.7.1 Outer Problem

The outer problem is to maximize (21) over Q_e, p_e, and v_e. The first-order condition for Q_e is identical to that for the basic extraction policy. The first order condition for p_e is the same as for the production-consumption case, except with $\partial Q_e^*/\partial p_e$ in place of $\partial Q_e^W/\partial p_e$. The first order condition for v_e is unchanged from the production-consumption case. Thus, we have:

$$\lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e - (1-\Lambda^*)\partial C_e^{z*}/\partial p_e - \Lambda^* \partial C_e^*/\partial p_e}.$$

C.7.2 Decentralization

In a market economy the optimal nominal extraction tax is $t_e^N = \varphi^W$, while the effective rate is:

$$t_e = \varphi^W - (\lambda_e - p_e) = \varphi^W \frac{(1-\Lambda^*)\left|\partial C_e^{z*}/\partial p_e\right| + \Lambda^* \left|\partial C_e^*/\partial p_e\right|}{\partial Q_e^*/\partial p_e + (1-\Lambda^*)\left|\partial C_e^{z*}/\partial p_e\right| + \Lambda^* \left|\partial C_e^*/\partial p_e\right|}.$$

The optimal consumption tax, applying to Home consumption of both domestically produced and imported goods is:

$$t_c = \lambda_e - p_e = \varphi^W \frac{\partial Q_e^*/\partial p_e}{\partial Q_e^*/\partial p_e + (1-\Lambda^*)\left|\partial C_e^{z*}/\partial p_e\right| + \Lambda^* \left|\partial C_e^*/\partial p_e\right|}.$$

The optimal production tax on Home exports of goods is:

$$t_p = v_e - p_e = (1-\Lambda^*)t_c.$$

D Solutions for Quantitative Illustration

Here we provide a list of equations for the parameterized version of the model that we use for the quantitative results in Section 6 of the paper. For each outcome, we start with the BAU competitive equilibrium value that we calibrate the model to. We then show how to express the optimal outcomes in terms of these BAU outcomes. To distinguished the two, we express outcomes under the optimal policy as functions of p_e and t_b (since $t_e^N = \varphi^W$ under the policy we don't need to include it in the notation). We eliminate these arguments to represent BAU outcomes. Thus for an outcome x we denote the optimal outcome as $x(p_e, t_b)$ (sometimes x' for short) and the BAU outcome as simply x. We impose the restrictions from Section 6.1.1.

D.1 Expressions to Compute the Optimal Policy

To avoid repetition, we state general solutions to integrals of interest

$$\int_{\bar{j}_1}^{\bar{j}_2} a_j^{1-\sigma} dj = \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta}(\bar{j}_2^{1+(1-\sigma)/\theta} - \bar{j}_1^{1+(1-\sigma)/\theta})$$

$$\int_{\bar{j}_1}^{\bar{j}_2} (a_j^*)^{1-\sigma^*} dj = \frac{(A^*)^{-(1-\sigma^*)/\theta}}{1+(1-\sigma^*)/\theta}((1-\bar{j}_1)^{1+(1-\sigma^*)/\theta} - (1-\bar{j}_2)^{1+(1-\sigma^*)/\theta}),$$

For example,

$$\begin{aligned} C_e^y(p_e, t_b) &= \int_0^{\bar{j}'_m} e_j(k) y_j dj = \eta g(p_e + t_b)^{-\sigma} g'(p_e + t_b) \int_0^{\bar{j}'_m} a_j^{1-\sigma} dj \\ &= D(p_e + t_b) \int_0^{\bar{j}'_m} a_j^{1-\sigma} dj = D(p_e + t_b)(\bar{j}'_m)^{1+(1-\sigma)/\theta} \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta}, \end{aligned}$$

so in BAU,

$$C_e^y = D(1)(\bar{j}_m)^{1+(1-\sigma)/\theta} \frac{A^{-(1-\sigma)/\theta}}{1+(1-\sigma)/\theta}.$$

Expressions for all other variables under the optimal policy are in table 14.

Table 14: Expressions for Variables under Optimal Policy

Under BAU	Under Unilateral Optimal in terms of BAU
$Q_e = E$	$Q'_e = \max\left\{\left(p_e + t_b - \varphi^W\right)^{\epsilon_S} Q_e, 0\right\}$
$Q^*_e = E^*$	$Q^{*\prime}_e = (p_e)^{\epsilon^*_S} Q^*_e$
$\bar{j}_m = \dfrac{C^y_e}{C_e}$	$\bar{j}'_m = \dfrac{C^y_e}{C_e}$
$\bar{j}_x = \dfrac{C^x_e}{C_e}$	$\bar{j}'_x = \dfrac{g(p_e+t_b)^{-\theta} C^x_e}{g(p_e+t_b)^{-\theta} C^x_e + (g(p_e) + t_b g'(p_e))^{-\theta} C^{y*}_e}$
$j_0 = \dfrac{C^x_e}{C_e}$	$\bar{j}'_0 = \dfrac{g(p_e+t_b)^{-\theta} C^x_e}{g(p_e+t_b)^{-\theta} C^x_e + g(p_e)^{-\theta} C^{y*}_e}$
$C^d_e = \eta_e$	$C^{d\prime}_e = \dfrac{C^d_e}{(p_e+t_b)^{\sigma_e}}$
$C^{d*}_e = \eta^*_e$	$C^{d*\prime}_e = \dfrac{C^{d*}_e}{p_e^{\sigma^*_e}}$
$C^y_e = D(1)(\bar{j}_m)^{1-\tilde{\sigma}} \dfrac{A^{\tilde{\sigma}}}{1-\tilde{\sigma}}$	$C^{y\prime}_e = \dfrac{D(p_e+t_b)}{D(1)} C^y_e$
$C^m_e = D(1)(1-\bar{j}_m)^{1-\tilde{\sigma}} \dfrac{(A^*)^{\tilde{\sigma}}}{1-\tilde{\sigma}}$	$C^{m\prime}_e = \dfrac{D(p_e+t_b)}{D(1)} C^m_e$
$C^{y*}_e = D^*(1)(1-\bar{j}_x)^{1-\tilde{\sigma^*}} \dfrac{(A^*)^{\tilde{\sigma^*}}}{1-\tilde{\sigma^*}}$	$C^{y*\prime}_e = \dfrac{D^*(p_e)}{D^*(1)} \left(\dfrac{1-\bar{j}'_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma^*}} C^{y*}_e$
$C^x_e = D^*(1)(\bar{j}_x)^{1-\tilde{\sigma^*}} \dfrac{(A)^{\tilde{\sigma^*}}}{1-\tilde{\sigma^*}}$	$C^{x\prime}_e = C^{x,1\prime}_e + C^{x,2\prime}_e$
$C^{x,1}_e = C^x_e$	$C^{x,1\prime}_e = \dfrac{D^*(p_e+t_b)}{D^*(1)} \left(\dfrac{j'_0}{j_0}\right)^{1-\tilde{\sigma^*}} C^x_e$
$C^{x,2}_e = 0$	$C^{x,2\prime}_e = \left(1-\tilde{\sigma^*}\right) \left(\dfrac{1-\bar{j}_x}{\bar{j}_x}\right)^{\sigma^*/\theta} \dfrac{g(p_e)^{-\sigma^*} g'(p_e+t_b)}{D^*(1)} \dfrac{\left(B\left(\bar{j}'_x, \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right) - B\left(j'_0, \frac{1+\theta}{\theta}, \frac{\theta-\sigma^*}{\theta}\right)\right)}{\bar{j}_x^{1-\sigma^*}} C^x_e$
$S = 0$	$S' = \dfrac{g(p_e+t_b)}{g'(p_e+t_b)} C^{x,2'}_e - \dfrac{g(p_e)^{1-\sigma^*}}{g(1)^{1-\sigma^*}} \dfrac{\left((1-j'_0)^{1-\tilde{\sigma^*}} - (1-\bar{j}'_x)^{1-\tilde{\sigma^*}}\right)}{\bar{j}_x\,(1-\bar{j}_x)^{-\tilde{\sigma^*}}} \dfrac{g(1)}{g'(1)} C^x_e$

$\tilde{\sigma} = (\sigma - 1)/\theta, \tilde{\sigma^*} = (\sigma^* - 1)/\theta$

Having solved for the optimal border adjustment and the corresponding change in the global energy price we can compute all other outcomes as well. A key outcome is Home's welfare in moving to the optimal unilateral policy from the BAU competitive equilibrium.

Home's Utility (dropping constants) can be expressed as:

1. Under BAU:

$$\begin{aligned} U &= u(C_g) + u^*(C_g^*) + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W(Q_e + Q_e^*) - L_g^W - L_e^W \\ &= \frac{\sigma}{\sigma-1}V_g + \frac{\sigma^*}{\sigma^*-1}V_g^* + v(C_e^d) + v^*(C_e^{d*}) - \varphi^W Q_e^W - L_g^W - L_e^W \end{aligned}$$

   where $V_g = u'(C_g)C_g, V_g^* = u^{*\prime}(C_g^*)C_g^*$ and $u(C_g) = \frac{\sigma}{\sigma-1}(V_g - 1)$. We use V_g instead of $u(C_g)$ both because it can be interpreted as spending on goods and because we use it in our measure of welfare.

2. The change in moving to the optimal unilateral policy from the BAU competitive equilibrium:

$$\begin{aligned} U(p_e, t_b) - U = &\frac{\sigma}{\sigma-1}(V_g(p_e,t_b) - V_g) + \frac{\sigma^*}{\sigma^*-1}(V_g^*(p_e,t_b) - V_g^*) \\ &+ (v(C_e^d(p_e,t_b)) - v(C_e^d) + (v^*(C_e^{d*}(p_e.t_b)) - v^*(C_e^{d*}) \\ &- \varphi^W(Q_e^W(p_e,t_b) - Q_e^W) - (L_g^W(p_e,t_b) - L_g^W) - (L_e^W(p_e,t_b) - L_e^W) \end{aligned}$$

   Our preferred measure of welfare is normalized by BAU spending on goods:

$$W = \frac{U(p_e,t_b) - U}{V_g}$$

We summarize the terms that enter welfare computation in table 16.

As an example, we show how much Home spends producing for itself.

$$\begin{aligned} V_g^y(p_e,t_b) &= \int_0^{\bar{j}'_m} p_j y_j dj = \eta g(p_e+t_b)^{1-\sigma}\int_0^{\bar{j}'_m} a_j^{1-\sigma} dj \\ &= \frac{g(p_e+t_b)}{g'(p_e+t_b)}\eta g(p_e+t_b)^{-\sigma} g'(p_e+t_b)\int_0^{\bar{j}'_m} a_j^{1-\sigma} dj = \frac{g(p_e+t_b)}{g'(p_e+t_b)} C_e^{y\prime}. \end{aligned}$$

Table 16: Expressions for Variables in Welfare Calculation

Change from BAU	Change when $\sigma = 1$ or $\sigma^* = 1$
$L'_e - L_e = \frac{\epsilon_S}{\epsilon_S + 1}((\max\{p_e + t_b - \varphi^W, 0\})^{\epsilon_S + 1} - 1)Q_e$	
$L^{*\prime}_e - L^*_e = \frac{\epsilon^*_S}{\epsilon^*_S + 1}(p_e^{\epsilon^*_S + 1} - 1)Q^*_e$	
$L'_g - L_g = \frac{C^{y\prime}_e + C^{x\prime}_e}{k(p_e + t_b)} - \frac{C^y_e + C^x_e}{k(1)}$	
$L^{*\prime}_g - L^*_g = \frac{C^{m\prime}_e}{k(p_e + t_b)} + \frac{C^{y*\prime}_e}{k(p_e)} - \frac{C^m_e + C^{y*}_e}{k(1)}$	
$v(C^{d\prime}_e) - v(C^d_e) = \frac{\sigma_e}{\sigma_e - 1}\left(C^{d\prime}_e \left(\frac{C^d_e}{C^{d\prime}_e}\right)^{1/\sigma_e} - C^d_e\right)$	$-C^d_e \sigma_e \log(p_e + t_b)$
$v^*(C^{d*\prime}_e) - v^*(C^{d*}_e) = \frac{\sigma^*_e}{\sigma^*_e - 1}\left(C^{d*\prime}_e \left(\frac{C^{d*}_e}{C^{d*\prime}_e}\right)^{1/\sigma^*_e} - C^{d*}_e\right)$	$-C^{d*}_e \sigma^*_e \log(p_e)$
$\frac{V'_g - V_g}{(\sigma - 1)/\sigma} = \frac{\sigma}{\sigma - 1}\left(\frac{g(p_e + t_b)^{1-\sigma}}{g(1)^{1-\sigma}} - 1\right)V_g$	$-\ln\left(\frac{g(p_e + t_b)}{g(1)}\right)V_g$
$\frac{V^{*\prime}_g - V^*_g}{(\sigma^* - 1)/\sigma^*} = \frac{\sigma^*}{\sigma^* - 1}\frac{g(p_e)^{1-\sigma^*}}{g(1)^{1-\sigma^*}}\frac{(1 - j'_0)^{1-\tilde{\sigma^*}}}{(1 - \bar{j}_x)^{-\tilde{\sigma^*}}}V^*_g$ $+\frac{\sigma^*}{\sigma^* - 1}\left(\frac{g(p_e + t_b)^{1-\sigma^*}}{g(1)^{1-\sigma^*}}\frac{(j'_0)^{1-\tilde{\sigma^*}}}{(\bar{j}_x)^{-\tilde{\sigma^*}}} - 1\right)V^*_g$	$-\left[\ln\left(\frac{g(p_e)}{g(1)}\right) + \frac{1}{\theta}\ln\left(\frac{1 - j'_0}{1 - j_0}\right)\right]V^*_g$

Under BAU

$$V^y_g = \frac{g(1)}{g'(1)}C^y_e.$$

Expressed in terms of BAU

$$V^y_g(p_e, t_b) = \frac{g(p_e + t_b)^{1-\sigma}}{g(1)^{1-\sigma}}V^y_g.$$

Total Home spending on goods is then

$$V_g(p_e, t_b) = V^{y\prime}_g + V^{m\prime}_g = \frac{g(p_e + t_b)^{1-\sigma}}{g(1)^{1-\sigma}}V_g.$$

We can apply L'Hopital's rule when $\sigma = 1$

$$\lim_{\sigma\to 1} \frac{\left(\frac{g(p_e+t_b)^{1-\sigma}}{g(1)^{1-\sigma}} - 1\right)}{(\sigma-1)/\sigma} V_g = -\ln\left(\frac{g(p_e+t_b)}{g(1)}\right) V_g.$$

Total Foreign spending on goods is

$$\begin{aligned} V_g^*(p_e,t_b) &= V_g^{x\prime} + V_g^{y*\prime} \\ &= \frac{g(p_e)^{1-\sigma^*}}{g(1)^{1-\sigma^*}} \left(\frac{1-j_0'}{1-\bar{j}_x}\right)^{1-\tilde{\sigma^*}} V_g^{y*} + \frac{g(v_e)^{1-\sigma^*}}{g(1)^{1-\sigma^*}} \left(\frac{j_0'}{\bar{j}_x}\right)^{1-\tilde{\sigma^*}} V_g^{x}. \end{aligned}$$

Given that

$$\bar{j}_x = C_e^x / C_e^* = V_g^x / V_g^* \implies 1 - \bar{j}_x = V_g^{y*}/V_g^*,$$

we can rewrite it as

$$V_g^*(p_e,t_b) = \left[\left(\frac{g(p_e)}{g(1)}\right)^{1-\sigma^*} \frac{(1-j_0')^{1-\tilde{\sigma^*}}}{(1-\bar{j}_x)^{-\tilde{\sigma^*}}} + \left(\frac{g(p_e+t_b)}{g(1)}\right)^{1-\sigma^*} \frac{(j_0')^{1-\tilde{\sigma^*}}}{(\bar{j}_x)^{-\tilde{\sigma^*}}}\right] V_g^*.$$

When $\sigma^* = 1$ the term that enters welfare change becomes

$$\begin{aligned} &\lim_{\sigma^*\to 1} \frac{\sigma^*}{\sigma^*-1}(V_g^*(p_e,t_b) - V_g^*) \\ &= -(1-j_0')\ln\left(\frac{g(p_e)(1-j_0')^{1/\theta}}{g(1)(1-\bar{j}_x)^{1/\theta}}\right) V_g^* - j_0'\ln\left(\frac{g(p_e+t_b)(j_0')^{1/\theta}}{g(1)(\bar{j}_x)^{1/\theta}}\right) V_g^* \\ &= -\ln\left(\frac{g(p_e)(1-j_0')^{1/\theta}}{g(1)(1-\bar{j}_x)^{1/\theta}}\right) V_g^* - j_0'\ln\left(\frac{g(1)(1-\bar{j}_x)^{1/\theta} g(p_e+t_b)(j_0')^{1/\theta}}{g(p_e)(1-j_0')^{1/\theta} g(1)(\bar{j}_x)^{1/\theta}}\right) V_g^* \\ &= -\left[\ln\left(\frac{g(p_e)}{g(1)}\right) + \frac{1}{\theta}\ln\left(\frac{1-j_0'}{1-j_0}\right)\right] V_g^*. \end{aligned}$$

The second term on the third line disappears because

$$\frac{j_0'(1-j_0)}{j_0(1-j_0')} = \frac{C_e^{y*}}{C_e^x} \frac{g(p_e+t_b)^{-\theta} C_e^x}{g(p_e)^{-\theta} C_e^{y*}} = \left(\frac{g(p_e)}{g(p_e+t_b)}\right)^{\theta}.$$

D.2 Expressions for Constrained-Optimal Policies

Many of the expressions needed for the constrained optimal policies are closely related to those for the unilateral optimal policy listed above. For policies involving a production tax, however, we need to incorporate the cost v_e of energy in Home.

The derivatives with respect to v_e are used for leakage computations for policies with a production tax. The derivatives with respect to p_e appear in equilibrium conditions for all constrained policies. Note, however, that for policies without a production tax, the partial derivatives for import/export margins are 0. All formulas for variables and derivatives are shown in tables 17 and 18, respectively.

We compute the partial derivative for $C_e^{y*\prime}$ as an example. Taking the derivative with respect to v_e yields

$$\begin{aligned} -\frac{\partial C_e^{y*\prime}}{\partial v_e} &= -\frac{D^*(p_e)}{D(1)}\left(\frac{1-\bar{j}_x'}{1-\bar{j}_x}\right)^{1-\tilde{\sigma^*}} \frac{1-\tilde{\sigma^*}}{1-\bar{j}_x'}\left(-\frac{\partial \bar{j}_x'}{\partial v_e}\right) \\ &= \frac{1-\tilde{\sigma^*}}{(1-\bar{j}_x')}\frac{\partial \bar{j}_x'}{\partial v_e} C_e^{y^*\prime}. \end{aligned}$$

Now with p_e

$$\begin{aligned} \frac{\partial C_e^{y^*}}{\partial p_e} &= \frac{D^{*\prime}(p_e)}{D(1)}\left(\frac{1-\bar{j}_x'}{1-\bar{j}_x}\right)^{1-\tilde{\sigma^*}} C_e^{y*} + \frac{1-\tilde{\sigma^*}}{1-\bar{j}_x'}\frac{D^*(p_e)}{D(1)}\left(\frac{1-\bar{j}_x'}{1-\bar{j}_x}\right)^{1-\tilde{\sigma^*}}\left(-\frac{\partial \bar{j}_x'}{\partial p_e}\right) C_e^{y*} \\ &= \frac{D^{*\prime}(p_e)}{D^*(p_e)} C_e^{y\prime} - \frac{1-\tilde{\sigma^*}}{1-\bar{j}_x'}\frac{\partial \bar{j}_x'}{\partial p_e} C_e^{y^*\prime}. \end{aligned}$$

The partial derivative of the export margin is

$$\begin{aligned} \frac{\partial \bar{j}_x{}'}{\partial p_e} &= -\frac{g(v_e)^{-\theta} C_e^x}{(g(v_e)^{-\theta} C_e^x + g(p_e)^{-\theta} C_e^{y*})^2}(-\theta g(p_e)^{-\theta-1} g'(p_e) C_e^{y^*}) \\ &= \frac{g(v_e)^{-\theta} C_e^x}{g(v_e)^{-\theta} C_e^x + g(p_e)^{-\theta} C_e^{y*}} \frac{g(p_e)^{-\theta} C_e^{y*}}{g(v_e)^{-\theta} C_e^x + g(p_e)^{-\theta} C_e^{y*}}\left(\theta \frac{g'(p_e)}{g(p_e)}\right) \\ &= \theta \frac{g'(p_e)}{g(p_e)} \bar{j}_x{}' \left(1-\bar{j}_x{}'\right). \end{aligned}$$

Table 17: Expressions for Variables under Constrained Policies

With Production Tax		Without Production Tax
$\bar{j}'_x = \dfrac{g(v_e)^{-\theta} C_e^x}{g(v_e)^{-\theta} C_e^x + g(p_e)^{-\theta} C_e^{y*}}$		$\dfrac{C_e^x}{C_e^*}$
$\bar{j}'_m = \dfrac{g(v_e)^{-\theta} C_e^y}{g(v_e)^{-\theta} C_e^y + g(p_e)^{-\theta} C_e^m}$	P, EP	
$\bar{j}'_m = \dfrac{C_e^y}{C_e}$	PC, EPC	
$C_e^{d\prime} = \dfrac{C_e^d}{v_e^{\sigma_e}}$		
$C_e^{d*\prime} = \dfrac{C_e^{d*}}{p_e^{\sigma_e^*}}$		
$C_e^{y\prime} = \dfrac{D(v_e)}{D(1)} \left(\dfrac{\bar{j}'_m}{\bar{j}_m}\right)^{1-\tilde{\sigma}} C_e^y$		
$C_e^{m\prime} = \dfrac{D(p_e)}{D(1)} \left(\dfrac{1-\bar{j}'_m}{1-\bar{j}_m}\right)^{1-\tilde{\sigma}} C_e^m$		
$C_e^{x\prime} = \dfrac{D^*(v_e)}{D^*(1)} \left(\dfrac{\bar{j}'_x}{\bar{j}_x}\right)^{1-\tilde{\sigma^*}} C_e^x$		$\dfrac{D^*(p_e+t_b)}{D^*(1)} \left(\dfrac{\bar{j}'_x}{\bar{j}_x}\right)^{1-\tilde{\sigma^*}} C_e^x$
$C_e^{y*\prime} = \dfrac{D^*(p_e)}{D^*(1)} \left(\dfrac{1-\bar{j}'_x}{1-\bar{j}_x}\right)^{1-\tilde{\sigma^*}} C_e^{y*}$		
$V'_g = \bar{j}'_m \left(\dfrac{g(p_e)}{g(1)} \left(\dfrac{\bar{j}'_m}{\bar{j}_m}\right)^{1/\theta}\right)^{1-\sigma} V_g + (1-\bar{j}'_m)\left(\dfrac{g(v_e)}{g(1)} \left(\dfrac{1-\bar{j}'_m}{1-\bar{j}_m}\right)^{1/\theta}\right)^{1-\sigma} V_g$	P, EP	$\dfrac{g(p_e+t_b)^{1-\sigma}}{g(1)^{1-\sigma}} V_g$
$V'_g = \dfrac{g(p_e+t_b)^{1-\sigma}}{g(1)^{1-\sigma}} V_g$	PC, EPC	
$V_g^{*\prime} = \bar{j}'_x \left(\dfrac{g(v_e)}{g(1)} \left(\dfrac{\bar{j}'_x}{\bar{j}_x}\right)^{1/\theta}\right)^{1-\sigma} V_g^* + (1-\bar{j}'_x)\left(\dfrac{g(p_e)}{g(1)} \left(\dfrac{1-\bar{j}'_x}{1-\bar{j}_x}\right)^{1/\theta}\right)^{1-\sigma} V_g^*$		$\dfrac{g(p_e)^{1-\sigma^*}}{g(1)^{1-\sigma^*}} V_g^*$
$\lim\limits_{\sigma\to 1} \dfrac{V'_g - V_g}{(\sigma-1)/\sigma} = -\left[\ln\left(\dfrac{g(p_e)}{g(1)}\right) + \dfrac{1}{\theta}\ln\left(\dfrac{1-\bar{j}'_m}{1-\bar{j}_m}\right)\right] V_g$		
$\lim\limits_{\sigma^*\to 1} \dfrac{V_g^{*\prime} - V_g^*}{(\sigma^*-1)/\sigma^*} = -\left[\ln\left(\dfrac{g(p_e)}{g(1)}\right) + \dfrac{1}{\theta}\ln\left(\dfrac{1-\bar{j}'_x}{1-\bar{j}_x}\right)\right] V_g^*$		$-\ln\left(\dfrac{g(p_e)}{g(1)}\right) V_g^*$

Blank implies it is identical to the expression under the unilateral optimal policy. P = Pure Production, EP = Extraction-Production, PC = Production-Consumption, EPC = Extraction-Production-Consumption

Table 18: Partial Derivatives of Interest for Policies Involving a Production Tax

Derivatives wrt v_e	Derivatives wrt p_e
$-\frac{\partial C_e^{y*\prime}}{\partial v_e} = h_2^*(v_e, \bar{j}_x')C_e^{y*\prime}$	$\frac{\partial C_e^{y*\prime}}{\partial p_e} = \left(\frac{D^{*\prime}(p_e)}{D^*(p_e)} - h_2^*(p_e, \bar{j}_x')\right) C_e^{y*\prime}$
$\frac{\partial C_e^{x\prime}}{\partial v_e} = \left(\frac{D^{*\prime}(v_e)}{D^*(v_e)} + h_1^*(v_e, \bar{j}_x')\right) C_e^{x\prime}$	$\frac{\partial C_e^{x\prime}}{\partial p_e} = h_1^*(p_e, \bar{j}_x')C_e^{x\prime}$
$-\frac{\partial C_e^{m\prime}}{\partial v_e} = h_2(v_e, \bar{j}_m')C_e^{m\prime}$	$\frac{\partial C_e^{m\prime}}{\partial p_e} = \left(\frac{D'(p_e)}{D(p_e)} - h_2(p_e, \bar{j}_m')\right) C_e^{m\prime}$
$\frac{\partial C_e^{y\prime}}{\partial v_e} = \left(\frac{D'(v_e)}{D(v_e)} + h_1(v_e, \bar{j}_m')\right) C_e^{y\prime}$	$\frac{\partial C_e^{y\prime}}{\partial p_e} = h_1(p_e, \bar{j}_m')C_e^{y\prime}$
$\frac{\partial \bar{j}}{\partial v_e} = -h_3(v_e, \bar{j})$	$\frac{\partial \bar{j}}{\partial p_e} = h_3(p_e, \bar{j})$

$h_1(p, \bar{j}) = \frac{1-\tilde{\sigma}}{\bar{j}}\frac{\partial \bar{j}}{\partial p}$, $h_2(p, \bar{j}) = \frac{1-\tilde{\sigma}}{1-\bar{j}}\frac{\partial \bar{j}}{\partial p}$ with $\tilde{\sigma^*}$ for h_1^* and h_2^*.
$h_3(x, \bar{j}) = \theta\frac{g'(x)}{g(x)}\bar{j}(1-\bar{j})$ for $\bar{j} \in \{\bar{j}_x', \bar{j}_m'\}$.

E Data and Calibration

E.1 Calibration

For our quantitative analysis we calibrate the model to fossil fuel extraction and the energy embodied in trade between the region that, in our model, will enact a carbon policy (Home) and the region that will remain with business as usual (Foreign). Our common unit for energy is gigatonnes of CO_2, based on the quantity released by its combustion.

We consider several scenarios for the regions representing Home and Foreign. In the first, the United States is Home and all other countries are Foreign. The alternative scenarios, respectively, are the European Union prior to Brexit (EU28) as Home (and all other countries as Foreign) and the members of the Organization for Economic Cooperation and Development (OECD37) as Home (and all others as Foreign).

Our data source for energy consumption is The Trade in Embodied CO_2 (TECO2) database from OECD. We use their measure of consumption-based CO_2 emissions embodied in domestic final demand and the country of origin of emissions. This database covers 83 countries and regional groups over the period 2005-2018. Carbon dioxide embodied in world consumption in 2018 is 33.63 gigatonnes. We cross-checked the results with a dataset from the Global Carbon Project. The overall difference is less than ten percent.

Extraction data are from the International Energy Agency (IEA), which provides the World Energy Statistics Database on energy supply from all energy sources, including fossil fuels, biofuels, hydro, geothermal, renewables and waste. This dataset covers 143 countries as well as regional and world totals. The data are provided in units terajoules. In order to keep the units consistent with the energy consumption data (gigatonnes of carbon dioxide), we apply emission factors to the five fossil fuel types to calculate CO_2 emissions. The five fossil fuel types considered are coal and coal products, natural gas, peat and peat products, oil shale and oil sands, as well as crude, NGL and feedstocks. The emission factors (listed in Table 19) are default emission factors for stationary combustion from the 2006 IPCC Guidelines for National Greenhouse Gas Inventories. Using this calculation, world extraction is 37.26 gigatonnes of carbon dioxide.

Table 19: Emission factors for fossil fuels

Type of fuel	Emission factor (kgCO_2/TJ)
Coal and coal products	94,600
Peat and peat products	106,000
Crude, NGL and feedstocks	73,300
Natural gas	56,100
Oil shale and oil sands	107,000

To explain the discrepancy between world consumption and world extraction, note that the OECD data for embodied carbon does not include non-energy use of fossil fuels. In other words, some fossil fuels extracted are not combusted to produce energy. Instead, they are consumed directly or as intermediate goods. For example, petroleum can be used as asphalt and road oil and as petrochemical feedstocks for agricultural land. However, given that combusted energy is the source of CO_2 emissions, non-energy use of fossil fuel extraction is excluded in our analysis.

To make this adjustment, we note that, according to EIA (2018), approximately 8 percent of fossil fuels are not combusted in the United States. Applying this rate to the world extraction, we get a number close to world consumption ($37.26 \cdot 0.92 = 34.28$, vs. 33.63). Thus, we can simply re-scale the world extraction data so that world extraction is equal to world consumption. To be specific, the original extraction data is divided by 1.019 (the ratio of world extraction to world consumption). Tables 5, 7, 8, and 9 display the resulting data we use for our calibration.

E.2 Parameter Values

For the key parameter in the goods production function α, the output elasticity of labor, we calibrate $(1-\alpha)/\alpha$ to the value of energy used in production $p_e G_e$

relative to the value added.[39] The data from TECO2 records the carbon emissions embodied by sector and country. We can convert to barrels of oil based on 0.43 metric tons of CO_2 per barrel of crude oil (from EPA, 2019). The price per barrel of oil is taken from the average closing price of West Texas Intermediate (WTI) crude oil in 2015, which is $48.66 per barrel. Value added data comes from OECD Input-Output Tables (2018). We consider three definitions of the goods sector, with both the numerator (value of energy) and the denominator (value added) computed for the same sector definition, either: (i) the manufacturing sector, (ii) manufacturing plus agriculture and construction, and (iii) manufacturing, agriculture, construction, wholesale, retail, and transportation. The values of α that we obtain are, respectively, 0.85, 0.79, and 0.84. Our preferred value is 0.85, very close to two of these three.

For the energy supply elasticities, ϵ_S and ϵ_S^*, we use data from Asker, Collard-Wexler, and De Loecker (2018) on the distribution across oil fields of extraction costs. The data come in the form of quantiles ($q = 0.05, 0.10, ..., 0.95$), separately for the EU, the US, OPEC, and ROW ($q\%$ of oil in the US is extracted at a cost below $\$a$ per barrel, for example). We approximate OECD countries by aggregating the EU and US while for the non-OECD region we aggregate OPEC and ROW. To aggregate the quantiles for two regions, we combine them, sort the combination by the cost level, and reassemble after taking account of total oil extraction for each region (available from the IEA). The data are plotted on log scales in Figures 8 and 9, to reveal the supply elasticity as the slope.

The most costly oil fields in either region would be the first to be abandoned under a carbon policy. Thus, the upper end of the cost distribution is the most relevant for calibrating the supply elasticities. Our baseline values of $\epsilon_S = 0.5$ and $\epsilon_S^* = 0.5$ are close to the slope shown in the figures when we consider only costs above the median. Our alternative value of $\epsilon_S^* = 1$ is closer to the slope if we were to use the upper 75% of costs or even all the data.

Lacking this distributional data for coal and natural gas fields, we assume that the distribution for oil extraction is representative of all fossil fuels.

[39]We think of value added as the closest proxy to labor cost in the model, since we interpret labor in the model as labor equipped with capital.

Figure 8: Calibration of the Extraction Supply Elasticity in Home

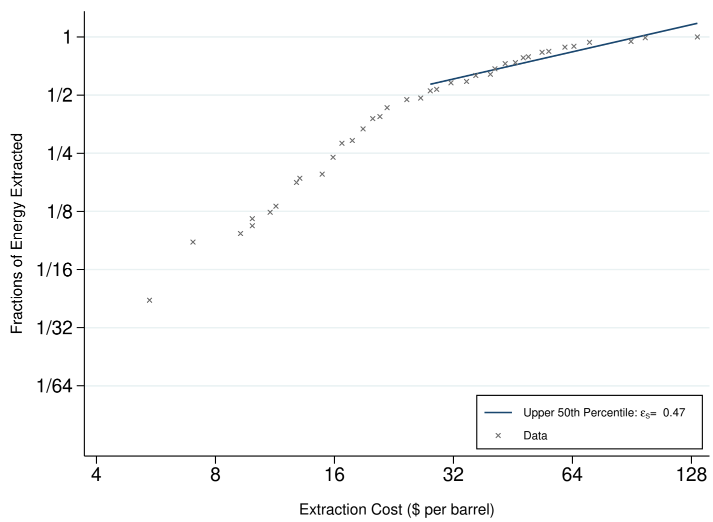

Figure 9: Calibration of the Extraction Supply Elasticity in Foreign