Fair Innings? The Utilitarian and Prioritarian Value of Risk Reduction over a Whole Lifetime

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Matthew D. Adler, Duke University*
Maddalena Ferranna, Harvard University
James Hammitt, Harvard University
Nicolas Treich, Toulouse School of Economics

* Corresponding author. adler@law.duke.edu.

Introduction

Is it socially more important to save the lives of younger individuals, than to save the lives of the old? It seems hard to dispute that younger individuals should take priority with respect to lifesaving measures to the extent that age inversely correlates with life expectancy remaining, at least if the younger and older individuals are similarly situated with respect to the non-lifespan determinants of well-being (health, income, etc.). If Anne is similarly situated to Bob, except for being younger, and a given reduction in Anne's current mortality risk produces a larger increase in her life expectancy than the same reduction in Bob's, the reduction for Anne is socially more valuable.

But some have argued that the young should take priority with respect to lifesaving measures, and health policy more generally, on fairness grounds—not merely on the utilitarian basis that lifesaving measures directed at the young tend to yield a greater increase in life expectancy and expected lifetime well-being. Harris (1985) introduced the idea of "fair innings" into the public health literature. "The fair innings argument requires that everyone be given an equal chance to have a fair innings, to reach the appropriate threshold but, having reached it, they have received their entitlement. The rest of their life is the sort of bonus which may be canceled when this is necessary to help others reach the threshold." Others who have endorsed some version of the fair innings concept include Williams (1997); Daniels (1988); Lockwood (1988); Nord (2005); and Bognar (2008, 2015). The notion that the young should receive priority with respect to lifesaving measures is reflected, not merely in the academic literature on fair innings, but also in surveys of citizen preferences regarding health policy. (See Bognar [2008] for references.)

Bognar (2015, p. 254) uses the following thought experiment to crystallize the fair innings concept.

> [Y]ou have only one drug and there are two patients who need it. The only difference between the two patients is their age. . . . You have to choose between saving: (C) a 20-year old patient who will live for 10 more years if she gets the drug; or (D) a 70-year old patient who will live for 10 more years if she gets the drug.
>
> Both patients would spend the remaining ten years of their life in good health. So there is no difference in expected benefit. The only difference is how much they have already lived when they receive the benefit.
>
> … [According to] the fairness-based argument for the fair innings view, you should … prefer C to D.

We'll build on the suggestion of Bognar (2015) in using the term "fair innings" to mean the following: as between a policy that produces a given gain in expected lifetime well-being for a younger person, and an otherwise-identical policy that produces the same gain in expected lifetime well-being for an older person, it is ethically better for society to undertake the first policy.

While fair innings in this sense is an intuitively appealing idea, it is *not* supported by the current economic literature regarding the valuation of lifesaving. That literature generally focuses on benefit-cost analysis (BCA), which is the dominant tool in governmental practice for assessing fatality risk-reduction policies. The methodology of BCA does *not* support the idea that gains to the young are socially more valuable than equal gains for the old.[1]

In this article, we examine the fair innings concept as part of a broader analysis of the use of social welfare functions (SWFs) to value risk reduction, and a comparison of the SWF framework to BCA. We show, in particular, that "prioritarian" SWFs place greater weight on gains to expected lifetime well-being accruing to younger rather than older individuals. We thus demonstrate, for the first time, that the fair innings concept has a rigorous basis in welfare economics—specifically in the SWF framework, not BCA.

BCA appraises government policies by summing individuals' monetary equivalents—an individual's monetary equivalent for a policy being the amount of money she is willing to pay or accept for it, relative to the status quo. In turn, the value of statistical life (VSL) is the concept that captures how BCA values fatality risk reduction. VSL is the marginal rate of substitution between an individual's survival probability in a period, and her income. Put differently, VSL is the coefficient that translates a change in someone's survival probability into a monetary equivalent. Individual *i*'s willingness to pay or accept for a small change Δp in survival is approximately $(\Delta p)\text{VSL}_i$.

BCA, although now widespread, is controversial. A different framework for evaluating policy—one that has strong roots in economic theory and plays a major role in various bodies of scholarship within economics—is the social welfare function (SWF). The SWF framework measures policy impacts in terms of interpersonally comparable *utilities*, not monetary equivalents. Each possible outcome is a vector of individual utilities, and a given policy is a probability distribution over such vectors. The SWF, abbreviated $W(.)$, assigns a social value to a policy P, $W(P)$, in light of the probability distribution over outcomes and, thus, utility vectors that P corresponds to.

In previous work (Adler, Hammitt and Treich [2014]), we analyzed the application of the SWF framework to risk policies and compared how it values risk reduction to VSL. The key construct in our analysis was the social value of risk reduction (SVRR). The SVRR for individual *i* is the social value per unit of risk reduction to individual *i*—social value as captured by the SWF $W(.)$. SVRR_i is just $\frac{\partial W}{\partial p_i}$, and the change in the SWF that occurs with a change Δp in individual *i*'s survival probability is approximately $(\Delta p)\text{SVRR}_i$.

[1] See below, Part II, explaining why BCA does not support fair innings.

Using the simple, one-period model that is often employed in the literature on SWFs, Adler, Hammitt and Treich [2014] calculated $SVRR_i$ for different types of SWFs: the utilitarian, "ex ante prioritarian," and "ex post prioritarian" SWFs. (Utilitarianism ranks outcomes by summing utility numbers, while prioritarianism does so by summing a strictly increasing and strictly concave transformation of utility numbers, thereby giving priority to those at lower utility levels. The ex ante and ex post prioritarian SWFs are two distinct specifications of prioritarianism for the case of uncertainty.) We analyzed the comparative statics of $SVRR_i$ and VSL_i with respect to individual wealth and baseline risk.

The current Article significantly expands the analysis of Adler, Hammitt and Treich (2014). We use a much richer model of individual resources and survival. An individual's life has multiple periods, up to a maximum T (e.g., 100 years). Each individual is characterized by a lifetime risk profile (a probability of surviving to the end of each period, conditional that she is alive at its beginning); a lifetime income profile (an income amount which she earns in each period if she survives to its end); and a current age. This multi-period setup permits a considerably more nuanced analysis of $SVRR_i$ and VSL_i. In particular, we can now examine the comparative statics of $SVRR_i$ and VSL_i with respect to an individual's *age* as well as income and baseline fatality risk.

Part I sets forth the model and the SWFs we will consider. Part II analyzes the comparative statics of $SVRR_i$ and VSL_i with respect to age. We provide an axiomatic statement of the fair innings concept, via an axiom which we term "Priority for the Young." We show that the ex ante prioritarian $SVRR_i$ and ex post prioritarian $SVRR_i$ both satisfy Priority for the Young.[2] By contrast, the utilitarian $SVRR_i$ and VSL_i do not satisfy Priority for the Young.

Part III analyzes the comparative statics of $SVRR_i$ and VSL_i with respect to income and baseline risk. Part IV undertakes an empirical exercise, based on the U.S. population survival curve and income distribution, to illustrate the $SVRR_i$ concept and to estimate the impact of age and (within each age cohort) income on $SVRR_i$ and VSL_i.

We find significant differences between the SWF framework and BCA as the basis for valuing risk reduction. Not only does the SWF framework (with a prioritarian SWF) provide a basis for the fair innings concept, a concept not supported by BCA. We also find that VSL_i differs from the utilitarian, ex ante prioritarian, and ex post prioritarian $SVRR_i$ in its comparative statics with respect to individual income and baseline risk. These analytic differences between the $SVRR_i$ concept and VSL_i are shown, in the empirical exercise, to translate into large differences in the relative value of risk reduction as between subgroups of the U.S. population differentiated by age and income.

[2] Indeed, as we demonstrate in Part II, the ex ante prioritarian and ex post prioritarian $SVRR_i$ both satisfy a stronger axiom, Ratio Priority for the Young.

I. **The Model**

There is a population of N individuals. The life of a given individual i is divided into periods 1, 2, …, t, … T, with T the maximum number of periods that any individual can live.

Death and survival are conceptualized as follows: An individual who is alive at the beginning of a given period may either die before the period ends, or survive to the end of the period (equivalently, be alive at the beginning of the following period). Let $p_i(t)$ denote individual i's probability of surviving to the end of period t, given that she is alive at the beginning of period t. We'll generally refer to $p_i(t)$ as a "survival probability." Individual i is characterized by a vector of such probabilities, one for each period up to T—for short, her "risk profile."

Government makes a policy choice (see below for more detail) at a point in calendar time, denoted "the present." The present is the *beginning* of the "current period."

For any individual now alive, the current period is some period in her life. (For example, if Betty has already lived 4 periods, the current period is number 5 in Betty's life.) Let A_i denote the number of the current period for individual i. We will also refer to this as the "age" of individual i (but please note that the present time is at the beginning of the current period, so that an individual "age 1" is at the beginning of period 1 of her life, an individual "age 2" at the beginning of period 2, and so forth).

Let $\pi_i(t; A_i)$ denote individual i's probability of surviving to the end of period t of her life, given that she is currently alive at the beginning of period A_i, with $t \geq A_i$. $\pi_i(t; 1)$ or, for short, $\pi_i(t)$, is just the individual's probability at birth of surviving until the end of period t. Then:

$$\pi_i(t; A_i) = \frac{\pi_i(t)}{\pi_i(A_i - 1)} = \prod_{s=A_i}^{t} p_i(s).$$

Finally, let $\mu_i(t; A_i)$ denote individual i's current probability of surviving to the end of period t and then dying during the next period $(t + 1)$. In other words, $\mu_i(t; A_i)$ is the individual's current probability of living exactly t periods. In the case of $t = (A_i - 1)$, this is the probability of dying before the end of the current period (A_i), i.e., $\mu_i(t; A_i) = 1 - p_i(A_i)$. For $t \geq A_i$, we have that:

$$\mu_i(t; A_i) = (1 - p_i(t+1))\pi_i(t; A_i).$$

The earnings process is as follows: if an individual survives to the end of period t, she earns an income amount $y_i(t)$. Individual i, thus, is characterized by a vector of incomes, ($y_i(1)$, …, $y_i(T)$)—her "income profile."

Period consumption, like period income, is modelled as occurring only if the individual survives to the end of the period. An individual's consumption during period t, if she survives to the end of period t, is denoted $c_i(t)$. We assume "myopic" consumption: $c_i(t) = y_i(t)$. The individual consumes in each period whatever she earns then, rather than saving earnings for future consumption or financing consumption by borrowing against future earnings.[3]

We assume that an individual's lifetime well-being is the discounted sum of period well-being, where $u(.)$ is the period utility function and $\beta = 1/(1+r)$, $r \geq 0$ the constant utility discount rate. $U_i(t)$ denotes the individual's lifetime well-being if she lives exactly t periods. $U_i(t) = \sum_{s=1}^{t} \beta^{s-1} u(c_i(s))$. We assume that $u(.)$ is continuously differentiable and that $u'(.) > 0$ and $u''(.) < 0$.

Further, our analysis assumes that $u(c_i(t)) > 0$ for all i, t. With myopic consumption, this becomes $u(y_i(t)) > 0$ for all i, t. Let c^{zero} be a cutoff level of consumption so low that the period well-being level of c^{zero} is equal to the well-being level of not being alive during the period. Then $u(c^{zero}) = 0$ and, by the assumption $u'(.) > 0$, $u(y) > 0$ iff $y > c^{zero}$. Thus the assumption that $u(y_i(t)) > 0$ implies that $y_i(t) > c^{zero}$ for all i, t.

We consider three different social welfare functions (SWFs): the utilitarian SWF, denoted W^U; the ex ante prioritarian SWF, denoted W^{EAP}; and the ex post prioritarian SWF, denoted W^{EPP}.

1. **Utilitarianism**. $W^U = \sum_{i=1}^{N} V_i$, where V_i is the expected lifetime well-being of individual i, given that his current age is A_i. $V_i = \sum_{t=1}^{A_i - 1} \beta^{t-1} u(c_i(t)) + \sum_{t=A_i}^{T} \pi_i(t; A_i) \beta^{t-1} u(c_i(t))$

2. **Ex ante prioritarianism**. Let $g(.)$ be a strictly increasing and strictly concave transformation function. Then $W^{EAP} = \sum_{i=1}^{N} g(V_i)$.

3. **Ex post prioritarianism**. Given that individual i is currently age A_i, the probability of attaining lifetime well-being $U_i(t)$ with $t \geq A_i$ is given by $\mu_i(t; A_i)$, as previously defined. Thus

$$W^{EPP} = \sum_{i=1}^{N} \left[(1 - p_i(A_i)) g(U_i(A_i - 1)) + \sum_{t=A_i}^{T} \mu_i(t; A_i) g(U_i(t)) \right]$$

Note that prioritarianism (in both ex ante and ex post versions) is a family of SWFs, corresponding to all the possible strictly increasing and strictly concave transformation functions $g(.)$. The choice of $g(.)$ defines a specific W^{EAP} and W^{EPP}. However, our analysis will be generic, holding true for any $g(.)$. We do assume that $g(.)$ is twice differentiable, so that $g'(.) > 0$ and $g''(.) < 0$.

[3] Extending the model to allow for endogenous consumption is an important research topic—one that we are currently undertaking.

In our empirical exercise (see Part IV), we use the "Atkinson" family of $g(.)$ functions[4], which have attractive axiomatic properties (Adler 2012) and are regularly used in the economic literature on prioritarianism. Atkinson $g(.)$ functions may be such that $g(0)$ is undefined.[5] In order for our analysis to accommodate the possibility that $g(0)$ is undefined, we assume that $A_i \geq 2$ for all i.[6] Because the period length can be arbitrarily short, this is not a significantly restrictive assumption.

Government's policy: As mentioned, government enacts a policy at the beginning of the current period. The policy changes individuals' current-period survival probabilities. That probability, for individual i, changes from $p_i(A_i)$ to $p_i(A_i) + \Delta p_i$, with $\Delta p_i > 0$. Her probability of surviving until the end of period $t > A_i$ is now as follows:

$$[p_i(A_i)+\Delta p_i]\prod_{s=A_i+1}^{t} p_i(s) = \pi_i(t;A_i)\left[1+\frac{\Delta p_i}{p_i(A_i)}\right]$$

The social value of risk reduction (SVRR$_i$) for a given SWF $W(.)$ is just $\frac{\partial W}{\partial p_i(t)}\Big|_{t=A_i}$, that is, $\lim_{\Delta p_i \to 0}\frac{\Delta W}{\Delta p_i}$. We denote the utilitarian SVRR$_i$ as S_i^U, the ex ante prioritarian SVRR as S_i^{EAP}, and the ex post prioritarian SVRR as S_i^{EPP}.

SVRR$_i$ is the marginal change in social welfare per unit of current risk reduction for individual i. To be sure, a governmental policy may well have effects other than changing individuals' current survival probabilities. It may also change their survival probabilities in future periods. And a risk-reduction policy will surely have costs, which will be reflected in a change to individuals' current or future incomes. The *total* effect of a policy on social welfare, ΔW, will be approximately equal to the sum, across individuals, of (SVRR$_i$)Δp_i plus corresponding terms for changes to future survival probabilities and to incomes.[7] SVRR$_i$ captures that *portion* of a policy's total impact on social welfare that result from changes to individual i's current survival probability.

Further, by comparing SVRR$_i$ to SVRR$_j$, for two individuals i and j—as we do below—we can determine the relative social impact of risk reductions for the two. Consider a change Δp

[4] $g(u) = (1-\gamma)^{-1}u^{1-\gamma}$, $\gamma > 0$, $\gamma \neq 1$; and $g(u) = \ln u$ if $\gamma = 1$.
[5] Specifically, $g(u)$ is undefined if $\gamma \geq 1$.
[6] Note that the expression below for the ex post prioritarian SVRR uses the $g(.)$ value of the lifetime well-being of a life with length $(A_i - 1)$. In order to avoid $(A_i - 1) = 0$, we assume $A_i \geq 2$ for all i.
[7] Assume that a policy changes individual i's survival probability in period t by Δp_i^t, with $t > A_i$; and her income (and consumption, given myopic consumption) by Δy_i^t, with $t \geq A_i$. As in the text, let Δp_i denote the policy change to the individual's current survival probability. Then, by the total-differential approximation from calculus, ΔW is approximately equal to: $\sum_i\left(SVRR_i\Delta p_i + \sum_{s=A_i+1}^{T}\frac{\partial W}{\partial p_i(t)}\Big|_{t=s}\Delta p_i^t + \sum_{s=A_i}^{T}\frac{\partial W}{\partial y_i(t)}\Big|_{t=s}\Delta y_i^t\right)$.

to someone's current survival probability. That risk change, if accruing to individual *i*, results in a change of social welfare by approximately $\text{SVRR}_i(\Delta p)$. If accruing to individual *j*, it results in a change of social welfare by approximately $\text{SVRR}_j(\Delta p)$. Thus (for a small Δp) the first social welfare change is larger than/smaller than/equal to the second iff SVRR_i is larger than/smaller than/equal to SVRR_j.

Calculating $\lim_{\Delta p_i \to 0} \frac{\Delta W}{\Delta p_i}$ for W^U, W^{EAP}, and W^{EPP} yields the formulas for SVRR_i, which are as follows:

$$S_i^U = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^{t-1} u(y_i(t))$$

$$S_i^{EAP} = g'(V_i) S_i^U$$

$$S_i^{EPP} = -g(U_i(A_i - 1)) + \sum_{t=A_i}^{T} \frac{\mu_i(t;A_i)}{p_i(A_i)} g(U_i(t))$$

Note that our assumption that $u(y_i(t)) > 0$ for all *i*, *t*—it is always better to survive a period than to die before its end—ensures that S_i^U, S_i^{EAP}, and $S_i^{EPP} > 0$ for all *i*. Risk reduction is always a social benefit.

Throughout this Article, as in Adler, Hammitt and Treich (2014), we assume that individuals have common preferences, represented by a common period utility function. This common function $u(.)$ is, at the same time, the basis for calculating individuals' lifetime well-being values, $U_i(t)$, for purposes of the various SWFs. In the standard analysis, a particular person's VSL is the change in her expected utility, per unit of survival probability, divided by the change in her expected utility, per unit of income (or wealth or consumption). In our model, given the above assumptions about $u(.)$ and $U_i(t)$, VSL can be defined more specifically as follows:

$$VSL_i = \frac{S_i^U}{p_i(A_i) u'(y_i(A_i)) \beta^{A_i - 1}} \ .$$

That is, VSL_i is the utilitarian SVRR divided by the expected marginal utility of *i*'s current consumption.

In the text below, to avoid clutter, we will often drop the individual subscript and use the terms "SVRR" and "VSL" to mean, respectively, SVRR_i and VSL_i.

II. The Effect of Age and "Priority for the Young"

The effect of age on the SVRR has never been addressed by the academic literature. The one-period model in Adler, Hammitt and Treich (2014) was not suited to tackle this question.

Here, we analyze what our model implies with respect to age effects on SVRR as well as VSL by considering two individuals *i* and *j*, with identical risk profiles and income profiles, but the first older than the second ($A_i > A_j$). In what follows, we drop subscripts on incomes or probabilities where these are the same for *i* and *j*, e.g., $y(t)$ indicates $y_i(t) = y_j(t)$.

<u>The Utilitarian SVRR</u>: Recall that $S_i^U = \sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^{t-1} u(y_i(t))$. Observe that S_i^U is equal to the difference between (1) individual *i*'s expected lifetime well-being conditional on surviving the current period and (2) her realized lifetime well-being if she dies during the current period (does not survive it). The intuition for this result is straightforward. Consider the simple case in which individual *i* would die for certain during the current period, absent governmental intervention, and intervention ensures that she survives the period. In this case, clearly, the change in utilitarian social welfare (ΔW^U) that results from the intervention is just the difference between individual *i*'s expected lifetime conditional on surviving the current period, and her realized lifetime well-being if she dies during the current period. For short, let's term this difference the "utilitarian gain from saving individual *i*."

More generally, consider a policy which increases individual *i*'s current survival probability by Δp_i. The change in utilitarian social welfare that results from the Δp_i increase is just $\Delta p \left(\sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^{t-1} u(y_i(t)) \right)$. Thus S_i^U, the change in utilitarian social welfare per unit of current-period risk reduction for individual *i*, is nothing other than $\sum_{t=A_i}^{T} \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^{t-1} u(y_i(t))$: the utilitarian gain from saving individual *i*.

What, then, are the relative magnitudes of S_i^U and S_j^U, for two individuals of different ages ($A_i > A_j$) but with identical risk and income profiles? In other words, how does the utilitarian gain from saving an individual depend upon her age?

It can be shown that $S_j^U - S_i^U$ equals:

$$\sum_{t=A_j}^{A_i-1} \pi(t;A_j+1)\beta^{t-1}u(y(t)) + \left(\pi(A_i;A_j+1)-1\right)\sum_{t=A_i}^{T} \pi(t;A_i+1)\beta^{t-1}u(y(t)) .$$

Thus the utilitarian SVRR decreases/is unchanged/increases with age iff the value of this formula is positive/zero/negative.

The first term in this formula (for short, the "duration term") is positive. By increasing the *younger* individual's current survival probability, we increase her chance of surviving the periods $A_j, A_j+1, \ldots, A_i - 1$ in her life, and that probability change for each such period yields an increment in expected lifetime well-being (by increasing her chance of accruing consumption utility with respect to that period). This increment to expected lifetime well-being with respect to periods $A_j, A_j+1, \ldots, A_i - 1$ does not occur if we increase the *older* individual's survival probability, since he has already survived those periods.

The second term in the formula above (for short, the "risk term") is negative. By increasing either individual's current survival probability, we increase that individual's chance of surviving periods $A_i, A_i + 1, \ldots, T$ in his or her life, and thereby increase his or her chance of accruing consumption utility with respect to those periods. The risk term captures the *difference* between the magnitude of this benefit for the younger individual and its magnitude for the older one. Since the older individual is sure to be alive at the beginning of period A_i, while the younger individual is not, this difference is negative.

Clearly, if income can increase with age, the magnitude of the risk term may exceed that of the duration term, and thus the utilitarian gain from saving the older individual may be greater than that of saving the younger one. What if constant income is assumed? With a constant income profile and a constant risk profile, the duration term predominates and the utilitarian SVRR decreases with age. More generally, it can be shown that if income is constant and the risk profile is such that survival probabilities do not increase with age, the utilitarian SVRR decreases with age. (See Appendix.)

<u>Ex ante Prioritarian SVRR</u>: A simple manipulation shows that $S_j^{EAP} - S_i^{EAP} = g'(V_j)\left(S_j^U - S_i^U\right) + S_i^U\left(g'(V_j) - g'(V_i)\right)$. We noted immediately above in discussing the utilitarian SVRR that the quantity $(S_j^U - S_i^U)$ equals a positive "duration" term plus a negative "risk" term. The first part of the formula here, namely $g'(V_j)\left(S_j^U - S_i^U\right)$, incorporates those terms. This part is positive iff $(S_j^U - S_i^U)$ is positive. The second part of the formula here, $S_i^U\left(g'(V_j) - g'(V_i)\right)$, is a third term ("priority for the young"), which is always positive. Because $V_i > V_j$ (the older individual has greater expected lifetime well-being) and $g(.)$ is strictly concave, $g'(V_i) < g'(V_j)$.

The intuition behind the formula is as follows. Ex ante prioritarian social welfare, W^{EAP}, is the sum of individuals' transformed expected lifetime well-beings— transformed by a strictly increasing and strictly concave $g(.)$ function. The effect of this transformation is to give greater social weight to changes in expected lifetime well-being that accrue to individuals at a lower level of expected lifetime well-being. The differential ex-ante-prioritarian benefit of saving a younger rather than older individual reflects the differential gains to expected lifetime well-being of saving the younger one $(S_j^U - S_i^U)$. But it also reflects the fact that the younger individual has a lower level of expected lifetime well-being and thus takes priority ($g'(V_j) > g'(V_i)$).

We now define "Priority for the Young" more formally. In defining this property, we incorporate a utilitarian baseline. The utilitarian social evaluation of risk reduction for a younger versus an older individual depends on a comparison of the gains to expected lifetime well-being from saving one or the other. Utilitarianism prefers to save the young only to the extent that doing so produces a larger increment in expected lifetime well-being. The ex ante prioritarian SWF satisfies "Priority for the Young," relative to the utilitarian baseline, defined as follows:

Proposition I-A : *The Ex Ante Prioritarian SWF Satisfies Priority for the Young*

$$S_j^U - S_i^U > 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0 \text{ and } S_j^U - S_i^U = 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$$

Priority for the Young is a precise expression, using the SVRR formalism, of the fair innings concept. Ex ante prioritarianism never assigns a smaller or equal per-unit value to risk reduction for the younger individual if the utilitarian per-unit value is larger than for the older individual. ($S_j^U - S_i^U > 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$.) Further, if the utilitarian per-unit values are equal, ex ante prioritarianism places a larger per-unit value on risk reduction for the younger individual. ($S_j^U - S_i^U = 0 \Rightarrow S_j^{EAP} - S_i^{EAP} > 0$.). Recall, finally, that the utilitarian per-unit value of risk reduction for any person, young or old, is just the gain to expected lifetime from saving her.

Actually, it's straightforward to prove a logically stronger result, namely that the relative social value of risk reduction for young versus old individuals is always greater with ex ante prioritarianism than with utilitarianism. $(S_j^{EAP} / S_i^{EAP}) > (S_j^U / S_i^U)$.[8] If utilitarianism prefers to reduce the younger individual's risk (the utilitarian gain from saving her is greater), ex ante prioritarianism has a yet greater degree of priority for the young. If utilitarianism is indifferent (the utilitarian gains are equal), ex ante prioritarianism gives priority to the young. Finally, although ex ante prioritarianism may prefer to reduce the risk of the older individual (if the utilitarian gain from saving her is sufficiently greater), in this case it always give less priority to the older individual than utilitarianism does.

Proposition I-B: *The Ex Ante Prioritarian SWF Satisfies Ratio Priority for the Young*

$$(S_j^{EAP} / S_i^{EAP}) > (S_j^U / S_i^U)$$

[8] $S_i^{EAP} = g'(V_i) S_i^U$, $S_j^{EAP} = g'(V_j) S_j^U$, and so we have that $\dfrac{S_j^{EAP}}{S_i^{EAP}} = \dfrac{g'(V_j)}{g'(V_i)} \dfrac{S_j^U}{S_i^U} > \dfrac{S_j^U}{S_i^U}$.

Note that Ratio Priority for the Young implies Priority for the Young (Proposition I-B implies Proposition I-A), but not vice versa.

Ex post prioritarian SVRR: It can be shown that $S_j^{EPP} - S_i^{EPP}$ equals:

$$\sum_{t=A_j}^{A_i-1} \mu(t; A_j+1) g(U(t)) + (\pi(A_i; A_j+1) - 1) \sum_{t=A_i}^{T} \mu(t; A_i+1) g(U(t)) + \left(g(U(A_i - 1)) - g(U(A_j - 1)) \right)$$

Although this formula is different from $S_j^{EAP} - S_i^{EAP}$, it nonetheless reflects the same three factors. The first term of the formula is a positive "duration term," reflecting the increased chance for the younger individual of surviving periods A_j through $A_i - 1$; the second term is a negative "risk term," reflecting the chance she will not survive to period A_i; and the third term is a positive "priority for the young" term.

We saw above that the ex ante prioritarian SWF satisfies "Priority for the Young": it prefers to reduce the younger individual's risk even when utilitarianism is indifferent, and prefers to do so whenever utilitarianism does. The same is true for the ex post prioritarian SWF.

Proposition II-A : *The Ex Post Prioritarian SWF Satisfies Priority for the Young*

$$S_j^U - S_i^U > 0 \Rightarrow S_j^{EPP} - S_i^{EPP} > 0 \text{ and } S_j^U - S_i^U = 0 \Rightarrow S_j^{EPP} - S_i^{EPP} > 0$$

See Appendix for proof.

The intuition for this result is as follows. Ex post prioritarian social welfare, W^{EPP}, is the sum of individuals' expected transformed lifetime well-beings—transformed by a strictly increasing and strictly concave $g(.)$ function.[9] The ex post prioritarian SVRR, $S_i^{EPP} = -g(U_i(A_i - 1)) + \sum_{t=A_i}^{T} \frac{\mu_i(t; A_i)}{p_i(A_i)} g(U_i(t))$, is the difference between individual i's expected transformed lifetime well-being conditional on surviving the current period, and her transformed lifetime well-being if she does not survive. Equivalently, it is the *expected difference* between her transformed lifetime well-being conditional on surviving the current period (given her possible lifespans if she does survive and their probabilities), and her transformed lifetime well-being if she does not survive.

Consider now two individuals, one (j) younger than the second (i), with a common income and risk profile. The ex post prioritarian SWF places less value on a risk reduction for i than for j because i's lifetime well-being if she dies during the current period, $U(A_i - 1)$, is greater than j's if she dies during the current period, $U(A_j - 1)$—and thus the very same increase in lifetime well-being for the two individuals translates into a smaller change in transformed

[9] While ex ante prioritarianism applies this function to expected lifetime well-being, ex post prioritarianism applies it to realized lifetime well-being.

lifetime well-being for *i*. Assume that *i*, if she survives the period, has probability μ of realizing a level of lifetime well-being which is ΔU greater than her level of lifetime well-being if she dies now. And assume that the same is true for *j*. The utilitarian value of a chance μ of increment ΔU is the same for both individuals, namely $\mu(\Delta U)$. The ex post prioritarian value of a chance μ for individual *j* of increment ΔU is $\mu\left(g(U(A_j-1)+\Delta U)-g(U(A_j-1))\right)$, while for *i* it is $\mu\left(g(U(A_i-1)+\Delta U)-g(U(A_i-1))\right)$. The first value is greater than the second by virtue of the strict concavity of $g(.)$, because $U(A_j-1) < U(A_i-1)$.

We saw above that ex ante prioritarianism satisfies not merely Priority for the Young but also the (logically stronger) Ratio Priority for the Young. The same is true for ex post prioritarianism (see Appendix for proof).

Proposition II-B: *The Ex Post Prioritarian SWF Satisfies Ratio Priority for the Young*

$$(S_j^{EPP}/S_i^{EPP}) > (S_j^U/S_i^U)$$

VSL. As is well known, the effect of age on VSL is ambiguous (Aldy and Viscusi 2007; Hammitt 2007). VSL reflects the influence of age on the utilitarian SVRR (the numerator of VSL), plus an additional effect: the change in expected marginal utility of consumption (the denominator of VSL) with age.

Let $C_i = p(A_i)\beta^{A_i-1}u'(y(A_i))$ and similarly for C_j. Then $\text{VSL}_j - \text{VSL}_i$ equals:

$$\frac{1}{C_j}\left(S_j^U - S_i^U\right) + S_i^U\left(\frac{1}{C_j}-\frac{1}{C_i}\right).$$

Note that the expected marginal utility of consumption for the younger individual (C_j) may be larger than for the older individual (C_i)—which can occur if the younger individual has less consumption and/or a greater current survival probability. If $C_j > C_i$, $1/C_j < 1/C_i$ and thus the second term in the above formula for $\text{VSL}_j - \text{VSL}_i$ will be negative even if $S_j^U = S_i^U$. Further, even if $S_j^U > S_i^U$, the magnitude of the second term may exceed that of the first.

In short, VSL does not satisfy either disjunct of Priority for the Young.

Proposition III-A: VSL does not Satisfy Priority for the Young (*either Disjunct*)

It is not the case that $S_j^U - S_i^U = 0 \Rightarrow S_j^{VSL} - S_i^{VSL} > 0$; and it is not the case that $S_j^U - S_i^U > 0 \Rightarrow S_j^{VSL} - S_i^{VSL} > 0$

In other words: BCA may prefer a risk reduction for the older individual even if the utilitarian gains are equal, indeed even if the utilitarian gain to saving the younger one is larger.

Because Ratio Priority for the Young implies Priority for the Young, the proposition that VSL fails to satisfy Priority for the Young implies (by contraposition) that it fails to satisfy Ratio Priority for the Young.

Proposition III-B VSL does not Ratio Priority for the Young

It is not the case that $(VSL_j / VSL_i) > (S_j^U / S_i^U)$

A Summary: The following table summarizes the results of our analysis of age effects on the utilitarian, ex ante prioritarian, and ex post prioritarian social value of risk reduction (SVRR), and on VSL.

	Priority for the Young	Ratio Priority for the Young
Utilitarian SVRR[10]	No	No
Ex Ante Prioritarian SVRR	Yes	Yes
Ex Post Prioritarian SVRR	Yes	Yes
VSL	No	No

One "takeaway" from our analysis is that the concept of prioritarianism, in both its ex ante and ex post variants, provides a rigorous basis for the fair innings concept—as formally expressed by Priority for the Young and Ratio Priority for the Young.

Second, the analysis extends an important finding of Adler, Hammitt and Treich (2014). That article, as mentioned, used a single-period model which was not suited to study age effects. What it *did* study was the effect of income and baseline risk on the utilitarian, ex ante prioritarian, and ex post prioritarian SVRRs and on VSL. Here, Adler, Hammitt and Treich (2014) found that BCA and the SWF framework value risk reduction in significantly different ways. The present analysis confirms that finding, now with respect to age affects. By contrast with ex ante and ex post prioritarian SVRRs, VSL does not satisfy Priority for the Young or Ratio Priority for the Young.

[10] Because Priority for the Young, and Ratio Priority for the Young are defined as a stronger preference for the young than the utilitarian preference, it is trivial that utilitarian SVRR doesn't have these properties. By contrast, our results for the ex ante and ex post prioritarian SVRRs and for VSL are not trivial, but require mathematical analysis.

III The Effects on Income and Baseline Risk on SVRR and VSL

We now consider how SVRR and VSL vary between individuals of the same age, but with different income or risk profiles.

A. *Sensitivity to Income*

We consider first whether SVRR and VSL increase, decrease, or are unchanged by a *single-period* difference in income. Two individuals *i* and *j* are identical in age ($A_i = A_j$), in their risk profiles, and in their income profiles except that $y_j(t) = y_i(t) + \Delta y$, $\Delta y > 0$, for some single period t. (The period in which the individuals' incomes differ can be the current period, in which case $t = A_i = A_j$) or it can be a past or future period.) We determine whether $\text{SVRR}_j > \text{SVRR}_i$, $\text{SVRR}_j = \text{SVRR}_i$, or $\text{SVRR}_j < \text{SVRR}_i$ by examining the sign of $\frac{\partial S_i}{\partial y_i(t)}$.

We find as follows. <u>Utilitarian SVRR</u>. The utilitarian SVRR is independent of a single-period change to past income (since the formula for S_i^U depends only on present and future income), while it is increasing with a single-period change to present or future income. $\frac{\partial S_i^U}{\partial y_i(t)} = \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^{t-1} u'(y_i(t)) > 0 \quad \forall t \geq A_i$. The intuition here is that preventing the current death of an individual with greater present or future income produces a larger gain in expected lifetime well-being.

<u>Ex Ante Prioritarian SVRR.</u> Unlike the utilitarian SVRR, the ex ante prioritarian SVRR is "history dependent"—sensitive to individuals' *past* characteristics. Specifically, the ex ante prioritarian SVRR decreases with a single-period increment to *past* income. $\frac{\partial S_i^{EAP}}{\partial y_i(t)} = g''(V_i) S_i^U \beta^{t-1} u'(y_i(t)) < 0 \quad \forall t < A_i$. (The intuition is the following: If two individuals are identical except that the first has lower past income, then preventing either of their deaths in the current period produces the same increment in expected lifetime well-being, but the first individual has a lower baseline level of expected lifetime well-being, thus takes priority under W^{EAP}).

As for a single-period increment to *present* or *future* income:

$$\frac{\partial S_i^{EAP}}{\partial y_i(t)} = \frac{\pi_i(t;A_i)}{p_i(A_i)} \beta^{t-1} u'(y_i(t)) \left(g''(V_i) \sum_{s=A_i}^{T} \pi_i(s;A_i) \beta^{s-1} u(y_i(s)) + g'(V_i) \right) \quad \forall t \geq A_i .$$

Thus $\frac{\partial S_i^{EAP}}{\partial y_i(t)}$ is positive/negative/zero iff $-\frac{g''(V_i)}{g'(V_i)}$ is less than/greater than/equal to $\frac{1}{\sum_{s=A_i}^{T} \pi_i(s;A_i) \beta^{s-1} u(y_i(s))}$.

Thus we have that the effect of a single-period increment to present income or future income on the ex ante prioritarian SVRR is ambiguous.[11] The ex ante prioritarian SVRR may increase, decrease, or even remain constant after that increment.

We can say a bit more about the determinants of the comparative statics. Note that $V_i > \sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^{s-1}u(y_i(s))$. Thus, manipulating the above equation, we have the following: if $-\frac{g''(V_i)V_i}{g'(V_i)} \leq 1$, then $\frac{\partial S_i^{EAP}}{\partial y_i(t)} > 0$. In short, if $g(\cdot)$ is such that coefficient of relative risk aversion is always less than or equal to 1, a one-time increase to present or future income will increase the ex ante prioritarian SVRR. The intuition for this result is as follows: The individual with lower present or future income has a lower *level* of expected lifetime well-being, so takes priority under W^{EAP}, but reducing her current risk produces a smaller increase in expected lifetime well-being, and so W^{EAP} prefers reducing the other individual's risk if $g(.)$ is not very concave.

Ex Post Prioritarian SVRR. The ex post prioritarian SVRR, too, is history dependent. Like the ex ante prioritarian SVRR, it decreases with a single-period increment to past income.

$$\frac{\partial S^{EPP}}{\partial y_i(t)} = -\beta^{t-1}u'(y_i(t))\left[g'(U_i(A_i - 1)) - \sum_{t=A_i}^{T}\frac{\mu_i(t;A_i)}{p_i(A_i)}g'(U_i(t))\right] < 0 \quad \forall t < A_i$$

However, unlike its ex ante counterpart, the ex post prioritarian SVRR *always* increases with a change to present or future income.

$$\frac{\partial S^{EPP}}{\partial y_i(t)} = \beta^{t-1}u'(y_i(t))\sum_{s=t}^{T}\frac{\mu_i(t;A_i)}{p_i(A_i)}g'(U_i(s)) > 0 \qquad \forall t \geq A_i$$

The reason for the divergence between S_i^{EAP} and S_i^{EPP} as regards sensitivity to present or future income is subtle. The social value, as per W^{EPP}, of preventing an individual from dying during the current period is the expected difference between the transformed lifetime well-being of the longer lives she might lead were she to survive the current period, and the transformed lifetime well-being of her life were it to end now. Increasing present or future income *increases* that expected difference in transformed lifetime well-being.

VSL Because VSL_i equals S_i^U divided by the expected marginal utility of *i*'s current income, the comparative statics of VSL with respect to past and future income are the same as

[11] By "ambiguous" we mean the following. The comparative statics of SVRR or VSL with respect to a parameter of interest (present income, future income, permanent income, age, etc.) are "ambiguous" if we can find some combination of the other parameters and strictly increasing and strictly concave $u(.)$ and $g(.)$ such SVRR or VSL is increasing in the parameter of interest, and some alternative combination of the other parameters and $u(.)$ and $g(.)$ such that SVRR or VSL is decreasing in the parameter of interest.

for the utilitarian SVRR. Further, because the utilitarian SVRR (the numerator of VSL) is increasing in current income, and the denominator is decreasing, VSL also increases in current income—indeed more quickly than the utilitarian SVRR.

Next, we consider the effect on SVRR and VSL of a change in *permanent income*. Two individuals i and j are identical except that $y_j(t) = y_i(t) + \Delta y$ *for all periods*. We find as follows; see Appendix for proofs. Utilitarian SVRR. Increasing in permanent income. Ex ante Prioritarian SVRR. Ambiguous. Will increase with permanent income if $g(.)$ is not too concave, i.e., the coefficient of relative risk aversion is less than or equal to one. Ex post prioritarian SVRR. Ambiguous. VSL Increasing in permanent income.

B. *Sensitivity to Baseline Risk*

We consider first whether SVRR and VSL increase, decrease, or are unchanged by a *one-period* difference in survival probability. Two individuals i and j are identical except that $p_j(t) = p_i(t) + \Delta q$, $\Delta q > 0$, for some single period t. We determine whether $\text{SVRR}_j > \text{SVRR}_i$, $\text{SVRR}_j = \text{SVRR}_i$, or $\text{SVRR}_j < \text{SVRR}_i$ by examining the sign of $\frac{\partial S_i}{\partial p_i(t)}$.

Neither SVRR nor VSL is sensitive to change in past survival probabilities—as is obvious—and so we discuss only the results for the case of a one-period change to present survival probability ($t = A_i = A_j$) or future survival probability.

We find as follows. Utilitarian SVRR. The utilitarian SVRR is insensitive to a one-period change in present survival probability. (Note that the formula for S_i^U is equivalent to $\beta^{A_i-1}u(y_i(A_i)) + \sum_{t=A_i+1}^{T} \beta^{t-1}u(y_i(t)) \prod_{s=A_i+1}^{t} p_i(s)$, so is not a function of $p_i(A_i)$.) It is increasing in a one-period change in future survival probability. $\frac{\partial S^U}{\partial p_i(t)} = \sum_{s=t}^{T} \frac{\pi_i(s;A_i)}{p_i(A_i)} \beta^{s-1}u(y_i(s)) \frac{1}{p_i(t)} > 0$ $\forall t > A_i$. (The intuition is that preventing a current death produces a bigger increase in expected lifetime well-being if the individual has a lower chance of dying in future periods.)

Ex Ante Prioritarian SVRR. The ex ante prioritarian SVRR is decreasing in current survival probability. (Note that $\frac{\partial S^{EAP}}{\partial p_i(t)} = g''(V_i)\left(S_i^U\right)^2 < 0$.) The effect of a one-period change in future survival probability is ambiguous.

$$\frac{\partial S^{EAP}}{\partial p_i(t)} = \sum_{s=t}^{T} \frac{\pi_i(s;A_i)}{p_i(A_i)} \beta^{s-1}u(y_i(s)) \frac{1}{p_i(t)} \left[g'(V_i) + g''(V_i) \sum_{s=A_i}^{T} \pi_i(s;A_i)\beta^{s-1}u(y_i(s)) \right]$$

. This is greater

than/equal to/less than 0 iff $\frac{-g''(V_i)}{g'(V_i)}$ is less than/equal to/greater than $\frac{1}{\sum_{s=A_i}^{T} \pi_i(s; A_i)\beta^{s-1}u(y_i(s))}$. Observing again that $V_i > \sum_{s=A_i}^{T} \pi_i(s; A_i)\beta^{s-1}u(y_i(s))$, we have a parallel result here as for the effect of present income, future income, and permanent income on the ex ante prioritarian SVRR (see above): a one-period change to future survival probability will increase the ex ante prioritarian SVRR if the coefficient of relative risk aversion for $g(.)$ is uniformly less than or equal to one.

The intuitions for these results are that an increase in current survival probability increases the individual's expected lifetime well-being, hence gives her lower priority as per W^{EAP}; while an increase in future survival probability has competing effects, both increasing the change to expected lifetime well-being of preventing the individual's current death, and increasing her level of expected lifetime well-being, with the first effect predominating if $g(.)$ is not too concave.

<u>Ex Post Prioritarian SVRR</u>. The ex post prioritarian SVRR is insensitive to a change to current survival probability. (Note that $\mu_i(t; A_i) = (1 - p_i(t+1))\prod_{s=A_i}^{t} p_i(s)$ for $t \geq A_i$, hence $p_i(A_i)$ drops out of the formula for S_i^{EPP}.) It is increasing in a one-period change to future survival probability. $\frac{\partial S^{EPP}}{\partial p_i(t)} = -\frac{\pi_i(t-1; A_i)}{p_i(A_i)} g(U_i(t-1)) + \sum_{s=t}^{T} \frac{\mu_i(s; A_i)}{p_i(A_i)p_i(t)} g(U_i(s)) > 0 \quad \forall t > A_i$.

<u>VSL.</u> VSL increases with a one-period change to future survival probabilities, and decreases with a change to current survival probability. This follows immediately from the results for the utilitarian SVRR—since VSL is the utilitarian SVRR divided by a denominator term that does not depend upon future survival probability, and increases as current survival probability does.

Next, we consider the effect on SVRR and VSL of a *permanent* difference in survival probability. Two individuals *i* and *j* are identical except that $p_j(t) = p_i(t) + \Delta q$, $\Delta q > 0$, for *every* period *t*. Our results are as follows; see Appendix for proofs. <u>Utilitarian SVRR</u>. The utilitarian SVRR increases with a permanent change in survival probability. <u>Ex ante Prioritarian SVRR</u>. Ambiguous. The ex ante prioritarian SVRR is increasing with a permanent change to survival probability if the coefficient of relative risk aversion of $g(.)$ is less than or equal to 1. <u>Ex Post Prioritarian SVRR</u>. The ex post prioritarian SVRR increases with a permanent change to survival probability. <u>VSL</u>. Ambiguous.

C. *A Summary*

The comparative statics of the SVRRs and VSL with respect to income and survival probability are summarized in Table 2 immediately below:

Table 2

	Income: Single-period difference	**Income: permanent difference**	**Survival probability: single-period difference**	**Survival probability: permanent difference**
Utilitarian SVRR	Past period: *Independent* Current period: *Increasing* Future period: *Increasing*	*Increasing*	Current period: *Independent* Future period: *Increasing*	*Increasing*
Ex Ante Prioritarian SVRR	Past period: *Decreasing* Current period: *Ambiguous* Future period: *Ambiguous* Note: The SVRR increases with a one-period increment to current or future income if *g*(.) is not too concave (coefficient of relative risk aversion ≤ 1)	*Ambiguous* Note: The SVRR increases with an increment to permanent income if *g*(.) is not too concave (coefficient of relative risk aversion ≤ 1)	Current period: *Decreasing* Future period: *Ambiguous* *Note*: The SVRR increases with a one-period increment to future survival probability if *g*(.) is not too concave (coefficient of relative risk aversion ≤ 1)	*Ambiguous* *Note*: The SVRR increases with a permanent change to survival probability if *g*(.) is not too concave (coefficient of relative risk aversion ≤ 1)
Ex Post Prioritarian SVRR	Past period: *Decreasing* Current period: *Increasing* Future period: *Increasing*	*Ambiguous*	Current period: *Independent* Future period: *Increasing*	*Increasing*
VSL	Past period: *Independent* Current period: *Increasing* Future period: *Increasing*	*Increasing*	Current period: *Decreasing* Future period: *Increasing*	*Ambiguous*

Much about this table is noteworthy. First, timing matters. Whether individuals who differ with respect to income, or with respect to survival probability, have divergent SVRRs or VSLs depends upon whether the income or survival probability difference occurs in the past, the present, or the future. Consider the columns for "income: single-period difference" and "survival probability: single-period difference." The following is true for each of the three SVRRs and for VSL: (1) its comparative statics (independent, increasing, decreasing, or ambiguous) are *not* the same for past, current, and future-period differences in income, and moreover (2) its comparative statics are *not* the same for current and future-period differences in survival probability.

Second, the prioritarian SVRRs, ex ante and ex post, are history-dependent—specifically, with respect to income. Each is decreasing with a one-period change to past income—by contrast with the utilitarian SVRR and VSL, which are independent of past income.

Third, this table confirms a key finding of Adler, Hammitt, and Treich (2014), using a simpler single-period model: the manner in which VSL values risk reduction is *not* robust to a change in social evaluation framework. VSL differs, in some significant way, from *each* of the SVRRs. VSL and the utilitarian SVRR have the same comparative statics with respect to income, but not survival probability. VSL and the prioritarian SVRRs have different comparative statics with respect to both income and survival probability.[12]

Fourth, the choice *within* the prioritarian family, between the ex ante and ex post prioritarian approaches, is seen to be significant. The ex ante prioritarian SVRR is decreasing in current survival probability and ambiguous with respect to future survival probability, while the ex post prioritarian SVRR is independent of current survival probability and increasing in future survival probability. Both SVRRs are decreasing in past income, but: the ex ante prioritarian SVRR is ambiguous with respect to current and future income, while the ex post prioritarian SVRR is *increasing* with current and future income.

This table, of course, concerns comparative statics: the direction of impact on VSL and the SVRRs of changes in risk and survival probability. It doesn't show the magnitude of impact—another type of difference between the various approaches. This difference will emerge in the following Part, where we empirically estimate VSL and the SVRRs for the US population.

III. SVRRs and VSL for the US Population

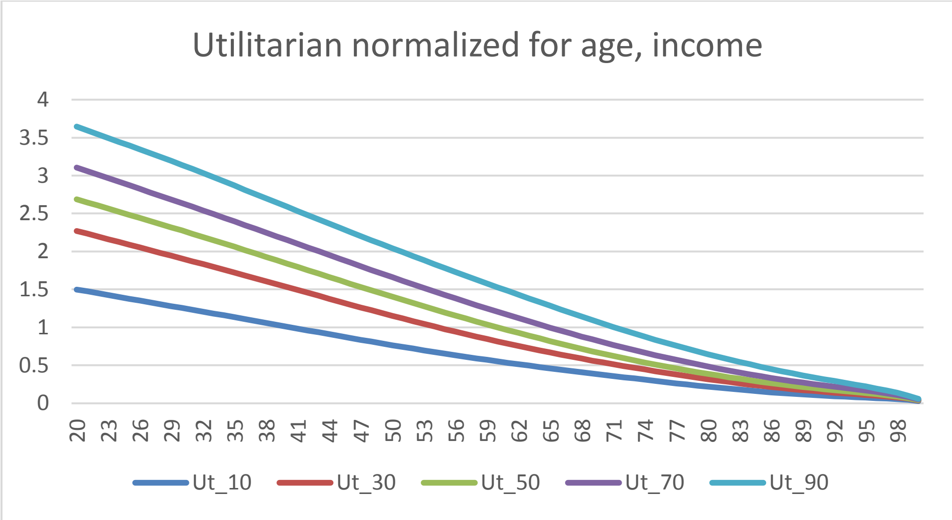
In this Part, we illustrate the SVRR and VSL concepts, and estimate their relative magnitudes, by calculating SVRR and VSL for cohorts of individuals characterized by varying risk profiles, income profiles and ages. The income and survival data for this exercise derive from the actual U.S. population. The U.S. Census Bureau collects data on the income distribution by age range. We used this to estimate the percentiles of the income distribution for each age. Assuming zero mobility (movement across percentiles), we determined a lifetime income profile for each percentile. The risk profile was based upon the actual U.S. population survival curve, and then adjusted by income percentile to reflect income differences in life expectancy.[13]

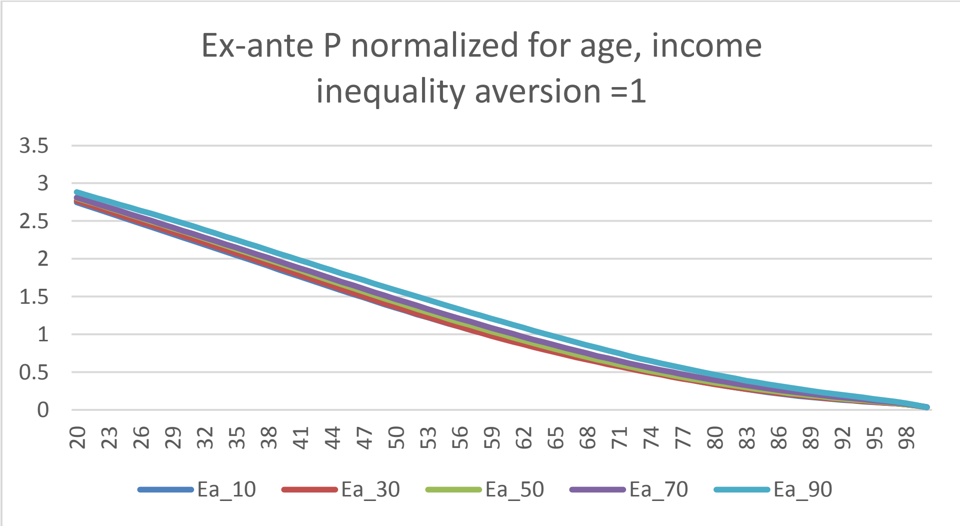
[12] Except that, if g(.) has a coefficient of relative risk aversion less than or equal to one, the comparative statics of ex ante prioritarianism with respect to survival probability are the same as BCA.

[13] Our income data was drawn from the U.S. Census Bureau, Current Population Survey, data on individual money income in 2016 (the most recent available when this empirical exercise was undertaken) . See https://www.census.gov/data/tables/time-series/demo/income-poverty/cps-pinc/pinc-01.html. The U.S. population survival curve was drawn from the United States Life Tables for 2014 (again the most recent available). https://www.cdc.gov/nchs/products/life_tables.htm. The methodology used to adjust the risk profile by income percentile to reflect income differences in life expectancy is the same used in Adler (2017).

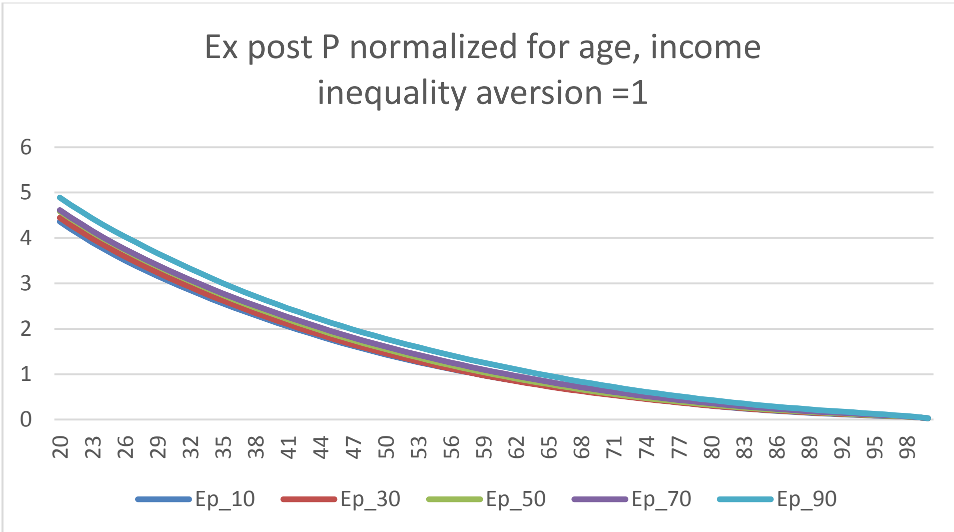
A logarithmic utility function was used: $u(c) = \ln c - \ln c^{zero}$, with c^{zero} set equal to $1000 (in the range of the World Bank's extreme poverty line[14]). The utility discount rate was set to 0. For the prioritarian SVRRs, we used an "Atkinson" (isoelastic) SWF with both a moderate inequality-aversion parameter (γ =1) and higher such parameter (γ = 2). This yields four different prioritarian SVRRs (namely ex post or ex ante, with γ =1 or 2).

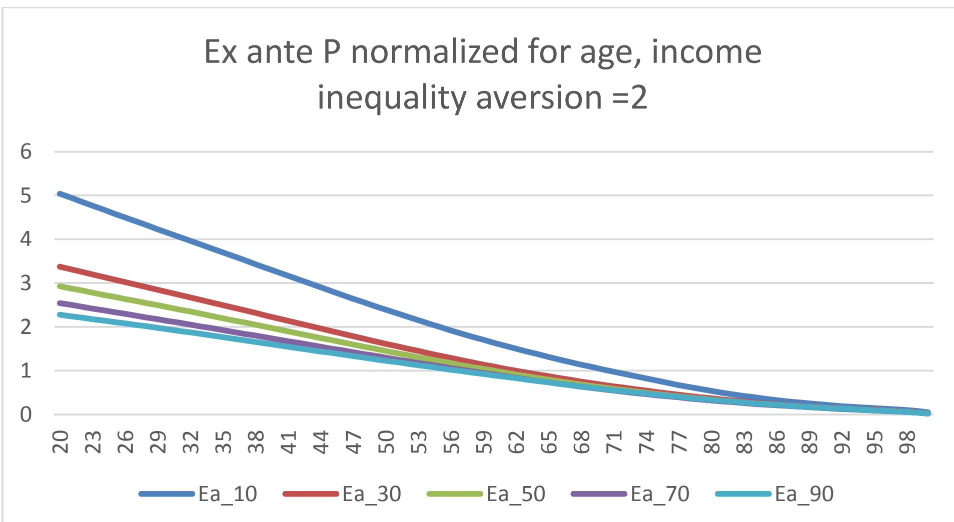
Figure 1





[14] The World Bank extreme poverty line is $1.90/day. (Ferreira et al. 2016).


Ex post P normalized for age, income inequality aversion =1
6
5
4
3
2
1
0
20 23 26 29 32 35 38 41 44 47 50 53 56 59 62 65 68 71 74 77 80 83 86 89 92 95 98
Ep_10 Ep_30 Ep_50 Ep_70 Ep_90


Ex ante P normalized for age, income inequality aversion =2
6
5
4
3
2
1
0
20 23 26 29 32 35 38 41 44 47 50 53 56 59 62 65 68 71 74 77 80 83 86 89 92 95 98
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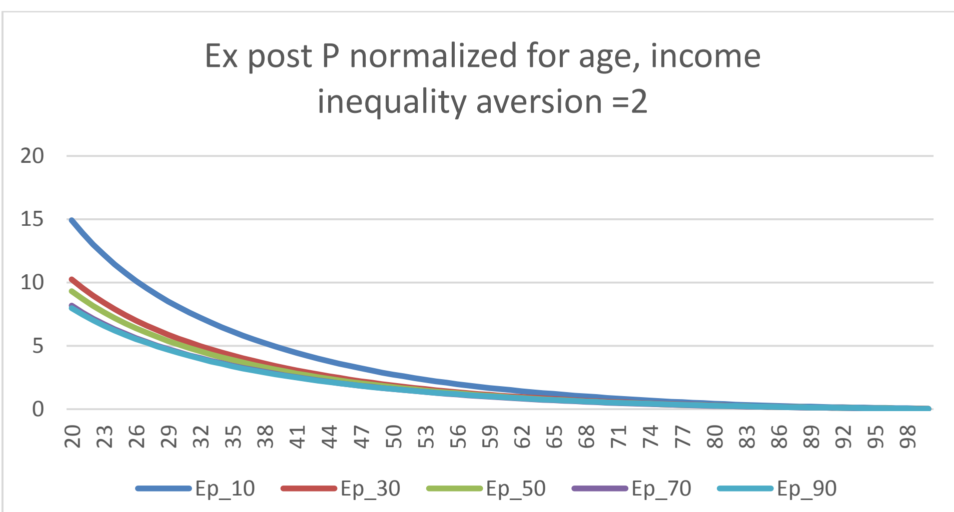
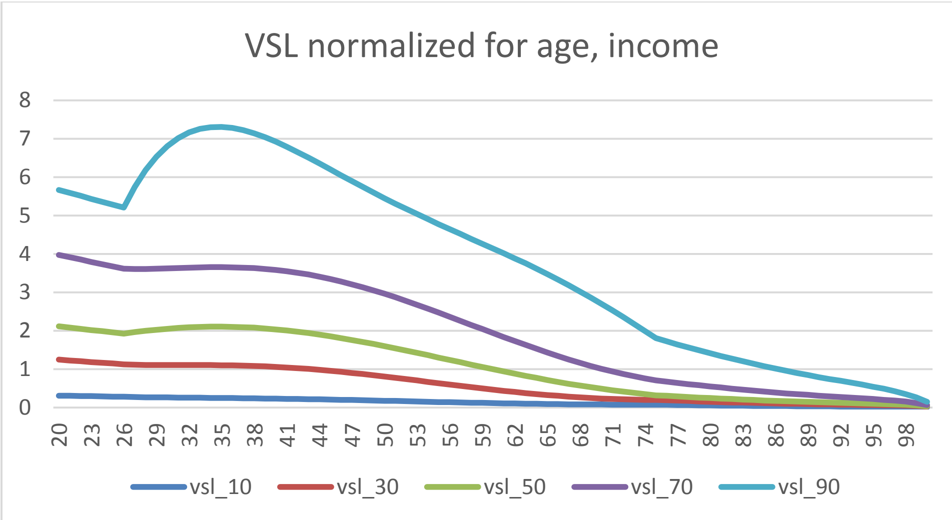

Ex post P normalized for age, income inequality aversion =2
20
15
10
5
0
20 23 26 29 32 35 38 41 44 47 50 53 56 59 62 65 68 71 74 77 80 83 86 89 92 95 98
Ep_10 Ep_30 Ep_50 Ep_70 Ep_90



The panels in Figure 1 display the utilitarian SVRR, the prioritarian SVRRs, and VSL as a function of age, for various percentiles of the income distribution. The results are normalized so that 1 represents the SVRR or VSL for a 60 year old, median income individual.

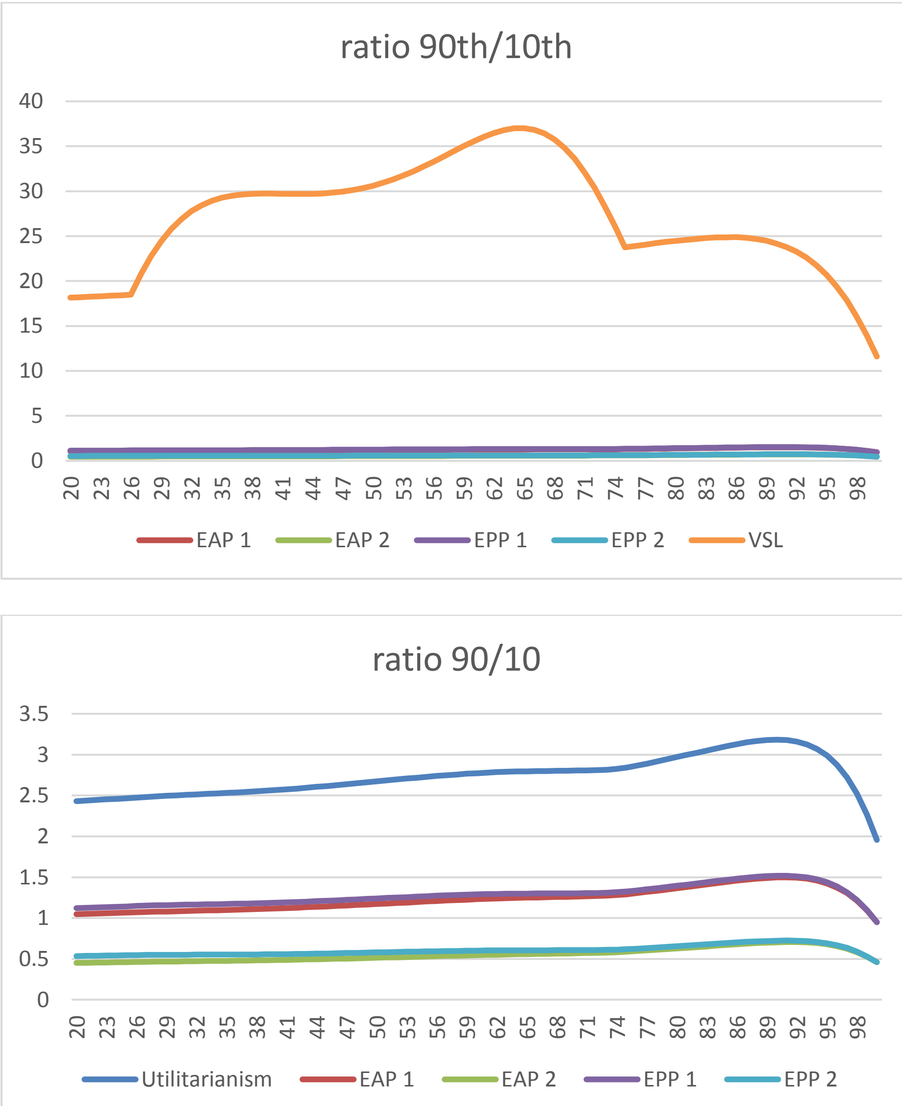
As the panels show, all the SVRRs decrease with age (even though this is not theoretically required—see Part II). The utilitarian SVRR and ex ante prioritarian SVRR have a similar degree of priority for the young at median income. Shifting from ex ante to ex post prioritarianism increases priority for the young.

The utilitarian SVRR increases with income: at every age, individuals in higher income percentiles have larger SVRRs. This is reversed for the prioritarian SVRRs with γ =2; at every age, SVRR decreases with income. γ =1 is an intermediate case, in which the utilitarian preference for income is almost neutralized but not reversed. Note here that the lines displaying the ex ante and ex post prioritarian SVRR as a function of age are virtually the same for all income percentiles. Thus the prioritarian SVRRs with moderate inequality aversion conform to lay moral judgments regarding lifesaving policies, namely that the young should take priority but income should make no difference.

VSL decreases with age for individuals above 40. At earlier ages, for some income percentiles, VSL displays the inverted U ("hump") shape often described in the literature.

The most striking difference between VSL and all the SVRRs concerns income effects: VSL increases with income at all ages, and much more steeply than even the utilitarian SVRR. This can be observed in Figure 1, and is displayed very clearly in Figure 2, which shows the ratio between SVRR or VSL at the 90^{th} and 10^{th} income percentiles as a function of age. That ratio is between 0.5 and 3 for all the SVRRs, while generally exceeds 20 for VSL.

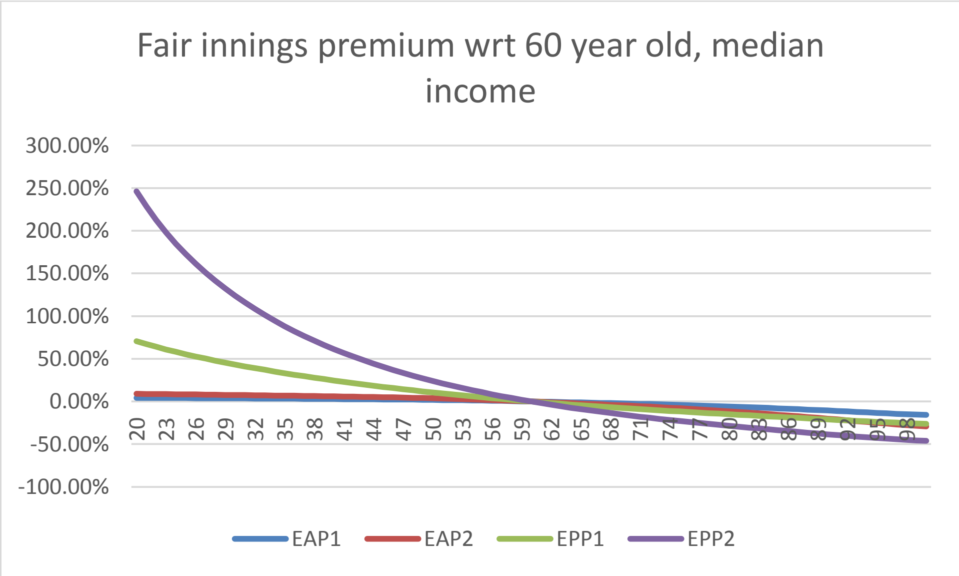
Figure 2

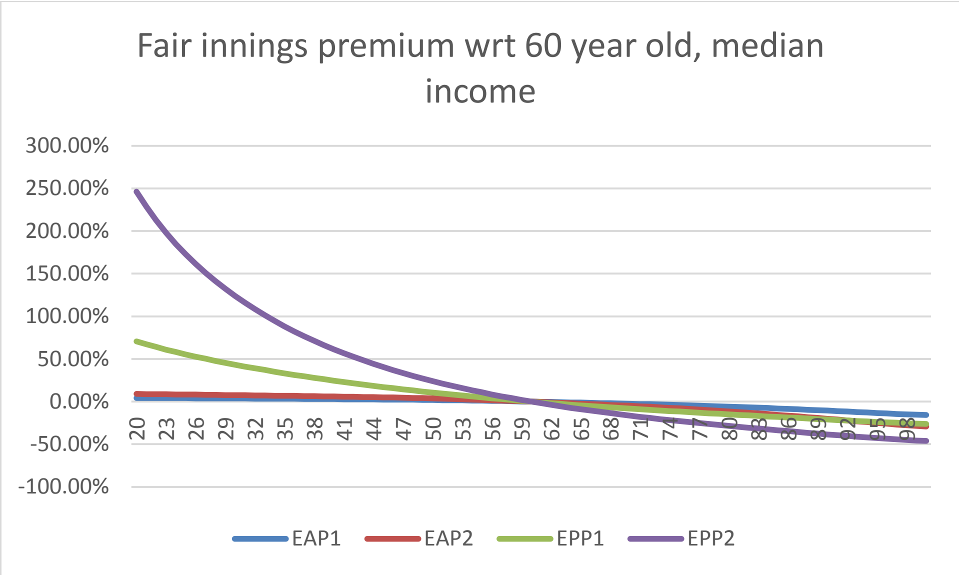


Our exercise here also sheds light on the U.S. government's practice of employing a single, population-average VSL, to value risk reduction. Such an approach is not only inconsistent with the theory of BCA—as Figure 1 shows, VSL varies by age and income—but also with the SWF framework. All of the SVRRs vary, at least, by age, and some by both age and income.

Finally (see Figure 3) we estimate a "fair innings premium." Recall that both ex ante prioritarian and ex post prioritarian SVRRs display a robust priority for the young, relative to utilitarianism: the ratio of prioritarian SVRRs, between a younger and older person with the same lifetime income and risk profile, is always larger than the ratio of utilitarian SVRRs. For individuals of the median income profile and associated risk profile, we calculate the percentage by which the prioritarian ratio between the SVRR of an individual of each age and a 60-year-old's SVRR exceeds the utilitarian young-to-old ratio. [15]

Figure 3





[15] That is, we calculate $[(S_j^{EAP} / S_{60}^{EAP}) / (S_j^U / S_{60}^U)] - 1$ and $[(S_j^{EPP} / S_{60}^{EPP}) / (S_j^U / S_{60}^U)] - 1$ for each age j.

Conclusion

The concept of the social value of risk reduction ($SVRR_i$) uses a social welfare function (SWF), rather than benefit-cost analysis (BCA), as the basis for quantifying the benefits of reducing an individual's fatality risk. $SVRR_i$—the social value per unit of risk reduction for individual i—is defined as $\frac{\partial W}{\partial p_i}$, with $W(.)$ the SWF and p_i individual i's current survival probability. By contrast, VSL_i, the construct that BCA uses to value risk reduction, is the marginal rate of substitution between current survival probability and income.

This Article significantly extends the work of Adler, Hammitt and Treich (2014) by using a multiperiod model—so that income, baseline risk and age all interact to determine $SVRR_i$. We consider utilitarian, ex post prioritarian, and ex ante prioritarian SWFs. We demonstrate that prioritarianism, in both variants, provides a rigorous basis for the notion of "fair innings": that a policy which produces a given increase in expected lifetime well-being for a younger person is preferable, ceteris paribus, to one that produces the same increase for an older person. This notion is formalized in two properties, Priority for the Young and Ratio Priority for the Young, that are satisfied by the ex ante prioritarian $SVRR_i$ and ex post prioritarian $SVRR_i$, but not VSL_i (or the utilitarian $SVRR_i$). We also show that each $SVRR_i$ differs from VSL_i in its comparative statics with respect to income and baseline risk. Finally, we undertake an empirical analysis, based upon the U.S. population survival curve and income distribution, which demonstrates the empirical significance of these differences between the SWF approach and BCA.

Appendix

What follows is a backup for the analysis in Parts II and III of the text. It shows the derivation of formulas/statements presented in the text, where that derivation is not mathematically or logically obvious.

A. *SVRR, VSL and Age*

In what follows, j is the younger person ($A_j < A_i$). Because the two individuals have common risk and income profiles, individual subscripts are dropped where possible (e.g., $p(t) = p_i(t) = p_j(t)$).

Utilitarian SVRR.

(a) The derivation of the formula stated in the text for $S_j^U - S_i^U$ is as follows.

$$S_j^U - S_i^U = \sum_{t=A_j}^{A_i-1} \frac{\pi(t;A_j)}{p(A_j)} \beta^{t-1} u(y(t)) + \sum_{t=A_i}^{T} \left(\frac{\pi(t;A_j)}{p(A_j)} - \frac{\pi(t;A_i)}{p(A_i)} \right) \beta^{t-1} u(y(t))$$

The first term on the RHS is $\sum_{t=A_j}^{A_i-1} \pi(t;A_j+1)\beta^{t-1}u(y(t))$, while the second term is equal to

$$\sum_{t=A_i}^{T} \left(\prod_{s=A_{j+1}}^{t} p(s) - \prod_{s=A_{i+1}}^{t} p(s) \right) \beta^{t-1} u(y(t)) = \sum_{t=A_i}^{T} \left(\pi(A_i;A_j+1)\pi(t;A_i+1) - \pi(t;A_i+1) \right) \beta^{t-1} u(y(t)) =$$

$$\left(\pi(A_i;A_j+1) - 1 \right) \sum_{t=A_i}^{T} \pi(t;A_i+1)\beta^{t-1} u(y(t)) \ .$$

(b) We note in the text that if income is constant and survival probabilities do not decrease with age, the utilitarian SVRR decreases with age. Let *u* be the (positive) utility of the constant income. Note that $S_i^U = u\beta^{A_i-1} + u\left(\sum_{t=A_i+1}^{T} \beta^{t-1} \prod_{s=A_i+1}^{t} p(s) \right)$.

$S_j^U = u\beta^{A_j-1} + u\left(\sum_{t=A_j+1}^{T-(A_i-A_j)} \beta^{t-1} \prod_{s=A_j+1}^{t} p(s) \right) + u\left(\sum_{t=T-(A_i-A_j)+1}^{T} \beta^{t-1} \prod_{s=A_j+1}^{t} p(s) \right)$. Note that $\left(\sum_{t=A_i+1}^{T} \beta^{t-1} \prod_{s=A_i+1}^{t} p(s) \right)$ and $\left(\sum_{t=A_j+1}^{T-(A_i-A_j)} \beta^{t-1} \prod_{s=A_j+1}^{t} p(s) \right)$ each have ($T - A_i$) terms and that, if $p(t)$ does not increase with time and $0<\beta\leq 1$, it must be the case that $\left(\sum_{t=A_j+1}^{T-(A_i-A_j)} \beta^{t-1} \prod_{s=A_j+1}^{t} p(s) \right) \geq \left(\sum_{t=A_i+1}^{T} \beta^{t-1} \prod_{s=A_i+1}^{t} p(s) \right)$. Because $u\left(\sum_{t=T-(A_i-A_j)+1}^{T} \beta^{t-1} \prod_{s=A_j+1}^{t} p(s) \right) > 0$, we have that $S_j^U > S_i^U$.

<u>Ex Post Prioritarian SVRR</u>.

(a) The derivation of the formula stated in the text for $S_j^{EPP} - S_i^{EPP}$ is as follows.

$$S_j^{EPP} - S_i^{EPP} = \sum_{t=A_j}^{A_i-1} \frac{\mu(t;A_j)}{p(A_j)} g(U(t)) + \sum_{t=A_i}^{T} \left(\frac{\mu(t;A_j)}{p(A_j)} - \frac{\mu(t;A_i)}{p(A_i)} \right) g(U(t)) + \left(g(U(A_i-1)) - g(U(A_j-1)) \right)$$

The first term on the RHS is $\sum_{t=A_j}^{A_i-1} \mu(t; A_j+1) g(U(t))$. The second term is equal to:

$$\sum_{t=A_i}^{T}\left((1-p(t+1))\prod_{s=A_{j+1}}^{t} p(s)-(1-p(t+1))\prod_{s=A_{i+1}}^{t} p(s)\right)g(U(t))=\sum_{t=A_i}^{T}\left(\pi(A_i; A_j+1)\mu(t; A_i+1)-\mu(t; A_i+1)\right)g(U(t))$$

$$=(\pi(A_i; A_j+1)-1)\sum_{t=A_i}^{T}\mu(t; A_i+1)g(U(t))$$

(b) *Ratio Priority for the Young*. We prove that the ex post prioritarian SVRR displays Ratio Priority for the Young: $(S_j^{EPP}/S_i^{EPP})>(S_j^U/S_i^U)$. It follows that the ex post prioritarian SVRR displays Priority for the Young, and we don't demonstrate that directly. In what follows, we'll abbreviate $g(U(A_j-1))$ and $g(U(A_i-1))$ as $g_{(A_j-1)}$ and $g_{(A_i-1)}$, respectively; and $U(A_j-1)$ and $U(A_i-1)$ as $U_{(A_j-1)}$ and $U_{(A_i-1)}$, respectively.

$S_j^{EPP}=-g_{(A_j-1)}+\sum_{t=A_j}^{A_i-1}\frac{\mu(t; A_j)}{p(A_j)}g(U(t))+\sum_{t=A_i}^{T}\frac{\mu(t; A_j)}{p(A_j)}g(U(t))$. Note now that $\sum_{t=A_i}^{T}\frac{\mu(t; A_j)}{p(A_j)}=\frac{\pi(A_i)}{\pi(A_j)}$ and $\sum_{t=A_j}^{A_i-1}\frac{\mu(t; A_j)}{p(A_j)}=1-\frac{\pi(A_i)}{\pi(A_j)}$. Thus we have that

$$S_j^{EPP}=\frac{\pi(A_i)}{\pi(A_j)}(g_{(A_i-1)}-g_{(A_j-1)})+\sum_{t=A_j}^{A_i-1}\frac{\mu(t; A_j)}{p(A_j)}\left(g(U(t))-g_{(A_j-1)}\right)+\sum_{t=A_i}^{T}\frac{\mu(t; A_j)}{p(A_j)}\left(g(U(t))-g_{(A_i-1)}\right).$$

$$S_i^{EPP}=-g_{(A_i-1)}+\sum_{t=A_i}^{T}\frac{\mu(t; A_i)}{p(A_i)}g(U(t))=\frac{\pi(A_j)}{\pi(A_i)}\sum_{t=A_i}^{T}\frac{\mu(t; A_j)}{p(A_j)}\left(g(U(t))-g_{(A_i-1)}\right).$$

Turning to the utilitarian SVRR: while the text provides the formula $S_k^U=\sum_{t=A_k}^{T}\frac{\pi_k(t; A_k)}{p_k(A_k)}\beta^{t-1}u(y_k(t))$ for $k = i, j$, it's straightforward to arrive at an alternative formula, parallel to that for the ex post prioritarian SVRR, namely: $S_k^U=-U_k(A_k-1)+\sum_{t=A_k}^{T}\frac{\mu_k(t; A_k)}{p_k(A_k)}U_k(t)$.

Thus, we can proceed by steps parallel to those immediately above to derive the following expressions for S_i^U and S_j^U.

$$S_j^U=\frac{\pi(A_i)}{\pi(A_j)}(U_{(A_i-1)}-U_{(A_j-1)})+\sum_{t=A_j}^{A_i-1}\frac{\mu(t; A_j)}{p(A_j)}\left(U(t)-U_{(A_j-1)}\right)+\sum_{t=A_i}^{T}\frac{\mu(t; A_j)}{p(A_j)}\left(U(t)-U_{(A_i-1)}\right).$$

$$S_i^U=\frac{\pi(A_j)}{\pi(A_i)}\sum_{t=A_i}^{T}\frac{\mu(t; A_j)}{p(A_j)}\left(U(t)-U_{(A_i-1)}\right).$$

Observe that $\dfrac{S_j^{EPP}}{S_i^{EPP}} = \dfrac{\dfrac{\pi(A_i)}{\pi(A_j)}(g_{(A_i-1)} - g_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\dfrac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)} + \dfrac{\pi(A_i)}{\pi(A_j)}$, and

that $\dfrac{S_j^U}{S_i^U} = \dfrac{\dfrac{\pi(A_i)}{\pi(A_j)}(U_{(A_i-1)} - U_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)}{\dfrac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)} + \dfrac{\pi(A_i)}{\pi(A_j)}$. Thus $\dfrac{S_j^{EPP}}{S_i^{EPP}} > \dfrac{S_j^U}{S_i^U}$ iff

$$\frac{\dfrac{\pi(A_i)}{\pi(A_j)}(g_{(A_i-1)} - g_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\dfrac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)} > \frac{\dfrac{\pi(A_i)}{\pi(A_j)}(U_{(A_i-1)} - U_{(A_j-1)}) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)}{\dfrac{\pi(A_j)}{\pi(A_i)} \sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)}$$

Equivalently, $\dfrac{S_j^{EPP}}{S_i^{EPP}} > \dfrac{S_j^U}{S_i^U}$ iff

$$\frac{\dfrac{\pi(A_i)}{\pi(A_j)}\left(g_{(A_i-1)} - g_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\dfrac{\pi(A_i)}{\pi(A_j)}\left(U_{(A_i-1)} - U_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)} > \frac{\sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)}{\sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)} .$$

Let $\theta = \dfrac{g_{(A_i)} - g_{(A_i-1)}}{U_{(A_i)} - U_{(A_i-1)}}$. Note that $\dfrac{\sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_i-1)}\right)}{\sum_{t=A_i}^{T} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_i-1)}\right)} \le \theta$. This is because—by the strict concavity of $g(\cdot)$—each term in the numerator of the preceding fraction is less than or equal to θ times the corresponding term in the denominator. Similarly,

$$\frac{\dfrac{\pi(A_i)}{\pi(A_j)}\left(g_{(A_i-1)} - g_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(g(U(t)) - g_{(A_j-1)}\right)}{\dfrac{\pi(A_i)}{\pi(A_j)}\left(U_{(A_i-1)} - U_{(A_j-1)}\right) + \sum_{t=A_j}^{A_i-1} \dfrac{\mu(t;A_j)}{p(A_j)}\left(U(t) - U_{(A_j-1)}\right)} > \theta$$

. QED.

B. *SVRR, VSL and Income*

As discussed in the text, the comparative statics of SVRR with respect to a single-period difference in income are determined by examining the sign of $\frac{\partial S_i}{\partial y_i(t)}$, while the comparative statics of VSL are straightforward in light of the definition of VSL in terms of S_i^U. What follows are derivations of the comparative statics with respect to changes to permanent income.

Utilitarian SVRR. Because the utilitarian SVRR is independent of a single-period change to past income, and increases with a single-period increment to present or future income, it clearly increases with an increment to permanent income.

Ex Ante Prioritarian SVRR. To be written.

Ex Post Prioritarian SVRR. To be written.

VSL. Because VSL is independent of a single-period change to past income, and increases with a single-period increment to present or future income, it clearly increases with an increment to permanent income.

C. *SVRR, VSL and Baseline Risk*

As discussed in the text, the comparative statics of SVRR with respect to a single-period difference in survival probability are determined by examining the sign of $\frac{\partial S_i}{\partial p_i(t)}$, while the comparative statics of VSL are straightforward in light of the definition of VSL in terms of S_i^U. What follows are derivations of the comparative statics with respect to a permanent (present and future) difference in survival probability.

Utilitarian SVRR. Because the utilitarian SVRR is insensitive to a change in current survival probability, and increases with a one-period increment in future survival probability, it clearly increases with a permanent increment in survival probability.

Ex ante prioritarian SVRR. To be written.

Ex post prioritarian SVRR. Because the ex post prioritarian SVRR is insensitive to a change in current survival probability, and increases with a one-period increment in future survival probability, it clearly increases with a permanent increment in survival probability.

References

Adler, Matthew D. 2012. *Well-Being and Fair Distribution: Beyond Cost-Benefit Analysis*. New York: Oxford University Press.

Adler, Matthew D. 2017. "A better calculus for regulators: From cost-benefit analysis to the social welfare function." Working paper, Duke Law School Public Law and Legal Theory Series No. 2017-19. Available at https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2923829.

Adler, Matthew D., James K. Hammitt, and Nicolas Treich. 2014. "The social value of mortality risk reduction: VSL versus the social welfare function approach." *Journal of Health Economics* 35: 82-93.

Aldy Joseph E. and W. Kip Viscusi. 2007. "Age differences in the value of statistical life: Revealed preference evidence." *Review of Environmental Economics and Policy*, 1:241–260.

Bognar, Greg. 2008. "Age-weighting." *Economics and Philosophy* 24: 167-189.

Bognar, Greg. 2015. "Fair innings." *Bioethics* 29:251-61.

Daniels, Norman. 1988. *Am I My Parents' Keeper? An Essay on Justice between the Young and the Old*. New York: Oxford University Press.

Ferreira, Francisco, et al. 2016. "A global count of the extreme poor in 2012." *Journal of Economic Inequality* 14: 141-72.

Hammitt, James K. 2007. "Valuing changes in mortality risk: Lives saved versus life years saved." *Review of Environmental Economics and Policy*, 1:228–240.

Harris, John. 1985. *The Value of Life*. London: Routledge and Kegan Paul.

Nord, Erik. 2005. "Concerns for the worse off: Fair innings versus severity." *Social Science and Medicine* 60: 257-63.

Lockwood, M. 1988. Quality of life and resource allocation. In Philosophy and Medical Welfare, ed. J. M. Bell and S. Mendus, 33–55. Cambridge: Cambridge University Press.

Williams, Alan. 1997. Intergenerational equity: An exploration of the ''fair innings'' argument. *Health Economics* 6: 117–132.