Trading and Shareholder Voting*

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Abstract

We study shareholder voting in a model in which trading affects the composition of the shareholder base. In this model, trading and voting are complementary, which gives rise to self-fulfilling expectations about proposal acceptance and multiple equilibria. We show three main results. First, increasing liquidity may reduce prices and welfare, because it allows shareholders with more extreme preferences to accumulate large positions and impose their views on more moderate shareholders through voting. Second, prices and welfare can move in opposite directions, which suggests that the former is an invalid proxy for the latter. Third, delegation of the decision to a board of directors may strictly improve shareholder value. However, the optimal board is generally biased, should not be representative of current shareholders, and may not always garner voting support from the majority of shareholders.

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"Shareholders express views by buying and selling shares; (...) The more shareholders govern, the more poorly the firms do in the marketplace. Shareholders' interests are protected not by voting, but by the market for stock (...)." (Easterbrook and Fischel (1983), pp. 396-397)

1 Introduction

Recent regulatory reforms in advanced economies have empowered shareholders and enhanced their voting rights in an effort to constrain managerial discretion.[1] As a result, shareholders not only elect directors, but frequently vote on executive compensation, corporate transactions, changes to the corporate charter, and social or environmental policies. This shift of power from boards to shareholder meetings takes for granted that shareholder voting increases welfare and firm valuations by aligning the preferences of those who make decisions with those for whom decisions are made – a form of "corporate democracy."[2] However, unlike the political setting, a key feature of the corporate setting is the existence of the market for shares, which allows investors to choose their ownership stakes based on their preferences and the stock price. Thus, who gets to vote on the firm's policies is fundamentally linked to voters' views on how the firm should be run. While the literature has looked at many important questions in the context of shareholder voting, it has so far not examined the effectiveness of voting when the shareholder base forms endogenously through trading.[3] The main goal of this paper is to examine the link between trading and voting and its implications for companies' valuations, and to highlight how the effectiveness of shareholder voting vis-a-vis board decision-making is affected by the firm's trading environment.

Specifically, we study the relationship between trading and voting in a context in which

[1]Cremers and Sepe (2016) make the same observation and review the large legal literature on the subject (see also Hayden and Bodie, 2008). The finance literature has assembled a wealth of empirical evidence on this shift, including the discussion on the effectiveness of say-on-pay votes, surveyed by Ferri and Göx (2018), reforms to disclose mutual fund votes in the United States (e.g., Davis and Kim, 2007; Cvijanovic, Dasgupta, and Zachariadis, 2016) and the introduction of mandatory voting on some takeover proposals in the UK (Becht, Polo, and Rossi, 2016).

[2]See, e.g., the speech by SEC Commissioner Luis A. Aguilar (Aguilar, 2009).

[3]Karpoff (2001) surveys the earlier and Yermack (2010) the later literature on shareholder voting.

shareholders differ in their attitudes toward proposals. We provide several key insights. First, trading aligns the shareholder base with the expected outcome, even if the expected outcome is not optimal. As a result, there can be multiple equilibria, so that similar firms can end up having very different ownership structures and taking very different strategic directions – a source of non-fundamental indeterminacy. Second, changes in the governance or trading environment of the firm can affect welfare and prices in opposite directions, which suggests that price reactions to voting outcomes may not be a valid empirical proxy for their welfare effects. Third, while higher market liquidity increases the ability of shareholders to gain from trade, they may nevertheless reduce welfare by allowing the shareholder base to become more extreme, so that the views of more extreme shareholders prevail over those with more moderate attitudes. Finally, shareholder welfare can be increased if, instead of voting, decisions are delegated to the board of directors. Moreover, the optimal choice between voting and delegation to the board crucially depends on market liquidity and potential shifts in the shareholder base.

We consider a model in which a continuum of shareholders first trade their shares in a competitive market and then vote on a proposal. Each shareholder's valuation of the proposal depends on an uncertain common value that all shareholders share, but also on a private value that reflects shareholders' different attitudes toward the proposal. After shareholders trade, but before they vote on the proposal, they observe a signal on the proposal's common value; the signal is public and there is no asymmetric information. Because of private values, some shareholders are biased toward the proposal and vote to accept it even if the common value is expected to be low; we call them *activist* shareholders, because they want to change the status quo. By contrast, other shareholders are biased against the proposal and have a higher bar for accepting it; we call them *conservative,* since they are biased in favor of the status quo. These different attitudes between shareholders may reflect private benefits from their ties with the company or ownership of other firms, different social or political views ("investor ideology"), time horizons, risk aversion, and tax considerations.[4] Some commentators even argue that

[4]Matvos and Ostrovsky (2010) analyze mutual fund votes and show that they differ systematically in their support for management. Cvijanovic, Dasgupta, and Zachariadis (2016) and He, Huang, and Zhao (2019) analyze the heterogeneity between mutual funds that arises, respectively, because they also run companies' pension funds, and because of differences in institutional cross-ownership. Some shareholders have interests that set them apart from other shareholders, e.g., unions (Agrawal, 2012), family shareholders and founders (Mullins

shareholder voting should be seen as a system to aggregate heterogeneous preferences (Hayden and Bodie, 2008).

We start by analyzing the setting in which shareholders can trade but cannot vote, e.g., if the decision on the proposal is taken by the board of directors. Because of heterogeneous preferences, shareholders differ in their valuation of the firm, which creates gains from trade. The equilibrium is unique and can be of two types, depending on the likelihood that the board will adopt the proposal. If the probability of adoption is above a certain threshold, then activist shareholders value the firm more than conservative shareholders and will buy shares from them, whereas in the opposite case, conservatives will buy and activists will sell. Thus, trading allows shareholders who do not agree with the company's decisions to sell to those shareholders who expect their preferred alternative to be chosen and to benefit from the higher price the buyers are willing to pay.

By contrast, we show that if the decision on the proposal is made by a shareholder vote, i.e., if shareholders first trade and then vote, then multiple equilibria can arise. An *activist* equilibrium, in which the proposal is accepted with a relatively high probability, can co-exist with a *conservative* equilibrium, in which the proposal is likely to be rejected.[5] Multiplicity arises because voting creates a strategic complementarity: If shareholders expect a high likelihood of proposal adoption, the more conservative shareholders sell to the more activist shareholders. As a result, the composition of the shareholder base after trading is more activist and proposals are approved more often, confirming the ex-ante expectations. Similarly, for the same parameters, if shareholders expect a low likelihood of proposal adoption, then trades occur in the opposite direction, creating a more conservative shareholder base, which approves the proposal less frequently. In both cases, expectations about the voting outcome are self-fulfilling.

and Schoar, 2016; Villalonga and Amit, 2006), CEOs, and governments. Bolton et al. (2019) and Bubb and Catan (2019) develop different classifications to capture the cross-sectional variation of shareholders' attitudes to optimal corporate governance. Bushee (1998) and Gaspar, Massa, and Matos (2005) analyze the implications of differences in time horizons between investors. Hayden and Bodie (2008) provide a comprehensive overview of different sources from which conflicts of interests may arise.

[5]Classic examples of multiple equilibrium models in financial economics include Diamond and Dybvig (1983) on bank runs; Calvo (1988) on debt repudiation; and Obstfeld (1996) on currency crises. Some researchers treat multiple equilibria as a modeling problem (Morris and Shin, 2000), whereas others suggest that the multiplicity of equilibria may be genuine based on theoretical and experimental results (Angeletos and Werning, 2006; Heinemann, Nagel, and Ockenfels, 2004).

The multiplicity of equilibria sheds light on a source of non-fundamental indeterminacy and highlights potential empirical challenges in analyzing shareholder voting, since firms with the same fundamental characteristics can have different ownership structures and adopt different policies. We show that such multiplicity is especially likely when the firm faces low trading frictions and high heterogeneity of the initial shareholder base. In the Conclusion we discuss how shareholders may coordinate if there are multiple equilibria.

Our second set of results explores price and welfare effects. Our analysis highlights that prices and welfare may react differently and in opposite directions to changes to the corporate governance or trading environment of the firm. Intuitively, the decision on the proposal depends on the identity of the marginal voter, which is determined by the post-trade shareholder base and the majority requirement. For example, under simple majority, the marginal voter is the median voter among the post-trade shareholders. The share price depends on how proposal adoption affects the valuation of the marginal trader, who is just indifferent between buying and selling shares. Hence, the share price decreases if the gap between the marginal voter and the marginal trader widens. By contrast, the aggregate welfare depends on how proposal adoption affects the valuation of the average shareholder who holds shares after trading. Thus, welfare decreases if the gap between the marginal voter and the average post-trade shareholder widens.

Prices and welfare react differently to policy changes if the marginal voter is more extreme than the marginal trader, but is less extreme than the average post-trade shareholder. In this case, a policy change, e.g., an increase in the majority requirement, shifts the marginal voter in a way that either moves him closer to the marginal trader but farther from the average post-trade shareholder, or the opposite. Hence, prices increase (decrease) exactly when welfare decreases (increases). This result challenges the notion that there is a close connection between welfare and prices, which the literature often relies on. It casts doubt on the validity of price reactions as an empirical proxy for the welfare effects of shareholder voting on proposals.

Our analysis also uncovers a novel effect of market liquidity on prices and welfare. In our model, liquidity summarizes all trading opportunities, e.g., from higher market depth or lower wealth constraints. If shareholders do not vote, e.g., if decisions over the proposal are made by the board, higher liquidity always results in higher prices and higher welfare: Shareholder heterogeneity creates gains from trade, and more liquid markets allow more gains from trade to be realized. However, when decisions are made by a shareholder vote, higher liquidity may be detrimental for both, prices and welfare. Intuitively, as opportunities to trade increase, the shareholder base becomes more extreme — e.g., the post-trade shareholder base becomes more activist in the activist equilibrium. This may widen the gap between the marginal voter and the average shareholder and thereby reduce welfare. Similarly, more trading can depress the stock price, because it widens the gap between the marginal voter and the marginal trader, whose valuation sets prices. Put differently, more liquidity allows more extreme investors to accumulate larger positions and impose their extreme views on more moderate shareholders through voting.

Finally, we examine the optimal allocation of power between boards and shareholder meetings by comparing welfare in the two settings described above – when shareholders trade and vote; and when shareholders trade but decisions are made by the board. The board, like each of the shareholders, is characterized by its attitude toward the proposal.

We define the optimal board as that which maximizes the initial shareholder welfare. We first show that the optimal board is biased and does not reflect the preferences of the initial shareholder base. Instead, the optimal board maximizes the average valuation of the post-trade shareholder base. Intuitively, the optimal board caters to the preferences of the shareholders with the highest willingness to pay, rather than to the average pre-trade shareholder. Indeed, if the board's preferences are aligned with those of more extreme shareholders, it also benefits shareholders with more moderate views, who can now sell their shares to those with more extreme views for a higher price. Essentially, the design of an optimal board accounts for gains from trade between shareholders with different views. Importantly, the optimal board, and even a "good enough" board that is sufficiently similar to the optimal board, increases shareholder welfare relative to decision-making via shareholder voting. In other words, the argument that whenever the board is biased, decisions should be delegated to shareholders, is not necessarily correct if shareholders can trade. Similarly, the objective of the optimal board should not be to maximize the share price, since the price reflects only the preferences of the marginal trader and not those of the average shareholder.

Even if it is optimal to delegate decision-making to the board, it is not guaranteed that the majority of shareholders will want to do so. To show this, we extend the model by adding a stage before trading in which shareholders vote on whether to delegate the decision on the proposal to the board. We show that shareholders may choose not to delegate decision-making to a board, not even an optimal board, because with voting before trading, a new externality arises: Shareholders who expect to buy shares after the vote on delegation consider not only the implications of delegation for the long-term value of the firm, but also for the short-term price at which they can buy shares from those shareholders who sell. As a result, short-term trading considerations may push these shareholders to vote against delegation to an optimal board in order to benefit from a lower price.

We discuss several extensions of our baseline model in Section 7. We show that our results are robust to allowing for general social preferences, which shareholders may have separately from their investment in the firm, to including a second round of trading after the vote, and to relaxing some simplifying assumptions in our baseline setup.

Overall, we strike a cautious note on the general movement to "shareholder democracy." Since shareholders can trade their shares, giving them voting rights creates a complementarity between voting and trading that gives rise to multiple equilibria. There is no guarantee that shareholders can always coordinate on the welfare-dominant equilibrium. Moreover, even the best voting equilibrium is dominated not only by delegation to an optimal board, but also by delegation to a "good enough" board. Finally, shareholders might make incorrect decisions when delegating their decision-making rights to the board if they give excessive weight to short-term trading considerations. As such, we resonate the critical stance of Easterbrook and Fischel (1983) in the opening vignette and expand on these issues in the Conclusion.

The remainder of the paper is organized as follows. Section 2 surveys the literature. Section 3 introduces the setup. Section 4 first analyzes two benchmarks that consider trading and voting separately, and then characterizes the equilibrium of the model with trading and voting. Section 5 discusses the implications for shareholder welfare and prices. Section 6 examines the benefits of delegating decision-making authority to the board of directors. Section 7 discusses several extensions of the baseline model. Section 8 concludes. All proofs are gathered in the Appendix. The Online Appendix presents the analysis of the model extensions.

2 Discussion of the literature

Our paper is related to the theoretical literature on shareholder voting (Maug and Rydqvist, 2009; Levit and Malenko, 2011; Van Wesep, 2014; Malenko and Malenko, 2019; and Bar-Isaac and Shapiro, 2019). These papers all assume an exogenous shareholder base and discuss strategic interactions between shareholders based on heterogeneous information, heterogeneous preferences, or both. By contrast, our analysis endogenizes the shareholder base and asks how the voting equilibrium changes if shareholders can trade before voting. Musto and Yilmaz (2003) analyze how adding a financial market changes political voting outcomes. However, in their model voters trade financial claims but not the votes, which is different from the corporate context. Overall, our paper contributes to this literature by overcoming an important theoretical challenge when analyzing shareholder voting: Shareholders' valuations and their trading decisions depend on expected voting outcomes, but voting outcomes depend in turn on the composition of the shareholder base, which is endogenous and changes through trading.

We are aware of three strands of literature that integrate the analysis of shareholder voting with trading. The first is the literature on general equilibrium economies with incomplete markets, including Gevers (1974), Drèze (1985), DeMarzo (1993), and Kelsey and Milne (1996). This literature recognizes that shareholders with different preferences will be unanimous and production decisions can be separated from consumption decisions (Fisher separation) only if markets are complete.[6] With incomplete markets, shareholders will generally disagree about the optimal production plans of the firm, since shareholders are not only interested in profit maximization but also in the influence of firms' decisions on product prices (e.g., Kelsey and Milne, 1996). Then conflicts of interest arise, governance mechanisms become necessary, and the objective of the firm becomes undefined. The models in this literature introduce mecha-

[6]Hirshleifer (1966) shows that Fisher separation obtains in an inter-temporal production economy with complete markets in a state-preference framework.

nisms such as voting, blockholders, or boards of directors to close this gap.[7] One important insight from this literature is that shareholder disagreement over companies' policies and governance mechanisms to resolve conflicts between shareholders both originate from incomplete markets. Compared to this earlier literature, we analyze a less general model, which allows us to characterize equilibria beyond existence, analyze the way in which voting and trading interact, derive implications for shareholder welfare, and characterize delegation decisions and their properties.

The second literature analyzes the issues that arise when financial markets allow traders to exercise voting rights without exposure to the firm's cash flows. Blair, Golbe, and Gerard (1989), Neeman and Orosel (2006), and Kalay and Pant (2009) show that vote-buying can enhance the efficiency of contests for corporate control. Brav and Mathews (2011) build a model of empty voting and conclude that the implications for efficiency are ambiguous and depend on transaction costs and shareholders' ability to evaluate proposals. Esö, Hansen, and White (2014) argue that empty voting may improve information aggregation. Our paper is complementary to this literature, since we abstract from derivatives markets and vote-trading and assume one-share-one-vote throughout.[8] Our theory builds on the fact that cash flow rights and voting rights are bundled in the same security, which is a fundamental feature of most publicly traded stocks.

The third literature analyzes blockholders who form large blocks endogenously through trading and affect governance through voice or exit (see Edmans (2014) and Edmans and Holderness (2017) for surveys). However, this literature does not focus on the complementarities and collective action problems that arise in our model, as the majority of this literature focuses on models with a single blockholder. Relative to existing governance models of multiple blockholders (Zwiebel, 1995; Noe, 2002; Edmans and Manso, 2011; and Brav, Dasgupta, and Mathews, 2017), our paper analyzes the feedback loop between voting and trading and how this affects the choice between delegation to a board and shareholder voting.[9]

[7] See Gevers (1974), Drèze (1985), and DeMarzo (1993) on majority voting. Drèze (1985) and DeMarzo (1993) conceptualize the board of directors as a group of controlling blockholders.

[8] Burkart and Lee (2008) provide a comprehensive survey of the theoretical literature on the one-share-one-vote structure.

[9] Garlappi, Giammarino, and Lazrak (2017; 2019) analyze group decision-making about investment projects

Broadly, our paper is also related to the literature on real effects of financial markets (see Bond, Edmans, and Goldstein (2012) for a survey). This literature focuses on price formation and information aggregation in financial markets and asks how information is transferred from markets to decision-makers, where the preferences of decision-makers are assumed to be exogenous. Our paper does not feature information aggregation and instead highlights a new force through which financial markets have real effects by allowing the shareholder base to shift: The preferences of decision-makers are endogenous and result from trading.

3 Model

Consider a firm with a continuum of measure one of risk-neutral shareholders, indexed by b. Each shareholder is endowed with $e > 0$ shares. There is a proposal on which shareholders vote. The proposal can be either accepted ($d = 1$) or rejected ($d = 0$). Each share has one vote. If a proportion of more than $\tau \in (0, 1)$ of all shares are cast in favor of the proposal, the proposal is accepted. Otherwise, the proposal is rejected.[10]

Preferences. Shareholders differ in their preferences regarding the proposal. The value of a share from the perspective of shareholder indexed by b depends on the state $\theta \in \{-1, 1\}$, on whether or not the proposal is accepted $d \in \{0, 1\}$, and on the shareholder's bias b:

$$v(d, \theta, b) = v_0 + (\theta + b)(d - \phi), \tag{1}$$

where $v_0 \geq 0$ is sufficiently large to ensure that shareholder value is non-negative under all circumstances. The state θ captures the part of value that is common to all shareholders: They are all more willing to accept the proposal if it is expected to increase value (i.e., $\theta = 1$ is more

and show how trade among group members may overcome inefficiencies from differences in beliefs. These papers focus on the dynamics of group decision-making and do not feature the mechanisms and results that arise in our model.

[10] There is heterogeneity across companies with respect to the majority requirement used in shareholder voting. While a large fraction of companies use a simple majority rule, many companies still have supermajority voting for issues such as mergers or bylaw and charter amendments, and supermajority requirements are often a subject of debate (see Papadopoulos, 2019, and Maug and Rydqvist, 2009).

likely). However, due to different attitudes toward the proposal, shareholders apply different hurdle rates for accepting it. Specifically, shareholder b would like the proposal to be accepted if and only if his expectation of $\theta + b$ is positive. Parameter b, which can be positive or negative, measures the shareholder's bias b toward proposal approval. We will refer to low (high) b as "conservatism" ("activism"). Differences in shareholders' preferences can stem from private benefits, common ownership, different social or political views, time horizons, risk aversion, or tax considerations. As noted in the introduction, the evidence for preference heterogeneity is prevalent. The initial shareholder base, i.e., the cross section of shareholders' biases b, is given by a differentiable cdf G, which is publicly known and has full support with positive density g on $\left[-\bar{b}, \bar{b}\right]$, where $\bar{b} > 0$ measures the heterogeneity among shareholders.

Parameter ϕ governs the relationship between the shareholder's attitude toward the proposal (i.e., bias b) and his valuation of the firm. The shareholder's valuation of the firm "as is" is $v(0, \theta, b) = v_0 - (\theta + b)\phi$, and his valuation under the new strategy is $v(1, \theta, b) = v_0 + (\theta + b)(1 - \phi)$. The added value of the proposal, defined as $v(1, \theta, b) - v(0, \theta, b)$, is equal to $\theta + b$. If $\phi < 0$ ($\phi > 1$), then activist shareholders value the firm more (less) than conservative shareholders regardless of whether or not the proposal is accepted, that is, both $v(1, \theta, b)$ and $v(0, \theta, b)$ increase (decrease) in b. However, if $\phi \in (0, 1)$, then activist shareholders value the firm more than conservative shareholders if and only if the proposal is sufficiently likely to be accepted. In those cases, the relationship between the shareholder's attitude toward the proposal and his valuation of the firm depends on the expectation of d, so the expected voting outcome is critical for whether activist or conservative shareholders value the firm more.

To illustrate the role of the heterogeneity parameter b, suppose b captures variation among shareholders' time horizons, where a larger b reflects a shorter horizon, i.e., more impatience. Suppose also that shareholders vote on a proposal that will shorten the horizon of the firm's projects (e.g., by inducing management to cut R&D). Then $\phi < 0$ corresponds to the situation when the existing projects of the firm are already very short-term, and thus impatient shareholders (i.e., activists) value the firm relatively more even if the proposal is rejected. The case $\phi > 1$ corresponds to the opposite situation when the existing projects of the firm are very long-term, and thus patient shareholders (i.e., conservatives) value the firm relatively more even if the proposal is approved. Finally, $\phi \in (0,1)$ corresponds to the situation when the horizon of the firm's existing projects is more balanced, and thus the relative effect of the proposal on shareholders' valuations is more significant: impatient shareholders value the firm relatively more if and only if the proposal is likely to be accepted. The role of ϕ will become clearer below, when we characterize the equilibria of the game.

Timeline. The game has two stages: first, trading and then, voting. This timing allows us to focus on the endogeneity of the voter base, which is crucial for our analysis. At the outset, all shareholders are uninformed about the value of θ; they all have the same prior on its distribution, which we specify below. Then trading takes place. Short sales are not allowed. In the baseline model, shareholders can either sell any amount of shares up to their entire endowment e, or buy any amount of shares up to a fixed quantity $x > 0$, or not trade. The quantity x captures trading frictions (e.g., illiquidity, transaction costs, wealth constraints), which limit shareholders' ability to build large positions in the firm.

In equilibrium the market must clear, and we denote the market clearing share price by p. To ease the notation in the analysis below, we define

$$\delta \equiv \frac{x}{x+e}, \tag{2}$$

which captures the relative strength with which shareholders can buy shares. We interpret δ as market liquidity, in particular, as market depth. We assume that shareholders do not trade if they are indifferent between trading at the market price p and not trading at all. This tie-breaking rule could be rationalized by adding arbitrarily small transaction costs to the model.[11]

After the market clears, but before voting takes place, all shareholders observe a public signal about the state θ. This public information may stem from disclosures by management, analysts, or proxy advisors. Let $q = \mathbb{E}[\theta|\text{public signal}]$ be the shareholders' posterior expecta-

[11] The purpose of this tie-breaking rule is to exclude equilibria that exist only in knife-edge cases. However, as the proof of Proposition 3 shows, other tie-breaking rules also eliminate these knife-edge equilibria — for example, rules under which indifferent shareholders always sell or always buy shares.

tion of the state following the signal. For simplicity and ease of exposition, we assume that the public signal is q itself, and that q is distributed according to a differentiable cdf F with mean zero and full support with positive density f on $[-\Delta, \Delta]$, where $\Delta \in (0,1)$. Thus, the ex-ante expectation of θ is zero. The symmetry of the support of q around zero is not necessary for any of the main results. To simplify the exposition, it is useful to introduce

$$H(q) \equiv 1 - F(q). \tag{3}$$

At the second stage, after observing the public signal q, each shareholder votes the shares he owns after the trading stage, based on his preferences and the realization of q. Shareholders vote either in favor or against the proposal (no abstentions). Hence, we assume that the record date, which determines who is eligible to participate in the vote, is after the trading stage.[12] We analyze subgame perfect Nash equilibria in undominated strategies of the induced voting game. The restriction to undominated strategies is common in voting games, which typically impose the equivalent restriction that agents vote as-if-pivotal.[13] This restriction implies that shareholder b votes his shares in favor of the proposal if and only if

$$b + q > 0. \tag{4}$$

Extensions. Our baseline model makes some simplifying assumptions for tractability and ease of exposition. In Section 7 we relax some of these assumptions to discuss the following extensions: (1) investors' social concerns, such that proposals can have an impact on investors' welfare irrespective of their ownership in the firm, e.g., for proposals with a social or environmental impact; (2) a second round of trade after the vote, which allows us to discuss price reactions to the voting outcome; (3) shareholders' endowment e and their ability to trade x that can vary with their bias b; (4) trading frictions that limit shareholders' ability to sell their

[12] If the record date were set prior to the trading stage, then shareholders who had sold their shares could still vote. We do not analyze such "empty voting."

[13] See, e.g., Baron and Ferejohn (1989) and Austen-Smith and Banks (1996). This restriction helps rule out trivial equilibria, in which shareholders are indifferent between voting for and against because they are never pivotal.

entire endowment e. In all four extensions we show that our main results continue to hold.

4 Analysis

We solve the model by backward induction. Before analyzing the full model with trading and voting, we first analyze two benchmark cases to build the intuition for this model, one in which shareholders vote but do not trade (Section 4.1) and one in which they trade but do not vote (Section 4.2).

We start by showing that regardless of trading, proposal approval at the voting stage takes the form of a simple cutoff rule:

Lemma 1. *If the proposal is decided by a shareholder vote, then in any equilibrium, there exists q^* such that the proposal is approved by shareholders if and only if $q > q^*$.*

Intuitively, this result follows because all shareholders, regardless of their biases, value the proposal more if it is more likely to increase value, i.e., if $\theta = 1$ is more likely.

4.1 Voting without trading

To begin, we develop the benchmark case in which shareholders vote but do not trade. Lemma 1 also applies in this case. The shareholder base at the voting stage is characterized by the pre-trade distribution G, and the proposal is approved if and only if at least fraction τ of the initial set of shareholders vote in favor. Since shareholders with a larger bias value the proposal more, it is approved if and only if the $(1-\tau)$-th shareholder, who has a bias of $G^{-1}(1-\tau)$, votes for the proposal. Hence, the cutoff q^* is given by the expression in Proposition 1:

Proposition 1 (voting without trading). *If the proposal is decided by a shareholder vote but trading is not allowed, there always exists a unique equilibrium. In this equilibrium, the proposal is approved by shareholders if and only if $q > q_{NoTrade}$, where*

$$q_{NoTrade} \equiv -G^{-1}(1-\tau). \tag{5}$$

Figure 1 illustrates the equilibrium of Proposition 1 and plots the cdf G against the private values (biases) b. The shareholder with bias $b = -q_{NoTrade}$ is the *marginal voter,* whose vote on the proposal determines whether it is approved. We will use the term "marginal voter" throughout the paper: The identity of this shareholder is crucial for the decision on the proposal. If $q = q_{NoTrade}$, there are $G(-q_{NoTrade}) = 1 - \tau$ shareholders for whom $b + q < 0$, who vote against ("Reject" region of the figure), and τ shareholders who vote in favor of the proposal ("Accept" region). Thus, the marginal voter is the shareholder who is indifferent between accepting and rejecting the proposal if exactly τ shareholders vote to accept it.

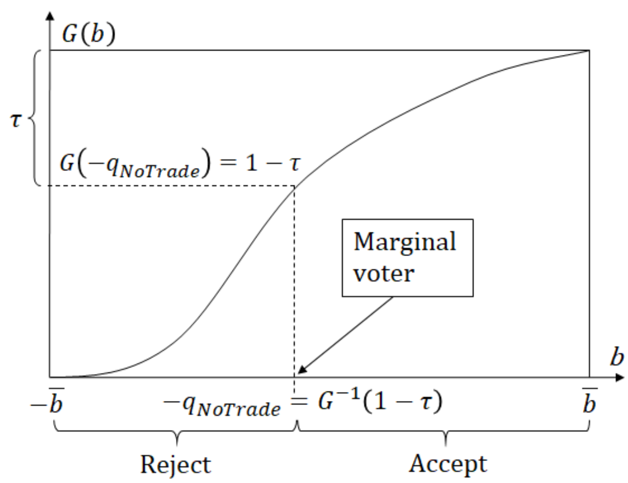

Figure 1 - Equilibrium characterization of the No-trade benchmark

4.2 Trading without voting

In the next step, we consider the second, complementary benchmark case, in which we have trading without voting. In this case, trading occurs as in the general model but then, after the public signal q is revealed, the decision on the proposal is exogenous. For concreteness, and to prepare for our later discussion of delegation in Section 6, we assume that the decision is made by the board of directors. We abstract from collective decision-making within the board and treat it as one single agent who acts like a shareholder with bias $b_m \in \left[-\overline{b}, \overline{b}\right]$ and valuation $v(d, \theta, b_m)$, so that it approves the proposal if and only if $b_m + q > 0$. Motivated by Lemma 1, we cast the following discussion in terms of a general exogenous decision rule q^*; for the decision rule of the board we have $q^* = -b_m$.

Denote by $v(b, q^*)$ the valuation of a shareholder with bias b prior to the realization of q, as a function of the cutoff q^*. Then

$$v(b, q^*) = \mathbb{E}\left[v\left(\mathbf{1}_{q>q^*}, \theta, b\right)\right], \tag{6}$$

where the indicator function $\mathbf{1}_{q>q^*}$ obtains a value of one if $q > q^*$ and zero otherwise, and $v(d, \theta, b)$ is defined by (1). When trading, the shareholder optimally buys x shares if his valuation exceeds the market price, $v(b, q^*) > p$, sells his endowment of e shares if $v(b, q^*) < p$, and does not trade otherwise. Notice that $v(b, q^*)$ can be rewritten as

$$v(b, q^*) = v_0 + b\left(H(q^*) - \phi\right) + H(q^*)\,\mathbb{E}\left[\theta | q > q^*\right], \tag{7}$$

and that $v(b, q^*)$ increases in b if and only if $H(q^*) > \phi$. In words, activist shareholders with a large bias toward the proposal value the firm more than conservative shareholders with a small bias if and only if the proposal is sufficiently likely to be approved. This observation will play a key role in the analysis below.

Proposition 2 (trading without voting). *There always exists a unique equilibrium of the game in which the proposal is decided by a board with decision rule q^*.*

*(i) If $H(q^*) > \phi$, the equilibrium is "**activist:**" a shareholder with bias b buys x shares if $b > b_a$ and sells his entire endowment e if $b < b_a$, where*

$$b_a \equiv G^{-1}(\delta). \tag{8}$$

The share price is given by $p = v(b_a, q^*)$.

*(ii) If $H(q^*) < \phi$, the equilibrium is "**conservative:**" a shareholder with bias b buys x shares if $b < b_c$ and sells his entire endowment e if $b > b_c$, where*

$$b_c \equiv G^{-1}(1 - \delta). \tag{9}$$

The share price is given by $p = v(b_c, q^*)$.

(iii) If $H(q^*) = \phi$, *no shareholder trades and the price is* $p = v_0 + \phi\mathbb{E}[\theta|q > q^*]$.

In equilibrium, the firm is always owned by investors who value it most, which gives rise to two different types of equilibria. In part (i) of Proposition 2, the equilibrium is "activist" in the sense that activist shareholders buy shares from conservatives and the post-trade shareholder base has a high preference b for the proposal. In part (ii), the equilibrium is "conservative" in the sense that conservative shareholders buy from activists, creating a post-trade shareholder base that has a low preference b for the proposal.

What determines which type of shareholders value the firm the most? Critically, according to expression (7), shareholders' valuation $v(b, q^*)$ increases in b if and only if $H(q^*) > \phi$, where $H(q^*) = \Pr[q > q^*]$ is the probability that the proposal is expected to be approved. Thus, if the proposal is approved with a relatively high (low) probability, activist shareholders value the firm more (less), and in equilibrium they buy (sell) shares from (to) conservative shareholders. Parameter ϕ, which governs the relationship between the shareholder's attitude toward the proposal and his valuation of the firm, determines how high (low) the likelihood of the proposal's approval must be in order for activists (conservatives) to be the shareholders with the highest valuation.

In the activist (conservative) equilibrium the market-clearing condition determines the "marginal trader" with bias b_a (b_c). In the activist equilibrium, the $1 - G(b_a)$ most activist shareholders with $b > b_a$ buy x shares each, whereas the remaining $G(b_a)$ more conservative shareholders sell e shares each. Hence, market clearing requires $x(1 - G(b_a)) = eG(b_a)$, or $G(b_a) = \delta$ from (2), which gives the marginal trader b_a as in (8). The equilibrium share price $p = v(b_a, q^*)$ is thus determined by the identity of the marginal trader and equals his valuation of the firm, which depends on the board's decision rule q^*. The marginal trader is indifferent between buying and selling shares given the market price. In contrast, any investor with $b \neq b_a$ values the firm differently from the marginal trader, and hence his valuation is either higher or lower than the market price, creating gains from trade. This equilibrium is illustrated in the left panel of Figure 2, which shows the location of the marginal trader who is indifferent between buying and selling.

The conservative equilibrium is analogous to the activist equilibrium, except that now the $1 - G(b_c)$ most activist shareholders sell their e shares to the $G(b_c)$ most conservative shareholders, which implies $G(b_c) = 1 - \delta$ by the same reasoning as for the activist equilibrium. It is displayed in the right panel of Figure 2. In what follows, we ignore the knife-edge case (iii), in which $H(q^*) = \phi$ and no shareholder trades.[14] Finally, we note that the equilibrium is unique, i.e., there is no set of parameters for which the conservative equilibrium and the activist equilibrium can coexist.

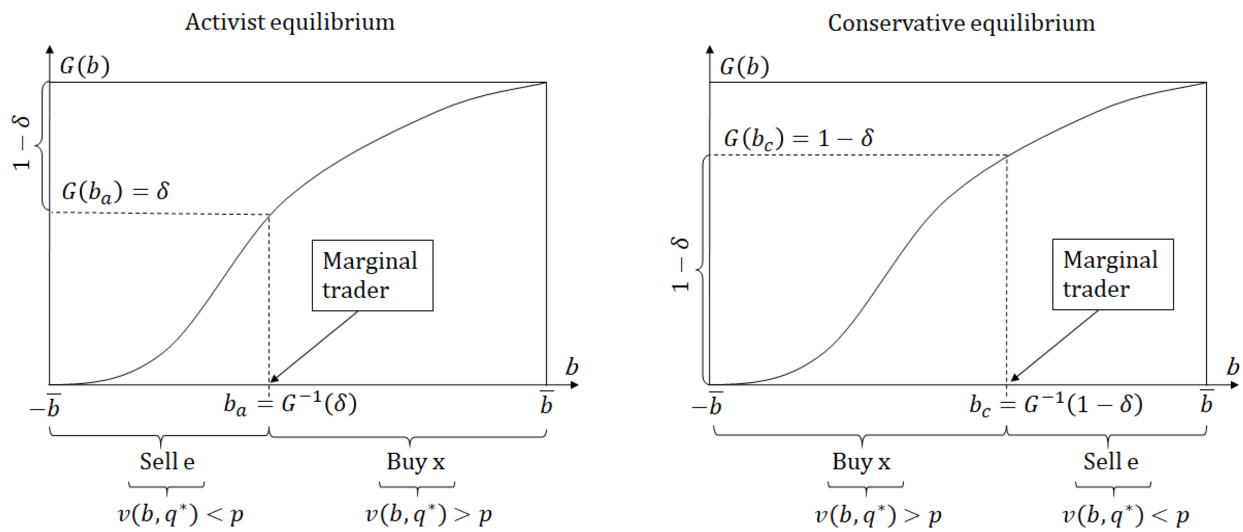

Figure 2 - Equilibrium characterization of the No-vote benchmark

The identity of the marginal trader depends on the trading frictions, as summarized in the next result.

Corollary 1. *The marginal trader becomes more extreme when trading frictions are relaxed, i.e., b_c decreases in δ and b_a increases in δ. In addition, $b_c < b_a$ if and only if $\delta > 0.5$.*

Corollary 1 follows directly from expressions (8) and (9). To see the intuition, notice that when trading frictions are small (δ is large), shareholders with the strongest preference for the likely outcome, i.e., those with a large bias in the activist equilibrium and those with a

[14] In Section 4.3 we show that when trade is allowed, this knife-edge equilibrium does not exist.

small bias in the conservative equilibrium, have the highest willingness to pay and buy the maximum number of shares. We sometimes refer to these shareholders as "extremists." Other shareholders with more moderate views (i.e., $b \in (b_c, b_a)$), take advantage of this opportunity and sell their shares to shareholders with extreme views. In contrast, when trading frictions are large (δ is small), only shareholders with the most extreme view *against* the likely outcome find it beneficial to sell their shares at a low price, while moderate shareholders (i.e., $b \in (b_a, b_c)$) always buy shares. This explains why the marginal trader in an activist equilibrium is more activist than in the conservative equilibrium if and only if trading frictions are relatively small ($\delta > 0.5$).

Overall, when trading frictions are small, the post-trade ownership structure is dominated by extremists, who can translate their strong views on the proposal into large positions in the firm. In contrast, when trading frictions are large, the post-trade shareholder base is relatively moderate and closer to the initial shareholder base. Below we show that this feature of the model has significant implications for prices and welfare when the decision on the proposal is made by a shareholder vote.

4.3 Equilibrium with trading and voting

We now analyze the general model, in which shareholders trade their shares, and those who own the shares after the trading stage vote those shares at the voting stage. In Section 4.3.1, we characterize the equilibria and discuss their properties. Then, in Section 4.3.2, we discuss the complementarity between trading and voting and derive the circumstances under which multiple equilibria exist.

4.3.1 Existence and characterization of equilibria

According to Lemma 1, the decision rule on the proposal takes the form of an endogenous cutoff q^*, and the proposal is approved if and only if $q > q^*$, i.e., with probability $H(q^*)$. The value of the firm for shareholder b as a function of q^* is again given by (7). As in the no-vote benchmark, $v(b, q^*)$ is increasing in b if and only if $H(q^*) > \phi$. At the trading stage, a shareholder with bias b buys x shares if $v(b, q^*) > p$, sells his endowment of e shares if $v(b, q^*) < p$, and does not trade otherwise. However, differently from the no-vote benchmark, the decision rule is now tightly linked to the trading outcome. In particular, the trading stage determines the composition of the shareholder base at the voting stage, which, in turn, determines the cutoff q^* and the probability that the proposal is approved. Therefore, there is a feedback loop between trading and voting: Shareholders' trading decisions depend on expected voting outcomes, and voting outcomes depend on how trading changes the shareholder base. The next result fully characterizes the equilibria of the game.

Proposition 3 (trading and voting). *An equilibrium of the game with trading and voting always exists.*

(i) *An* ***activist*** *equilibrium exists if and only if* $H(q_a) > \phi$, *where*

$$q_a \equiv -G^{-1}(1 - \tau(1 - \delta)). \tag{10}$$

In this equilibrium, a shareholder with bias b *buys* x *shares if* $b > b_a$ *and sells his entire endowment* e *if* $b < b_a$, *where* $b_a \equiv G^{-1}(\delta)$. *The proposal is accepted if and only if* $q > q_a$, *and the share price is given by* $p_a = v(b_a, q_a)$.

(ii) *A* ***conservative*** *equilibrium exists if and only if* $H(q_c) < \phi$, *where*

$$q_c \equiv -G^{-1}((1 - \delta)(1 - \tau)). \tag{11}$$

In this equilibrium, a shareholder with bias b *buys* x *shares if* $b < b_c$ *and sells his entire endowment* e *if* $b > b_c$, *where* $b_c = G^{-1}(1 - \delta)$. *The proposal is accepted if and only if* $q > q_c$, *and the share price is given by* $p_c = v(b_c, q_c)$.

(iii) *Other equilibria do not exist.*

Note that $q_c > q_a$: the cutoff for accepting the proposal is higher in the conservative equilibrium than in the activist equilibrium. Accordingly, the probability of accepting the proposal is higher in the activist equilibrium, i.e., $H(q_a) > H(q_c)$. Figure 3 illustrates both equilibria and combines the respective elements from Figures 1 and 2.

The logic behind both equilibria is the same as in the no-vote benchmark in Proposition 2. In the activist equilibrium displayed in the left panel of Figure 3, the cutoff q_a is relatively low ($-q_a$, the bias of the marginal voter, is high) and the proposal is likely to be approved. Hence, the term $H(q_a) - \phi$ in (7) is positive, so conservative shareholders who are biased against the proposal, $b < b_a$, sell their endowment to shareholders who are biased toward the proposal, $b > b_a$. The marginal trader b_a is determined by the exact same market clearing condition described in Proposition 2. Hence, $1 - G(b_a) = 1 - \delta$ shareholders own the firm after trading, and of these, at least $\tau(1-\delta)$ need to approve the proposal to satisfy the majority requirement, so that $1 - G(-q_a)$ shareholders vote in favor, with q_a defined by (10). Importantly, and differently from the no-vote benchmark, the cutoff q_a is now *endogenously* low: the fact that the post-trade shareholder base consists of shareholders who are biased toward the proposal, $b > b_a$, implies that the post-trade shareholders will optimally vote in favor of the proposal unless their expectation q is sufficiently low to offset their bias. Hence, the expectations about the high likelihood of proposal approval become self-fulfilling.

Similarly, in the conservative equilibrium displayed in the right panel of Figure 3, shareholders expect a low probability of approval (i.e., q_c is high). Hence, the term $H(q_c) - \phi$ in (7) is negative, and shareholders with $b < b_c$ value the firm more and buy shares from shareholders with $b > b_c$. Since the post-trade shareholder base consists of shareholders who are biased against the proposal and are more likely to reject it, expectations about the low probability of approval are self-fulfilling.

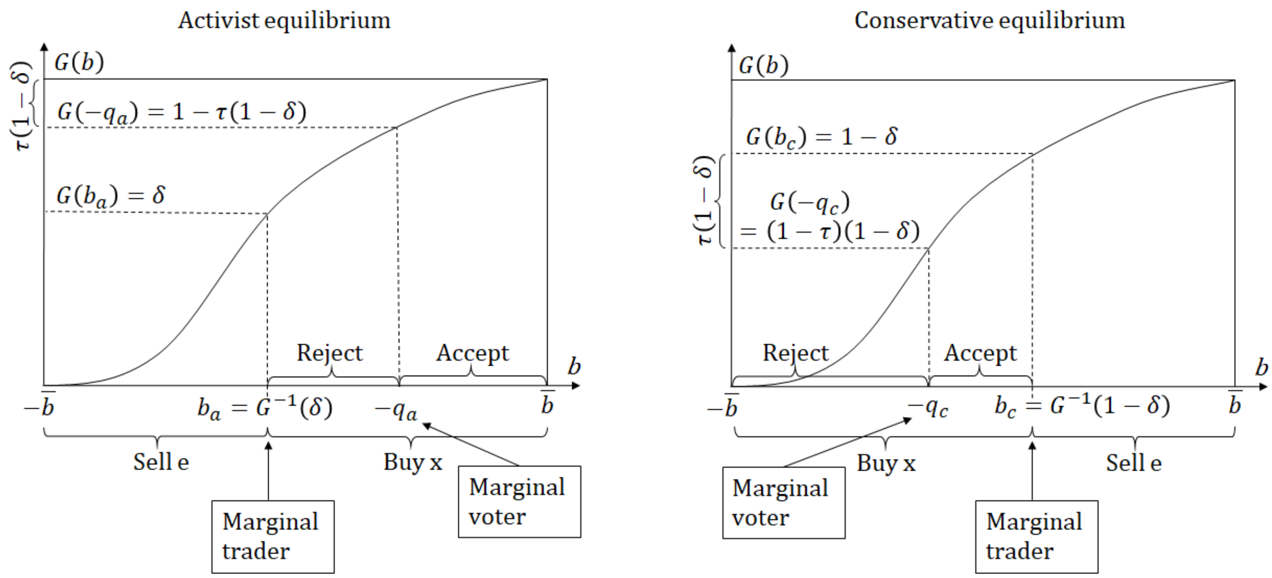

Figure 3 - Equilibrium characterization of the model with trading and voting

Figure 3 also shows that the marginal voter is always more extreme than the marginal trader, i.e., in the activist (conservative) equilibrium, the marginal voter is more activist (conservative) than the marginal trader: $-q_a > b_a$ $(-q_c < b_c)$. These relationships, which play a key role in the analysis of welfare and prices in Section 5, can be easily verified from the expressions in Proposition 3.

Similar to Lemma 1 in the no-vote benchmark, the marginal trader becomes more extreme when trading frictions are relaxed, i.e., b_a (b_c) increases (decreases) in δ. In addition, it follows from the expressions (10) and (11) that $-q_a$ $(-q_c)$ increases (decreases) in δ. Thus, both the marginal trader and the marginal voter become more extreme as trading frictions are relaxed. However, the extreme to which they converge as trading frictions disappear depends on the type of equilibrium, that is, whether it is activist or conservative:

Corollary 2. *The marginal voter becomes more extreme when trading frictions are relaxed. In the activist (conservative) equilibrium, $-q_a$ increases in δ, and both $-q_a$ and b_a converge to $\bar{b}$ as $\delta \to 1$ ($-q_c$ decreases in δ, and both $-q_c$ and b_c converge to $-\bar{b}$ as $\delta \to 1$).*

The intuition is similar to the intuition of the no-vote benchmark: When trading frictions are relaxed, the post-trade shareholder base is dominated by extremists who hold larger positions in the firm, whereas the more moderate shareholders sell. The more extreme preferences of the post-trade shareholder base then push the firm's decision-making to the extreme. This analysis uncovers a new effect of liquidity on governance through voice: Higher liquidity makes the firm's decision rule more extreme and increases the turnover of the shareholder base before important decisions.

4.3.2 Multiple equilibria

As the above discussion shows, the introduction of the voting stage creates self-fulfilling expectations: Shareholders with a preference for the expected outcome buy shares, which in turn makes their preferred outcome more likely. Voting also creates strategic complementarities at the trading stage between agents with similar preferences. For example, if an activist shareholder with a large bias towards the proposal is more likely to buy shares and, therefore, more likely to vote for the proposal, this increases the likelihood of proposal acceptance and hence the payoff from buying for another activist shareholder. This complementarity, and the presence of self-fulfilling expectations, suggest that the two equilibria—conservative and activist—can coexist. Indeed, according to Proposition 3, both equilibria exist whenever

$$H(q_c) < \phi < H(q_a). \tag{12}$$

The multiplicity of equilibria can be interpreted as an additional source of volatility if agents change expectations for exogenous reasons. Hence, without any change in the fundamentals of the firm, prices and voting outcomes may change if agents form different expectations and, accordingly, coordinate on a different equilibrium. Therefore, we treat multiple equilibria as a source of non-fundamental uncertainty or indeterminacy. The indeterminacy associated with multiple equilibria underscores potential empirical challenges in analyzing shareholder voting and could explain the mixed evidence about the effect of voting on proposals on shareholder value.[15] The same proposal voted on at two firms with similar characteristics and fundamentals could have very different voting outcomes and valuation effects.

The next result highlights the factors that contribute to the multiplicity of equilibria.

Proposition 4. *The conservative and the activist equilibria coexist if the market is liquid (sufficiently high δ); if the voting requirement is in an intermediate interval, $\tau \in (\underline{\tau}, \overline{\tau})$; if the expected voting outcome is critical for whether activist or conservative shareholders value the firm more, $\phi \in (H(q_c), H(q_a))$; and only if heterogeneity of the initial shareholder base is large (sufficiently large $\overline{b}$).*

Intuitively, the multiplicity of equilibria arises from the possibility that expectations become self-fulfilling. If shareholders can take larger positions, i.e., δ is large, then extreme shareholders accumulate larger positions in the firm. The firm experiences larger shifts in the shareholder base, and the direction of these shifts depends on shareholders' expectations about the proposal outcome. As the post-trade shareholder base and the marginal voter in each equilibrium become more extreme, the interval in (12) in which the two equilibria coexist expands, so that (12) is more easily satisfied. Conversely, for small δ, i.e., large trading frictions, both types of equilibria converge to the no-trade benchmark as $\delta \to 0$ ($q_a \to q_{NoTrade}$ and $q_c \to q_{NoTrade}$), so the interval in (12) in which multiple equilibria exist vanishes.

Multiple equilibria are also less likely to exist if the governance structure requires either very large or very small majorities to approve a decision: If τ is sufficiently large (small), then an activist (conservative) equilibrium is unlikely to exist because approval of the proposal requires almost all shareholders to vote in its favor (against). Since most firms have simple majority voting rules, the non-fundamental indeterminacy we point out seems important.

Activist and conservative equilibria are more likely to coexist if ϕ is neither too large nor too small. That is, the effect of the proposal's approval must be critical for whether activist or conservative shareholders value the firm more. If ϕ is too large (too small), then the activist

[15] Karpoff (2001) surveys the earlier literature, and Yermack (2010) and Ferri and Göx (2018) review some of the later studies focused on say-on-pay votes on executive compensation. Cunat, Gine, and Guadalupe (2012) also summarize that "(...) the range of results in the existing literature varies widely, from negative effects of increased shareholder rights (...) to very large and positive effects on firm performance (...)" (pp. 1943-44).

(conservative) shareholders value the firm less regardless of the expected decision and have low incentives to buy. Hence, the shareholder base does not shift toward activist (conservative) shareholders, so the activist (conservative) equilibrium cannot exist, and multiplicity vanishes.

Finally, the heterogeneity among shareholders has to be sufficiently large, since only then are there enough shareholders with extreme views or preferences regarding the proposal who can give rise to both types of equilibria.

5 Welfare and prices

In this section we analyze the welfare and price effects of trading and voting. We start by deriving general properties that form the basis for our discussion. Then, in Section 5.1, we show that shareholder welfare and prices may move in opposite directions in response to changes in parameters, and in Section 5.2, we show that greater opportunities to trade can be detrimental for both prices and welfare.

The equilibrium share price is characterized by Proposition 3, which shows that the price depends on the identities of the marginal voter and the marginal trader, $p_a = v(b_a, q_a)$ and $p_c = v(b_c, q_c)$. The marginal voter determines the firm's decision rule regarding the proposal, and the marginal trader's valuation given this decision rule determines the market price.

We now derive the aggregate expected welfare of all shareholders (hereafter, expected welfare). In the activist equilibrium, whenever it exists, the expected welfare is

$$W_a = ep_a \Pr[b < b_a] + \mathbb{E}[(e + x) v(b, q_a) - xp_a | b > b_a] \Pr[b > b_a]. \tag{13}$$

Similarly, in the conservative equilibrium, the expected welfare is

$$W_c = ep_c \Pr[b > b_c] + \mathbb{E}[(e + x) v(b, q_c) - xp_c | b < b_c] \Pr[b < b_c]. \tag{14}$$

In both expressions, the first term captures the value of shareholders who sell their endowment e in equilibrium, whereas the second term is the expected value of shareholders who buy shares in equilibrium: it equals the value of their post-trade stake in the firm minus the price paid for the additional shares acquired through trading. To simplify the notations, we define

$$\beta_a \equiv \mathbb{E}\left[b|b > b_a\right] \text{ and } \beta_c \equiv \mathbb{E}\left[b|b < b_c\right], \tag{15}$$

which denotes the average bias of the post-trade shareholder base for, respectively, the activist and the conservative equilibrium. The average bias of the post-trade shareholder base plays a critical role in the following welfare analysis. Indeed, while the share price is determined by the valuation of the *marginal* trader, the next result shows that the expected welfare is determined by the valuation of the *average* post-trade shareholder.

Lemma 2. *In any equilibrium, the expected welfare of the shareholder base pre-trade is equal to the valuation of the average post-trade shareholder. In particular,*

$$W_a = e \cdot v\left(\beta_a, q_a\right) \text{ and } W_c = e \cdot v\left(\beta_c, q_c\right). \tag{16}$$

To understand Lemma 2, notice first that the expected welfare of the pre-trade shareholder base equals the expected welfare of the shareholder base post-trade, $\mathbb{E}\left[v\left(b, q_a\right)|b > b_a\right]$ in the activist equilibrium and $\mathbb{E}\left[v\left(b, q_c\right)|b < b_c\right]$ in the conservative equilibrium. Intuitively, market clearing implies that all the gains of the shareholders who sell shares are offset by the losses of the shareholders who buy shares. Since selling shareholders sell their entire endowment, their valuations are fully captured by the transfers from buying shareholders. The linearity of $v\left(b, q^*\right)$ in b in turn implies that the expected welfare of the shareholder base post-trade is equal to the valuation of the average post-trade shareholder.

Before deriving the main results of this section, we analyze the conditions under which the expected welfare and the share price are maximized. For this purpose, we consider the following thought experiment: Holding everything else equal, when does $v\left(b, q^*\right)$ obtain its maximum as a function of the marginal voter's bias $-q^*$? Expression (7) implies

$$\frac{\partial v\left(b, q^*\right)}{\partial q^*} > 0 \Leftrightarrow -q^* > b. \tag{17}$$

Therefore, the valuation $v\left(b, q^*\right)$ of a shareholder with bias b is maximized if $-q^* = b$, i.e., if the marginal voter, who determines the decision, represents the shareholder's view.

Since in the activist equilibrium $p_a = v(b_a, q_a)$ and $W_a = e \cdot v(\beta_a, q_a)$, and in the conservative equilibrium $p_c = v(b_c, q_c)$ and $W_c = e \cdot v(\beta_c, q_c)$, this insight gives the following result, which plays a central role in the analysis below.

Lemma 3.

(i) *The share price obtains its maximum when the bias of the marginal voter equals the bias of the marginal trader (b_a in the activist equilibrium and b_c in the conservative equilibrium).*

(ii) *The expected welfare obtains its maximum when the bias of the marginal voter equals the bias of the average post-trade shareholder (β_a in the activist equilibrium and β_c in the conservative equilibrium).*

By implication, the share price increases (decreases) if the marginal voter moves toward (away from) the position of the marginal trader. Similarly, welfare increases (decreases) if the marginal voter moves toward (away from) the position of the average post-trade shareholder. In the following subsections, we use these insights to explore the welfare and price effects.[16]

5.1 Opposing effects on welfare and prices

The literature in financial economics often draws a parallel between welfare and prices and uses stock returns to approximate effects on welfare. This parallel is natural if shareholders have homogeneous preferences. The next result highlights that if shareholders have heterogeneous preferences, shareholder welfare and prices may in fact move in opposite directions in response to exogenous changes to the firm's governance structure or trading environment.

Proposition 5. *Suppose the marginal voter is less extreme than the average post-trade shareholder (i.e., $-q_a < \beta_a$ in the activist equilibrium and $-q_c > \beta_c$ in the conservative equilibrium), and consider a small exogenous change in parameters that affects the position of the marginal*

[16] In an empirical study of proxy contests, Listokin (2008) also observes the difference between the preferences of marginal traders, who set prices, and marginal voters, who determine voting outcomes, and concludes that marginal voters value management control more than marginal traders in his sample.

voter without affecting the marginal trader or the average post-trade shareholder. Then, if such a change in parameters increases (decreases) shareholder welfare, it also necessarily decreases (increases) the share price.

The intuition for Proposition 5 is best explained with the help of Figure 4, which focuses on the activist equilibrium.

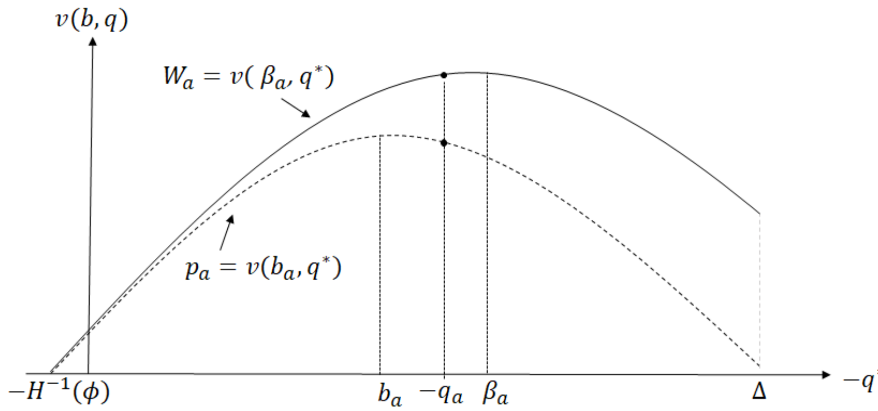

Figure 4 - Opposing effects on welfare and prices in the activist equilibrium

Recall that, for any given decision rule q^*, the share price equals the valuation of the marginal trader, $p_a = v(b_a, q^*)$, which is maximized at $-q^* = b_a$ by Lemma 3. Similarly, shareholder welfare is the valuation of the average post-trade shareholder, $W_a = v(\beta_a, q_a)$, which is maximized at $-q^* = \beta_a$, again by Lemma 3. Both functions are displayed in Figure 4. Since the average post-trade shareholder is always more extreme than the marginal trader, $\beta_a > b_a$, shareholder welfare is higher than the share price for any decision rule q^*: graphically, the function $W_a = v(\beta_a, q^*)$ (solid line) lies above the function $p_a = v(b_a, q^*)$ (dashed line). Given the assumptions of the proposition, the bias of the marginal voter, $-q_a$, is located between that of the marginal trader and that of the average post-trade shareholder, i.e. $b_a < -q_a < \beta_a$. However, in this interval, the welfare function is increasing in $-q^*$, whereas the price function is decreasing. Intuitively, when $-q^*$ increases, the distance of the marginal voter from the average post-trade shareholder decreases, whereas its distance from the marginal trader increases. Hence, any change that affects only the location of the marginal voter moves prices and welfare in opposite directions.

An exogenous change to the majority requirement τ is an example of a parameter change in our setting that affects the marginal voter without affecting the position of the marginal trader or the average post-trade shareholder, as required by Proposition 5.

Corollary 3. *Suppose in equilibrium the marginal voter is less extreme than the average post-trade shareholder. Then, a small change in the majority requirement τ that increases (decreases) shareholder welfare, necessarily decreases (increases) the share price.*

Indeed, based on expressions (10) and (11) in Proposition 3, an increase in τ implies that the marginal voter becomes more conservative in both equilibria (i.e., $-q_a$ and $-q_c$ decrease in τ). This is because an increase in τ requires shareholders with a lower preference for the proposal to vote in favor. At the same time, τ has no effect on the marginal trader (b_a and b_c), and hence, on the average post-trade shareholder (β_a and β_c). Corollary 3 is then a direct consequence of Proposition 5.[17]

The opposing welfare and price effects are not unique to changes in the majority requirement or, more generally, to parameters that only affect the identity of the marginal voter: any parameter shift that moves the marginal voter closer to the marginal trader but farther from the average post-trade shareholder will have opposing effects on welfare and prices. In Section 7.3, we analyze an extension of the baseline model with an additional round of trade post-voting, and show that the logic above also implies that price and welfare reactions to voting outcomes can have opposite signs.

Overall, Proposition 5 highlights a potential limitation to prices as a measure of shareholder welfare in the context of shareholder voting. By using prices as a proxy for welfare, the researcher may sometimes not only obtain a biased estimate of the real effect of the proposal, but even get the wrong sign of the effect.

[17] Proposition 15 in the Online Appendix characterizes the majority requirement that maximizes the expected shareholder welfare. In general, the optimal majority requirement is not a simple majority and depends on trading frictions.

5.2 Trading frictions

Trade in our model enables shareholders with different views and preferences to exchange shares with each other in order to improve their welfare. In particular, larger opportunities to trade allow shareholders to build larger positions, so that the post-trade ownership structure becomes more concentrated among the most extreme shareholders. Therefore, when decisions on the proposal are not themselves affected by trade, e.g., when the decision is made by the board as in the no-vote benchmark of Section 4.2, the ability to trade always increases the share price and shareholder welfare:

Lemma 4. *When the proposal is decided by a board with decision rule q^*, the share price and the expected welfare increase when trading frictions are relaxed (i.e., larger δ).*

By contrast, the next result demonstrates that when shareholders vote, then greater opportunities to trade can in fact reduce the share price and expected welfare.

Proposition 6. *Suppose the proposal is decided by a shareholder vote and $|q_{NoTrade}| < \Delta$. There exist $\underline{\delta}$ and $\overline{\delta}$, $0 < \underline{\delta} < \overline{\delta} < 1$, such that in any equilibrium:*

(i) *The share price increases in δ if $\delta > \overline{\delta}$, and decreases in δ if $\delta < \underline{\delta}$ and $|H(q_{NoTrade}) - \phi|$ is sufficiently small.*

(ii) *The expected welfare increases in δ if $\delta > \overline{\delta}$, and decreases in δ if $\delta < \underline{\delta}$, $|H(q_{NoTrade}) - \phi|$ is sufficiently small, and the marginal voter in this equilibrium is more extreme than the average post-trade shareholder.*

Consider first the price effect in part (i). From Proposition 3, the share price reflects the valuation of the marginal trader, which depends on the decision of the marginal voter. Since the marginal trader is always less extreme than the marginal voter, the voting outcome is never optimal from his point of view, since shareholders in the activist (conservative) equilibrium vote in favor of (against) the proposal too often. The stock price increases with liquidity if and only if the distance between the marginal trader and the marginal voter declines. When liquidity

δ is large, then increasing it further implies that both, the marginal trader and the marginal voter, converge to the most extreme shareholder, and since the wedge between them shrinks to zero, the share price necessarily increases in δ. This explains the cutoff $\overline{\delta}$.

In the opposite case, if δ is small and close to zero, the wedge between the marginal trader and the marginal voter can be large. For example, in the activist equilibrium, $\lim_{\delta \to 0} b_a = -\overline{b}$, while $\lim_{\delta \to 0} q_a = q_{NoTrade}$. Based on expression (7), $|H(q^*) - \phi|$ is the sensitivity of the shareholder's valuation to his attitude b towards the proposal. Thus, when this sensitivity is small, the marginal voter becomes extreme at a faster rate than the marginal trader as δ increases,[18] and as a result, the share prices decreases as liquidity increases. Overall, this result highlights that more trading opportunities can be detrimental to the share price because they make the marginal voter relatively more extreme and thereby decrease the value of the marginal trader.

The intuition behind the effect of δ on welfare in part (ii) is similar, with one exception. Recall that the key difference between welfare and the share price is that the former is the valuation of the average post-trade shareholder, while the latter is the valuation of the marginal trader. Whereas the marginal voter is always more extreme than the marginal trader, he is not necessarily more extreme than the average post-trade shareholder. Thus, relative to the conditions in part (i) for prices, the negative effect of trading opportunities δ on welfare also requires the marginal voter to be more extreme than the average post-trade shareholder—only in those circumstances can the wedge between the marginal voter and the average shareholder increase.[19]

Proposition 6 reveals a new force through which financial markets have real effects, which could be detrimental. In our setting financial markets do not aggregate or transmit investors' information to decision-makers. Instead, financial markets affect the ability of shareholders to accumulate large positions in the firm and then use their votes to impose their views. This

[18] The condition $|q_{NoTrade}| < \Delta$ ensures that the marginal voter changes with δ when δ is small. Intuitively, this condition means that in the no-trade benchmark, the outcome of the vote is uncertain.

[19] Note that the conditions in Proposition 5, which are necessary to obtain opposing effects on welfare and prices, require the marginal voter to be *less* extreme than the average post-trade shareholder. Thus, these conditions are violated by the assumptions of Proposition 6 part (ii), which require the marginal voter to be *more* extreme.

effect can be detrimental to the ex-ante shareholder value, both to those shareholders who buy shares and to those who sell their shares in equilibrium. Intuitively, if more trade makes the marginal voter too extreme, then even shareholders who buy shares are worse off if their bias is moderate. Since the willingness to pay of these shareholders decreases, the price at which shareholders can sell their shares decreases as well. Therefore, both shareholders who sell their shares and the moderate shareholders who buy shares may be worse off if δ is higher. Only the most extreme shareholders are always better off when trading frictions are relaxed.[20]

6 Delegation

As shown in the previous section, when decisions are made by a shareholder vote, shareholders with extreme views can accumulate large positions and then use their voting power to impose their views on more moderate shareholders, which can be detrimental to aggregate welfare. This raises the question of whether shareholders would be better off if decision-making were instead delegated to the company's board of directors.

6.1 Optimal board

To study this question, we return to the game from Section 4.2 in which the decision is made unilaterally by a board of directors with bias b_m and decision rule $q^* = -b_m$, which reflects the incentives and preferences of board members. We are interested in the effect of b_m on shareholder welfare. For example, if $b_m = b_a$ ($b_m = b_c$), then the board's objective is to maximize the value of the marginal trader in the activist (conservative) equilibrium, that is, to maximize the share price.

As shown in Proposition 2, the equilibrium is unique and either activist, if the board is biased toward the proposal, or conservative, if it is biased against the proposal. Lemma 2 holds in this context as well, so the expected welfare of the initial shareholder base equals the

[20] Shareholders who are more extreme than the marginal voter (i.e., $b < -q_c$ in the conservative equilibrium and $b > -q_a$ in the activist equilibrium) benefit from a larger δ. This is because the marginal voter always becomes more extreme as δ increases, and hence his preferences become more aligned with these extreme shareholders.

expected welfare of the post-trade shareholder base, and is given by

$$W_{m,a} = e \cdot v\left(\beta_a, -b_m\right) \text{ and } W_{m,c} = e \cdot v\left(\beta_c, -b_m\right) \tag{18}$$

if the board is activist and conservative, respectively. We call the board optimal if it maximizes the expected shareholder welfare. The next result characterizes the bias of the optimal board and compares it to the welfare outcome with shareholder voting.

Proposition 7. *The bias of the optimal board and the expected welfare with the optimal board are given by*

$$b_m^* = \begin{cases} \beta_c & \text{if } v\left(\beta_c, -\beta_c\right) > v\left(\beta_a, -\beta_a\right) \\ \beta_a & \text{otherwise} \end{cases}, \quad W_m^* = e \cdot \max\left\{v\left(\beta_c, -\beta_c\right), v\left(\beta_a, -\beta_a\right)\right\}. \tag{19}$$

(i) *If $v\left(\beta_c, -\beta_c\right) < (>)\, v\left(\beta_a, -\beta_a\right)$, then the optimal board is more activist (conservative) than the average bias of the initial shareholder base, i.e., $b_m^* > \mathbb{E}[b]$ ($b_m^* < \mathbb{E}[b]$) and the induced delegation equilibrium is activist (conservative).*

(ii) *The expected welfare under the optimal board, W_m^*, is increasing in δ.*

(iii) *If the marginal voter in either equilibrium with shareholder voting is not given by b_m^* (i.e., $q_a \neq -b_m^*$ and $q_c \neq -b_m^*$), then there exists $\varepsilon > 0$ such that if $|b_m - b_m^*| < \varepsilon$, the induced delegation equilibrium generates a strictly higher expected welfare than any voting equilibrium.*

The main implication of Proposition 7 is that it is optimal to have a biased board. According to part (i), the optimal board is always either more conservative or more activist relative to the initial shareholder base, i.e., $b_m^* \neq \mathbb{E}[b]$, even though it maximizes the welfare of the *initial* shareholder base. The intuition is similar to the one behind the welfare analysis in Section 5. Recall from Lemma 2 that the value of the selling shareholders is the price they receive for their shares, which is a transfer from the buying shareholders. Thus, the aggregate welfare of the initial shareholder base is exactly equal to the aggregate welfare of post-trade shareholders, which, in turn, is maximized by a biased board: The bias of the optimal board always equals the average bias of the *post-trade* shareholder base (β_a or β_c). Our prior analysis also implies that the optimal board is tightly linked to the firm's trading environment: As opportunities for trade (δ) increase, the post-trade shareholder base becomes more extreme, so the optimal board becomes more biased. The optimal board is unbiased only if there is no trading between shareholders, i.e., $b_m^* \to \mathbb{E}[b]$ as $\delta \to 0$.

Overall, among all boards that induce an activist (conservative) equilibrium, the board that gives the highest shareholder welfare is one with a bias exactly equal to β_a (β_c). In particular, since $\beta_a \neq b_a$ ($\beta_c \neq b_c$) and prices are determined by the valuation of the marginal trader, the objective of the optimal board should not be to maximize the share price. The optimal choice between an activist and a conservative board is determined by the welfare comparison of the activist and conservative equilibria induced by these two boards.

To see part (ii), recall that as δ increases, the post-trade shareholder base has more extreme preferences. Since the decisions of the optimal board are fully aligned with the preferences of the post-trade shareholders, a more extreme post-trade shareholder base values the firm more and welfare increases. Put differently, the optimal design of the board eliminates the detrimental effect of trade (δ) by aligning the decision on the proposal with the preferences of the average post-trade shareholder.

Finally, in part (iii), we compare the benefits from delegation to the board with decision-making via shareholder voting, which results in a decision rule q_a or q_c. We note that a board with bias $b_m = -q_a$ ($b_m = -q_c$) implements the outcome of the activist (conservative) voting equilibrium. Therefore, shareholders cannot be worse off with an optimally chosen board than with a shareholder vote. Moreover, shareholders are strictly better off with an optimal board except for the knife-edge cases in which the voting equilibrium already yields the highest expected welfare, i.e., if the marginal voter just happens to equal the post-trade average shareholder ($q_a = -b_m^*$ or $q_c = -b_m^*$). In all other cases, the board does not have to be optimal, but just has to be good enough in the sense of being in the interval around b_m^* to increase welfare relative to decision-making via voting. In the Online Appendix, we examine how the comparison between delegation to an optimal board and decision-making via shareholder voting depends on liquidity.

6.2 Voting to delegate to a board

Due to the heterogeneity of the shareholder base, even the optimal board, which maximizes the aggregate welfare of all shareholders, may nevertheless harm some of them. Those shareholders would prefer to retain their voting rights. This raises the question whether shareholders would delegate decision-making to a board that improves aggregate welfare, i.e., whether a fraction of at least τ of the initial shareholders would give up their right to vote on the proposal and leave the choice to the board. In other words, can we expect shareholders to reach a consensus on delegation?

To answer this question, in this section we analyze the following extension. Suppose that at the outset of the game, i.e., before the trading stage, shareholders choose between two alternatives: (i) all shareholders retain their voting rights, as in the baseline model; and (ii) all shareholders delegate decision-making authority to a board with an exogenously given bias b_m, which then decides on the proposal. Decision-making is delegated to the board only if at least fraction τ of the shareholders supports it. Hence, we ask: Assuming the firm has a board with bias b_m, would at least τ of the initial shareholders ever vote in favor of surrendering their choice over the proposal to the board, rather than voting on the proposal themselves? Below, we show that the optimal board may not always be in the set of boards that can garner support from at least τ initial shareholders.

Proposition 8. *Suppose shareholders expect the activist (conservative) equilibrium in the voting game and the optimal board is activist (conservative) as well. Then, there exists* $\overline{\tau} \in (0, 1)$ *such that if* $\tau \in (\overline{\tau}, 1)$*, then at least* $1-\tau$ *initial shareholders strictly prefer retaining their voting rights over delegation to the optimal board.*

Hence, if τ is too high, shareholders will not delegate to the board, not even to the board that maximizes ex ante shareholder welfare. To see the intuition, consider the activist equilibrium (the intuition for the conservative equilibrium is similar). The initial shareholders' preferences over the board crucially depend on whether or not they plan to sell their stake in the firm. Indeed, shareholders who sell their shares ($b < b_a$) obtain a payoff proportional to the price p_a and hence would like to maximize the share price. Recall that the share price is given by the valuation of the marginal trader: $p_a = v(b_a, q^*)$, where q^* is the corresponding decision cutoff. From the marginal trader's perspective, delegation to a board with bias b_m is preferred to the conservative voting equilibrium whenever $b_m \in (b_a, -q_a)$, i.e., the board's position over the proposal is closer to his own position than that of the marginal voter. Therefore, all shareholders with $b < b_a$ would vote for a board with bias $b_m \in (b_a, -q_a)$. In contrast, consider shareholders with bias $b > b_a$, who buy shares. These shareholders have two reasons to prefer a board that is more activist than the marginal trader. First, because they are more activist than the marginal trader, they favor the proposal more and hence would intrinsically benefit from a more activist board. Second, because they pay $p_a = v(b_a, q^*)$ for each share they buy, they have incentives to support boards that the marginal trader dislikes. This consideration amplifies their incentives to support activist boards. Essentially, buying shareholders support boards that are more activist than they are, since they internalize the negative effect that such boards will have on the value of the marginal trader, and thereby, on the share price.

In general, the set of boards that obtain the support of at least τ of the initial shareholders is limited.[21] In particular, Proposition 8 shows that the optimal board, as characterized by Proposition 7, is not always within this set. This is true especially if τ is large. In this case, the marginal voter is more conservative than the average post-trade shareholder in the conservative equilibrium (i.e., $-q_c < \beta_c$), and therefore, there are welfare gains from delegating the decision rights to a less conservative board, and in particular, to the optimal conservative board. In fact, notice that $1-(1-\tau)(1-\delta) > \tau$ of the initial shareholders are less conservative than the marginal voter, and yet, they cannot agree to delegate their voting rights to even a marginally less conservative board when τ is large. The reason for this collective action failure stems from the externality mentioned above: some moderate shareholders who are less conservative

[21] The proof of Proposition 8 in fact shows a more general result: Suppose shareholders expect the activist (conservative) equilibrium in the voting game. For *any* board with an activist (conservative) bias, i.e., $b_m > -H^{-1}(\phi)$ ($b_m < -H^{-1}(\phi)$), there exists $\overline{\tau} \in (0,1)$ such that if $\tau \in (\overline{\tau}, 1)$, then at least $1-\tau$ initial shareholders strictly prefer retaining their voting rights over delegation to a board with bias b_m.

than the marginal voter are not willing to delegate their decision rights to a less conservative board (and in particular to the optimal conservative board) because doing so will also benefit the marginal trader and thereby increase the price they have to pay to buy the shares in the delegation equilibrium. As a result, welfare-improving boards, and in particular the optimal board, cannot garner sufficient support from initial shareholders when τ is large.

Overall, our analysis demonstrates that when voting occurs prior to trading, short-term trading considerations impose an externality and may push shareholders to make suboptimal delegation decisions in order to gain from trading.

7 Extensions and robustness

In this section we discuss several extensions of the baseline model. The complete analysis of these extensions is presented in the Online Appendix, and we only summarize the key conclusions here.

7.1 Social concerns

Consider a variation of the model in which shareholders care about the proposal beyond its impact on the value of their shares. Shareholders may have such preferences if the proposal has environmental or social implications, which shareholders care about even after selling their entire endowment. Specifically, consider a shareholder with bias b who trades $t \in [-e, x]$ shares and owns $e + t$ shares after trading, and assume that his preferences are given by

$$(e+t)\left[v_0 + (\theta + b)(d - \phi)\right] + \gamma b d. \tag{20}$$

Parameter $\gamma \geq 0$ captures the weight the shareholder assigns to the proposal beyond his ownership in the firm, and in this respect it measures social concerns. The case $\gamma = 0$ is the baseline model. We fully develop this extension in Section A.1 in the Online Appendix.

Since shareholders do not expect their own vote to be pivotal for the voting outcome, social concerns do not affect their trading decisions. Hence, the marginal trader remains unchanged and, as a result, the marginal voter is unchanged as well. However, social concerns affect the *preferences* of the marginal voter because they amplify all shareholders' attitudes to the proposal. In particular, a shareholder who buys x shares votes for the proposal if and only if $q > -b(1+(\gamma/e)(1-\delta))$. Hence, conservative shareholders ($b < 0$) become even more conservative in that they apply an even higher hurdle toward accepting the proposal, whereas activist shareholders ($b > 0$) become even more activist. Despite this modification, the qualitative properties of the equilibria do not change.

The presence of shareholders' social concerns also affects the welfare functions W_a and W_c, which now represent the valuation of investors with attitudes $\beta_a+(\gamma/e)\,\mathbb{E}[b]$ and $\beta_c+(\gamma/e)\,\mathbb{E}[b]$, respectively. Intuitively, with social concerns, shareholders are affected by the proposal even if they sell their shares, and hence the welfare function must put some weight on $\mathbb{E}[b]$, the average bias of the initial, pre-trade, shareholder base. However, and for the same reasons as in the baseline model, we still obtain opposing effects on welfare and prices for certain parameter ranges, the optimal board is still biased, and shareholders may still not wish to delegate decision-making to the optimal board (see Propositions 5, 7, and 8 above).

7.2 Heterogeneous endowments and trading frictions

We also extend the baseline model by allowing shareholders to differ with respect to their endowments and their ability to buy shares (see Section A.2 of the Online Appendix for the complete analysis). Specifically, we assume that a shareholder with bias b has an endowment $e(b) > 0$ and can buy up to $x(b) > 0$ shares. We do not restrict the correlations between $x(b)$, $e(b)$ and b in any way. For example, we allow endowments and trading opportunities to be higher for activist shareholders, for conservative shareholders, or for extremist (high-$|b|$) shareholders. We denote by $\mathbf{e} \equiv \int_{-\bar{b}}^{\bar{b}} e(b)\,dG(g)$ the total endowment.

The trading equilibrium is very similar to the baseline case, i.e., there is an activist and a conservative equilibrium. Consider the activist equilibrium. The marginal trader b_a, who is indifferent between buying and selling, is determined by market clearing, i.e., by the unique solution of $\int_{b_a}^{b} x(b)dG(b) = \int_{-b}^{b_a} e(b)dG(b)$. All shareholders with a bias higher (lower) than that of the marginal trader buy (sell), so post-trading, a shareholder with bias $b > b_a$ holds $x(b) + e(b)$ shares. Thus, we define a new density function and cdf for the distribution of post-trade shareholders as

$$g_a(b) \equiv g(b) \frac{x(b) + e(b)}{\mathbf{e}}, \quad G_a(b) \equiv \int_{b_a}^{b} g_a(b)\, db, \tag{21}$$

which allows us to apply the arguments of the baseline model to this extension. In particular, the marginal voter is given by $-q_a = G_a^{-1}(1-\tau)$ and is more extreme than the marginal trader, i.e., $-q_a > b_a$. The welfare functions have the same characteristics and reflect the welfare of the post-trade shareholders (as in Lemma 2), so our results on the opposing effects on welfare and prices (Proposition 5) and the optimal board (Propositions 7 and 8) continue to hold.

7.3 Trading after voting

The baseline model features one round of trading *prior to* the vote. In a further extension, we introduce a second round of trading *after* the vote, but before state θ is realized. The purpose of this extension is to explicitly analyze the reactions of the share price and welfare to the voting outcome. In addition, this analysis demonstrates the robustness of our main insights to a dynamic trading environment. For simplicity, in this discussion, we focus on the case $\phi = 0$, when the equilibrium is activist. The complete analysis of this case and the discussion of cases with $\phi \neq 0$ are in Section A.3 of the Online Appendix.

The pre-vote trading stage is similar to that in the baseline model: conservative shareholders with $b < b_a$ sell to activist shareholders, so the shareholder base at the voting stage consists of shareholders with $b > b_a$, where b_a is given by (8). However, additional trading now takes place after the vote: If the proposal is accepted, the more moderate shareholders among those with $b > b_a$ sell to the more activist shareholders. The anticipation of this post-vote trading implies that the pre-vote share price is the expected post-vote price, i.e., the expected valuation of the post-vote marginal trader. Therefore, the price reaction to proposal approval is positive if and only if proposal approval benefits the post-vote marginal trader.

We next show that the average price and welfare reactions to proposal approval can have opposite signs. The intuition is similar to the intuition for opposing price and welfare effects in Section 5.1. If the marginal voter has more activist (i.e., more extreme) preferences than the post-vote marginal trader, then on average, this marginal trader's valuation and hence the share price react negatively to proposal approval. In contrast, shareholder welfare can on average react positively to proposal approval if the marginal voter has less activist (i.e., less extreme) preferences than the *average* shareholder after the post-vote trading stage. Overall, this extension further supports our conclusion in Section 5.1 that price reactions may be an imperfect proxy for welfare effects of shareholder votes.

7.4 Trading with partial sales of endowments

The baseline model treats sales and purchases of shares asymmetrically by assuming that shareholders can buy only a limited number x of shares, but can always sell their entire endowment. In Section A.4 of the Online Appendix, we introduce partial sales of endowments by assuming that shareholders cannot sell more than $y \in (0, e)$ shares. The baseline model corresponds to $y = e$, and the no-trade benchmark corresponds to $y = 0$. The resulting equilibrium is similar to that in the baseline model with the marginal traders now given by $b_a = G^{-1}(\delta(y))$ and $b_c = G^{-1}(1 - \delta(y))$, where $\delta(y) \equiv \frac{x}{y+x}$ is the analog of δ in equation (2). The voting equilibrium with partial sales of endowments differs in two respects from the baseline case. First, the marginal voter can now be less extreme than the marginal trader if y is sufficiently close to zero. Intuitively, when y is very small, the supply of shares is very low, and only the most extreme shareholders with the highest willingness to pay will buy shares in equilibrium. That is, the marginal trader is extreme. At the same time, the post-trade shareholder base is very similar to the initial shareholder base because the volume of trade is low, and thus the marginal voter is relatively moderate. Second, the welfare function now becomes a weighted average of the welfare of the selling shareholders and that of the buying shareholders, where the weight of the selling shareholders is always smaller than that of the buying shareholders and decreases in y. Despite these differences, our main results about the price and welfare implications and delegation to the board continue to hold.

8 Conclusion

In this paper we study the relationship between trading and voting in a model in which shareholders have identical information but heterogeneous preferences. They trade with each other before those who end up owning the shares vote on a proposal. One of our main conclusions is that the complementarity between trading and voting gives rise to multiple equilibria. Multiple equilibria arise with self-fulfilling expectations, in our case about the likelihood that the proposal is accepted: If shareholders expect a high likelihood that the proposal is accepted, then the activist equilibrium obtains, and vice versa for a low likelihood. This leaves us with the question of how shareholders coordinate on a particular equilibrium. One way of addressing this issue is to root expectation formation in the economic environment.[22] In our context there are multiple potential sources in the economic environment that may influence expectation formation. For example, some shareholders may be more visible, have better access to the media, or have other characteristics not included in our model that put them into a position to influence the expectations of other shareholders. Proxy advisory firms may perform a similar function and may have an influence on voting outcomes by coordinating shareholders' expectations. We hope that the future empirical literature will study how shareholders form expectations about governance outcomes, how these expectations affect trading before shareholder votes, and how these changes in the shareholder base affect voting outcomes.

The second important conclusion is that shareholder voting may not lead to optimal outcomes. First, there is no guarantee that shareholders coordinate on the welfare-maximizing equilibrium if there are multiple equilibria. Second, we show that delegation to a board of directors can improve shareholder welfare even if shareholders can coordinate on the welfare-maximizing voting equilibrium. Third, the welfare of current shareholders is not maximized with a board that best represents their preferences. Rather, it is maximized by a board that represents the interests of those shareholders who own the firm after trading, and thus the

[22]This is ultimately the reasoning behind the notion of a focal point (Schelling, 1960), which rests on the argument that economic agents rely on additional reasoning to coordinate on a particular equilibrium. See Sugden and Zamarron (2006) and Myerson (2009) for positive evaluations of this "pragmatic" approach to equilibrium selection, and Morris and Shin (2003) for a more critical stance on leaving expectation formation outside the model.

optimal board needs to be biased. Hence, observing that the board pursues interests different from those of the average shareholder is not sufficient for making a case for "shareholder democracy:" Such a divergence can indeed be optimal. The parallelism to political democracy breaks down in one important respect: Shareholders can trade, and trading aligns the shareholder base with the expected outcomes.[23]

The model in this paper relies on heterogeneous preferences. However, the model could be easily modified to accommodate homogeneous preferences if we assume that shareholders have differences of opinions. In such a model, all shareholders would have the same bias, but each shareholder would have a different interpretation of the public signal about the proposal that all shareholders observe in our model.[24] The characterization of the equilibrium would remain similar, but the welfare analysis would require some adjustments, since models with differences of opinions lack objectively correct probability distributions. Exploring such an extension is left for future research.

[23] Easterbrook and Fischel (1983) already pointed out this important difference when they argued that the ability to sell shares serves the same purpose as voting in a polity, which is designed to "elicit the views of the governed and to limit powerful states." (p. 396). The issue is still debated vigorously in the law literature, see Bebchuk (2005) and Bainbridge (2006).

[24] Some papers have explored differences of opinions in relation to corporate governance theoretically (Boot, Gopalan, and Thakor, 2006; Kakhbod et al., 2019) and empirically (Li, Maug, and Schwartz-Ziv, 2019).

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Appendix - Proofs

This appendix presents the proofs of all results in the paper. Throughout the appendix, the cutoff q^* can potentially fall out of the support of the distribution of q, $[-\Delta, \Delta]$. In this case, if $q^* \geq \Delta$, we set $H(q^*) = 0$, $H(q^*) \mathbb{E}[\theta | q > q^*] = 0$, and $f(q^*) = 0$. Similarly, if $q^* \leq -\Delta$, we set $H(q^*) = 1$, $H(q^*) \mathbb{E}[\theta | q > q^*] = \mathbb{E}[\theta] = 0$, and $f(q^*) = 0$.

Proof of Lemma 1. Given the realization of q, a shareholder indexed by b votes his shares for the proposal if and only if $q > -b$. Denote the fraction of post-trade shares voted to approve the proposal by $\Lambda(q)$. Note that $\Lambda(q)$ is weakly increasing (everyone who votes "for" given a smaller q will also vote "for" given a larger q, and there might be a non-negative mass of new shareholders who start voting "for"). If, for the lowest possible $q = -\Delta$, we have $\Lambda(-\Delta) > \tau$, then q^* in the statement of the lemma is equal to -1 (because the proposal is always approved). Similarly, if for the highest possible $q = \Delta$, we have $\Lambda(\Delta) \leq \tau$, then q^* in the statement of the lemma is equal to Δ (because the proposal is never approved). Finally, if $\Lambda(-\Delta) \leq \tau < \Lambda(\Delta)$, there exists $q^* \in [-\Delta, \Delta)$ such that the fraction of votes voted in favor of the proposal is greater than τ if and only if $q > q^*$. Hence, the proposal is approved if and only if $q > q^*$. ■

Proof of Proposition 2. We consider three cases. First, suppose $H(q^*) > \phi$. In this case, $v(b, q^*)$ increases in b, and a shareholder with bias b buys x shares if

$$v(b, q^*) > p \Leftrightarrow b > b_a \equiv \frac{p - v_0 - H(q^*) \mathbb{E}[\theta | q > q^*]}{H(q^*) - \phi},$$

and sells e shares if $v(b, q^*) < p$. Therefore, the total demand for shares is $D(p) = x \Pr[b > b_a]$ and the total supply of shares is $S(p) = e \Pr[b < b_a]$. The market clears if and only if $D(p) = S(p) \Leftrightarrow$

$$\Pr[b < b_a] = \frac{x}{x + e} = \delta \Leftrightarrow b_a = G^{-1}(\delta).$$

Since $\delta \in (0, 1)$, we have $b_a \in \left(-\bar{b}, \bar{b}\right)$. The price that clears the market is the valuation of the marginal trader b_a, and therefore, $p = v(b_a, q^*)$, as required.

Second, suppose $H(q^*) < \phi$. In this case, $v(b, q^*)$ decreases in b, and a shareholder with bias b buys x shares if

$$v(b, q^*) > p \Leftrightarrow b < b_c \equiv \frac{p - v_0 - H(q^*)\,\mathbb{E}[\theta | q > q^*]}{H(q^*) - \phi},$$

and sells e shares if $v(b, q^*) < p$. Therefore, the total demand for shares is $D(p) = x \Pr[b < b_c]$ and the total supply of shares is $S(p) = e \Pr[b > b_c]$. The market clears if and only if $D(p) = S(p) \Leftrightarrow$

$$\Pr[b < b_c] = \frac{e}{x + e} = 1 - \delta \Leftrightarrow b_c = G^{-1}(1 - \delta).$$

Since $\delta \in (0, 1)$, we have $b_c \in \left(-\bar{b}, \bar{b}\right)$. The price that clears the market is the valuation of the marginal trader b_c, and therefore, $p = v(b_c, q^*)$, as required.

Finally, suppose $H(q^*) = \phi$. In this case, the expected value of each shareholder is

$$v(b, q^*) = v_0 + H(q^*)\,\mathbb{E}[\theta | q > q^*] = v_0 + \phi \mathbb{E}[\theta | q > q^*].$$

The market can clear only if $p = v_0 + \phi \mathbb{E}[\theta | q > q^*]$, since otherwise, either all shareholders would want to buy shares or all shareholders would want to sell their shares. Notice that shareholder value does not depend on b, and that market clearing implies that all shareholders are indifferent between buying and selling shares. Based on the tie-breaking rule we adopt, shareholders will not trade. ■

Proof of Proposition 3. According to Lemma 1, any equilibrium is characterized by some cutoff q^* at the voting stage. We consider three cases.

First, suppose that $H(q^*) > \phi$ (activist equilibrium). The arguments in the proof of Proposition 2 can again be repeated word for word. In particular, the marginal trader is b_a as given by (8), and after the trading stage, the shareholder base consists entirely of shareholders with $b > b_a$. Consider a realization of q. If $q > -b_a$, the proposal is accepted ($b > b_a > -q$ for all shareholders of the firm). If $q < -b_a$, then shareholders who vote in favor are those with $b \in (-q, \bar{b}]$ out of $b \in (b_a, \bar{b}]$, which gives a fraction of $\Pr[-q < b | b_a < b]$ affirmative votes. Hence, the proposal is accepted if and only if either (1) $q > -b_a$ or (2) $q < -b_a$ and $\Pr[-q < b | b_a < b] > \tau$, where the condition in (1) is equivalent to $q > -G^{-1}(\delta)$, and the conditions in (2) are together equivalent to

$$\begin{aligned}
\Pr\left[-q < b | b_a < b, q < -b_a\right] &> \tau \Leftrightarrow 1 - G\left(-q\right) > \tau\left(1 - G\left(b_a\right)\right) = \tau\left(1 - \delta\right) \\
&\Leftrightarrow q > -G^{-1}\left(1 - \tau\left(1 - \delta\right)\right).
\end{aligned}$$

Hence, the proposal is accepted if and only if $q > q_a = \min\{-G^{-1}(\delta), -G^{-1}(1 - \tau(1 - \delta))\}$, and since $\delta < 1 - \tau(1 - \delta)$, the cutoff in this "activist" equilibrium is q_a as given by (10). Similarly to the proof of Proposition 2, the share price is $p_a = v(b_a, q_a)$.

Second, suppose that $H(q^*) < \phi$ (conservative equilibrium). The arguments in the proof of Proposition 2 can again be repeated here. In particular, the marginal trader is b_c as given by (9), and after the trading stage, the shareholder base consists entirely of shareholders with $b < b_c$. Consider a realization of q. Recall that shareholder b votes for the proposal if and only if $q > -b$. Hence, if $q < -b_c$, all shareholders of the firm vote against ($b < b_c < -q$), so the proposal is rejected. If $q > -b_c$, then shareholders who vote in favor are those with $b \in (-q, b_c)$ out of $b \in [-\bar{b}, b_c)$, which gives a fraction of $\Pr[-q < b < b_c | b < b_c]$ affirmative votes. Hence, the proposal is accepted if and only if $-q < b_c$ and $\tau < \Pr[-q < b < b_c | b < b_c]$, which are together equivalent to

$$\begin{aligned}
\tau &< \frac{\Pr\left[b < b_c\right] - \Pr\left[b < -q\right]}{\Pr\left[b < b_c\right]} \Leftrightarrow \Pr\left[b < -q\right] < \left(1 - \tau\right) \Pr\left[b < b_c\right] \\
&\Leftrightarrow G\left(-q\right) < \left(1 - \tau\right)\left(1 - \delta\right) \Leftrightarrow q > -G^{-1}\left(\left(1 - \tau\right)\left(1 - \delta\right)\right).
\end{aligned}$$

Hence, the cutoff in this "conservative" equilibrium is q_c, given by (11). Similarly to the proof of Proposition 2, the share price is $p_c = v(b_c, q_c)$.

Third, suppose $H(q^*) = \phi$. In this case, the value of each shareholder is

$$v\left(b, q^*\right) = v_0 + H\left(q^*\right) \mathbb{E}\left[\theta | q > q^*\right] = v_0 + \phi \mathbb{E}\left[\theta | q > q^*\right].$$

Therefore, the market can clear only if $p = v_0 + \phi \mathbb{E}[\theta | q > q^*]$. Notice that shareholder value does not depend on b, and that market clearing implies that all shareholders are indifferent between buying and selling shares. Based on the tie-breaking rule we adopt, shareholders will not trade. Therefore, the post-trade shareholder base is identical to the pre-trade shareholder base. Next, note that $H(q^*) = \phi$ implies that the proposal is accepted if and only if $q > F^{-1}(1 - \phi)$. Since a shareholder votes for the proposal if and only if $q > -b$, it must be that the fraction of initial shareholders with $F^{-1}(1-\phi) > -b$ is exactly τ, which is equivalent to $1 - G(-F^{-1}(1-\phi)) = \tau$, or $G^{-1}(1-\tau) = -F^{-1}(1-\phi)$. This is a knife-edge case that we ignore, since it does not hold generically.

Finally, notice that $q_a < q_c$, and therefore, either $H(q_c) < \phi$, or $H(q_a) > \phi$, or both. Therefore, an equilibrium always exists (but may be non-unique if $H(q_c) < \phi < H(q_a)$). This completes the proof.

As a side note, notice also that many other tie-breaking rules, those in which all shareholders follow the same strategy upon indifference (e.g., buy $r \in [-e, x]$ shares), would also eliminate this type of equilibrium. Indeed, if all shareholders buy or sell a certain (the same across shareholders) amount of shares upon indifference, the market is unlikely to clear. For the market to clear, shareholders with different biases would need to behave differently when they are indifferent between buying and selling shares, that is, the tie-breaking rule has to differ across shareholders in a particular way. Since such a tie-breaking rule is somewhat arbitrary, we ruled it out as an unlikely outcome. ■

Proof of Proposition 4. Note that condition (12) can be written as

$$(1-\delta)(1-\tau) < G\left(-F^{-1}(1-\phi)\right) < 1 - \tau(1-\delta). \tag{22}$$

To see the point about δ, note that (22) is equivalent to

$$\delta > \max\left\{1 - \frac{G(-F^{-1}(1-\phi))}{1-\tau}, 1 - \frac{1 - G(-F^{-1}(1-\phi))}{\tau}\right\}.$$

To see the point about τ, note that (22) is equivalent to

$$1 - \frac{G(-F^{-1}(1-\phi))}{1-\delta} < \tau < \frac{1 - G(-F^{-1}(1-\phi))}{1-\delta}.$$

To see the point about ϕ, note that (22) is equivalent to

$$1 - F\left(-G^{-1}((1-\delta)(1-\tau))\right) < \phi < 1 - F\left(-G^{-1}(1-\tau(1-\delta))\right).$$

Finally, notice that as $\bar{b} \to 0$, the bias of the post-trade shareholder base becomes homogeneous at zero, and in particular, the marginal voter must converge to zero as well. This implies

$\lim_{\bar{b}\to 0} q^* = 0$ in any equilibrium, and thus, the voting equilibrium must be unique: it is an activist equilibrium if and only if $H(0) < \phi$. Therefore, condition (12) can be satisfied only if $\bar{b}$ is sufficiently large, as required. ■

Proof of Lemma 2. Recall that in the conservative equilibrium, market clearing implies $\Pr[b > b_c]\, e = \Pr[b < b_c]\, x$, where $\Pr[b < b_c] = 1 - \delta = \frac{e}{x+e}$. Therefore,

$$\begin{aligned} W_c &= \Pr[b > b_c]\, ep_c + \Pr[b < b_c]\, \mathbb{E}[(e+x)\, v(b, q_c) - xp_c | b < b_c] \\ &= \Pr[b < b_c]\, xp_c + \Pr[b < b_c]\, \mathbb{E}[(e+x)\, v(b, q_c) - xp_c | b < b_c] \\ &= \Pr[b < b_c]\, \mathbb{E}[(e+x)\, v(b, q_c)\, | b < b_c] = (1-\delta)(e+x)\, \mathbb{E}[v(b, q_c)\, | b < b_c] \\ &= e\mathbb{E}[v(b, q_c)\, | b < b_c] = ev(\mathbb{E}[b | b < b_c], q_c) = ev(\beta_c, q_c), \end{aligned}$$

where the second to last equality follows from the linearity of $v(b, q_c)$ in b.

Similarly, in the activist equilibrium, market clearing implies $\Pr[b < b_a]\, e = \Pr[b > b_a]\, x$, where $\Pr[b > b_a] = 1 - \delta = \frac{e}{x+\varepsilon}$. Therefore,

$$\begin{aligned} W_a &= \Pr[b < b_a]\, ep_a + \Pr[b > b_a]\, \mathbb{E}[(e+x)\, v(b, q_a) - xp_a | b > b_a] \\ &= \Pr[b > b_a]\, xp_a + \Pr[b > b_a]\, \mathbb{E}[(e+x)\, v(b, q_a) - xp_a | b > b_a] \\ &= \Pr[b > b_a]\, \mathbb{E}[(e+x)\, v(b, q_a)\, | b > b_a] = (1-\delta)(e+x)\, \mathbb{E}[v(b, q_a)\, | b > b_a] \\ &= e\mathbb{E}[v(b, q_a)\, | b > b_a] = ev(\mathbb{E}[b | b > b_a], q_a) = ev(\beta_a, q_a). \end{aligned}$$

■

Proof of Proposition 5. Consider the conservative equilibrium. Recall that in this equilibrium $W_c = e \cdot v(\beta_c, q_c)$ and $p_c = v(b_c, q_c)$. Then, a change in parameters that affects the marginal voter (q_c) without changing the marginal trader only affects W_c and p_c through its effect on q_c. Also recall that based on (17), $v(\beta_c, q^*)$ is a hump-shaped function in q^* with a maximum at $q^* = -\beta_c$, and $v(b_c, q^*)$ is a hump-shaped function in q^* with a maximum at $q^* = -b_c$. Since $-b_c < q_c - \beta_c$ by assumption of the proposition, any small enough change in parameters that leaves this order unchanged ($-b_c < q_c - \beta_c$) either increases the distance of q_c to $-\beta_c$ but decreases the distance to $-b_c$, or vice versa. Hence, this change of parameters necessarily moves prices and welfare in opposite directions.

Consider the activist equilibrium. Recall that in this equilibrium $W_a = e \cdot v(\beta_a, q_a)$ and $p_a = v(b_a, q_a)$. Then, a change in parameters that affects the marginal voter (q_a) without changing the marginal trader only affects W_a and p_a through its effect on q_a. Also recall that based on (17), $v(\beta_a, q^*)$ is a hump-shaped function in q^* with a maximum at $q^* = -\beta_a$, and $v(b_a, q^*)$ is a hump-shaped function in q^* with a maximum at $q^* = -b_a$. Since $-b_a < q_a - \beta_a$ by assumption of the proposition, any small enough change in parameters that leaves this order unchanged $(-b_a < q_a - \beta_a)$ either increases the distance to $-\beta_a$ but decreases the distance to $-b_a$, or vice versa. Hence, this change of parameters necessarily moves prices and welfare in opposite directions. ■

Proof of Lemma 4. Based on Proposition 2, the share price is

$$p_{NoVote}(q^*) = v_0 + H(q^*)\,\mathbb{E}[q|q > q^*] + \begin{cases} b_c(H(q^*) - \phi) & \text{if } H(q^*) < \phi \\ b_a(H(q^*) - \phi) & \text{if } H(q^*) > \phi, \end{cases}$$

and the expected shareholder welfare is

$$W_{NoVote}(q^*) = e \cdot \left[v_0 + H(q^*)\,\mathbb{E}[q|q > q^*] + \begin{cases} \beta_c(H(q^*) - \phi) & \text{if } H(q^*) < \phi \\ \beta_a(H(q^*) - \phi) & \text{if } H(q^*) > \phi. \end{cases} \right]$$

Recall that $b_c = G^{-1}(1-\delta)$, $\beta_c = \mathbb{E}[b|b < b_c]$, $b_a = G^{-1}(\delta)$, and $\beta_a = \mathbb{E}[b|b > b_a]$. Thus, $p_{NoVote}(q^*)$ and $W_{NoVote}(q^*)$ depend on δ only through their effect on b_c and b_a. Since, by Corollary 1, b_c and β_c are decreasing in δ, and b_a and β_a are increasing in δ, both $p_{NoVote}(q^*)$ and $W_{NoVote}(q^*)$ increase in δ. ■

Proof of Proposition 6. First, consider the conservative equilibrium, which exists if and only if $H(q_c) < \phi$. Recall $p_c = v(b_c, q_c)$ and $W_c = e \cdot v(\beta_c, q_c)$, where $b_c = G^{-1}(1-\delta)$, $\beta_c = \mathbb{E}[b|b < b_c] = \frac{1}{G(b_c)} \int_{-\bar{b}}^{b_c} b dG(b)$, and $q_c = -G^{-1}((1-\delta)(1-\tau))$. Using (7),

$$\frac{\partial p_c}{\partial \delta} = \frac{\partial b_c}{\partial \delta}(H(q_c) - \phi) - (b_c + q_c)\frac{\partial q_c}{\partial \delta} f(q_c) \tag{23}$$

and

$$\frac{1}{e}\frac{\partial W_c}{\partial \delta} = \frac{\partial \beta_c}{\partial \delta}(H(q_c) - \phi) - (\beta_c + q_c)\frac{\partial q_c}{\partial \delta} f(q_c). \tag{24}$$

More precisely, (23)-(24) hold when $q_c \in (-\Delta, \Delta)$, and when q_c is outside these bounds, the second term in both of these expressions is equal to zero (as noted above, we set $f(q^*) = 0$ for $q^* \notin (-\Delta, \Delta)$).

Using (11) and (9), we get $\frac{\partial q_c}{\partial \delta} = \frac{1-\tau}{g(-q_c)} > 0$, $\frac{\partial b_c}{\partial \delta} = -\frac{1}{g(b_c)} < 0$, and

$$\frac{\partial \beta_c}{\partial \delta} = \frac{\frac{\partial b_c}{\partial \delta} b_c g(b_c) G(b_c) - \left[\int_{-\bar{b}}^{b_c} b g(b)\, db\right] g(b_c) \frac{\partial b_c}{\partial \delta}}{[G(b_c)]^2} = \frac{\partial b_c}{\partial \delta} \frac{g(b_c)}{G(b_c)} (b_c - \beta_c) = -\frac{b_c - \beta_c}{G(b_c)} < 0.$$

Plugging into (23) and (24), we get

$$\begin{aligned} \frac{\partial p_c}{\partial \delta} &= -\frac{H(q_c) - \phi}{g(b_c)} - (1-\tau)(b_c + q_c) \frac{f(q_c)}{g(-q_c)} \\ \frac{1}{e} \frac{\partial W_c}{\partial \delta} &= -\frac{H(q_c) - \phi}{G(b_c)} (b_c - \beta_c) - (1-\tau)(\beta_c + q_c) \frac{f(q_c)}{g(-q_c)}, \end{aligned}$$

where again, the second term is zero if $q_c \notin (-\Delta, \Delta)$. Notice that as $\delta \to 1$, then b_c, β_c, and $-q_c$ all converge to $-\bar{b}$, and $H(q_c) - \phi \to H(\bar{b}) - \phi$. Suppose the conservative equilibrium exists in the limit (which is the case if $H(\bar{b}) < \phi$). Since g is positive on $[-\bar{b}, \bar{b}]$,

$$\lim_{\delta \to 1} \frac{\partial p_c}{\partial \delta} = -\frac{H(\bar{b}) - \phi}{g(-\bar{b})} > 0.$$

In addition, $\lim_{\delta \to 1} \frac{1}{e} \frac{\partial W_c}{\partial \delta} = -\left(H(\bar{b}) - \phi\right) \lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)}$. Using l'Hopital's rule,

$$\lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)} = \lim_{\delta \to 1} \frac{\frac{\partial b_c}{\partial \delta} - \frac{\partial \beta_c}{\partial \delta}}{g(b_c) \frac{\partial b_c}{\partial \delta}} = \frac{1}{g(-\bar{b})} - \lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)},$$

which implies $\lim_{\delta \to 1} \frac{b_c - \beta_c}{G(b_c)} = \frac{1}{2} \frac{1}{g(-\bar{b})} > 0$, and hence $\lim_{\delta \to 1} \frac{\partial W_c}{\partial \delta} > 0$.

Also notice that as $\delta \to 0$, then $b_c \to \bar{b}$, $\beta_c \to \mathbb{E}[b]$, and $q_c \to q_{NoTrade} = -G^{-1}(1-\tau) > -\bar{b}$. Suppose the conservative equilibrium exists in this limit (which is the case if $H(q_{NoTrade}) < \phi$). Then, using (23),

$$\lim_{\delta \to 0} \frac{\partial p_c}{\partial \delta} = -\frac{H(q_{NoTrade}) - \phi}{g(\bar{b})} - (1-\tau)\left(\bar{b} + q_{NoTrade}\right) \frac{f(q_{NoTrade})}{g(-q_{NoTrade})},$$

where the second term is strictly negative because (1) by assumption, $q_{NoTrade} \in (-\Delta, \Delta)$, and (2) $\bar{b} + q_{NoTrade} > 0$, as shown above. Hence, $\lim_{\delta \to 0} \frac{\partial p_c}{\partial \delta} < 0$ if $|H(q_{NoTrade}) - \phi|$ is sufficiently small.

Also notice that

$$\lim_{\delta\to 0}\frac{1}{e}\frac{\partial W_c}{\partial \delta}=-\frac{H\left(q_{NoTrade}\right)-\phi}{G\left(\bar{b}\right)}\left(\bar{b}-\mathbb{E}\left[b\right]\right)-\left(1-\tau\right)\left(\mathbb{E}\left[b\right]+q_{NoTrade}\right)\frac{f\left(q_{NoTrade}\right)}{g\left(-q_{NoTrade}\right)}.$$

Thus, if $\lim_{\delta\to 0}\left(\beta_c+q_c\right)=\mathbb{E}\left[b\right]+q_{NoTrade}>0$ (i.e., the marginal voter is more extreme than the average post-trade shareholder) and $\left|H\left(q_{NoTrade}\right)-\phi\right|$ is small enough, then $\lim_{\delta\to 0}\frac{\partial W_c}{\partial\delta}<0$.

Second, consider the activist equilibrium, which exists if and only if $H\left(q_a\right)-\phi>0$. Similarly to the above, recall $p_a=v\left(b_a,q_a\right)$ and $W_a=e\cdot v\left(\beta_a,q_a\right)$, where $b_a=G^{-1}\left(\delta\right)$, $\beta_a=\mathbb{E}\left[b|b>b_a\right]=\frac{1}{G(1-b_a)}\int_{b_a}^{\bar{b}}bdG\left(b\right)$, and $q_a=-G^{-1}\left(1-\tau\left(1-\delta\right)\right)$. Using (7),

$$\frac{\partial p_a}{\partial\delta}=\frac{\partial b_a}{\partial\delta}\left(H\left(q_a\right)-\phi\right)-\left(b_a+q_a\right)\frac{\partial q_a}{\partial\delta}f\left(q_a\right) \tag{25}$$

and

$$\frac{1}{e}\frac{\partial W_a}{\partial\delta}=\frac{\partial\beta_a}{\partial\delta}\left(H\left(q_a\right)-\phi\right)-\left(\beta_a+q_a\right)\frac{\partial q_a}{\partial\delta}f\left(q_a\right). \tag{26}$$

More precisely, (25)-(26) hold when $q_a\in\left(-\Delta,\Delta\right)$, and when q_a is outside these bounds, the second term in both of these expressions is equal to zero (as noted above, we set $f\left(q^*\right)=0$ for $q^*\notin\left(-\Delta,\Delta\right)$).

Using (10) and (8), we get $\frac{\partial q_a}{\partial\delta}=-\frac{\tau}{g(-q_a)}<0$, $\frac{\partial b_a}{\partial\delta}=\frac{1}{g(b_a)}>0$, and

$$\begin{aligned}\frac{\partial\beta_a}{\partial\delta} &= \frac{-\frac{\partial b_a}{\partial\delta}b_a g\left(b_a\right)\left[1-G\left(b_a\right)\right]+\left[\int_{b_a}^{\bar{b}}bg\left(b\right)db\right]g\left(b_a\right)\frac{\partial b_a}{\partial\delta}}{\left[1-G\left(b_a\right)\right]^2}\\ &= \frac{\partial b_a}{\partial\delta}\frac{g\left(b_a\right)}{1-G\left(b_a\right)}\left(\beta_a-b_a\right)=\frac{\beta_a-b_a}{1-G\left(b_a\right)}>0.\end{aligned}$$

Plugging into (25) and (26), we get

$$\begin{aligned}\frac{\partial p_a}{\partial\delta} &= \frac{H\left(q_a\right)-\phi}{g\left(b_a\right)}+\tau\left(b_a+q_a\right)\frac{f\left(q_a\right)}{g\left(-q_a\right)}\\ \frac{1}{e}\frac{\partial W_a}{\partial\delta} &= \frac{H\left(q_a\right)-\phi}{1-G\left(b_a\right)}\left(\beta_a-b_a\right)+\tau\left(\beta_a+q_a\right)\frac{f\left(q_a\right)}{g\left(-q_a\right)},\end{aligned}$$

where again, the second term is zero if $q_a\notin\left(-\Delta,\Delta\right)$. Notice that as $\delta\to 1$, then b_a, β_a, and $-q_a$ all converge to $\bar{b}$, and $H\left(q_a\right)-\to H\left(-\bar{b}\right)-\phi$. Suppose the activist equilibrium exists in the limit (which is the case if $H\left(-\bar{b}\right) > \phi$). Since g is positive on $\left[-\bar{b}, \bar{b}\right]$,

$$\lim_{\delta\to 1}\frac{\partial p_a}{\partial\delta} = \frac{H\left(-\bar{b}\right)-\phi}{g(\bar{b})} > 0.$$

In addition, $\lim_{\delta\to 1}\frac{1}{e}\frac{\partial W_a}{\partial\delta} = \left(H\left(-\bar{b}\right)-\phi\right)\lim_{\delta\to 1}\frac{\beta_a - b_a}{1-G(b_a)}$. Using l'Hopital's rule,

$$\lim_{\delta\to 1}\frac{\beta_a - b_a}{1-G\left(b_a\right)} = \lim_{\delta\to 1}\frac{\frac{\partial\beta_a}{\partial\delta}-\frac{\partial b_a}{\partial\delta}}{-g\left(b_a\right)\frac{\partial b_a}{\partial\delta}} = \frac{1}{g\left(\bar{b}\right)} - \lim_{\delta\to 1}\frac{\beta_a - b_a}{1-G\left(b_a\right)}$$

which implies $\lim_{\delta\to 1}\frac{\beta_a - b_a}{1-G(b_a)} = \frac{1}{2}\frac{1}{g\left(\bar{b}\right)} > 0$. Therefore, $\lim_{\delta\to 1}\frac{\partial W_a}{\partial\delta} > 0$.

Also notice that as $\delta\to 0$, then $b_a\to -\bar{b}$, $\beta_a\to\mathbb{E}\left[b\right]$, and $q_a\to q_{NoTrade} = -G^{-1}\left(1-\tau\right) < \bar{b}$. Suppose the activist equilibrium exists in this limit (which is the case if $H\left(q_{NoTrade}\right) > \phi$). Then

$$\lim_{\delta\to 0}\frac{\partial p_a}{\partial\delta} = \frac{H\left(q_{NoTrade}\right)-\phi}{g\left(-\bar{b}\right)} + \tau\left(-\bar{b}+q_{NoTrade}\right)\frac{f\left(q_{NoTrade}\right)}{g\left(-q_{NoTrade}\right)},$$

where the second term is strictly negative because (1) by assumption, $q_{NoTrade}\in\left(-\Delta,\Delta\right)$, and (2) $-\bar{b}+q_{NoTrade} < 0$, as shown above. Hence, $\lim_{\delta\to 0}\frac{\partial p_a}{\partial\delta} < 0$ if $\left|H\left(q_{NoTrade}\right)-\phi\right|$ is sufficiently small. Also notice that

$$\lim_{\delta\to 0}\frac{1}{e}\frac{\partial W_a}{\partial\delta} = \frac{H\left(q_{NoTrade}\right)-\phi}{1-G\left(b_a\right)}\left(\mathbb{E}\left[b\right]+\bar{b}\right) + \tau\left(\mathbb{E}\left[b\right]+q_{NoTrade}\right)\frac{f\left(q_{NoTrade}\right)}{g\left(-q_{NoTrade}\right)}.$$

Thus, if $\lim_{\delta\to 0}\left(\beta_a + q_a\right) = \mathbb{E}\left[b\right]+q_{NoTrade} < 0$ (i.e., the marginal voter is more extreme than the average post-trade shareholder) and $\left|H\left(q_{NoTrade}\right)-\phi\right|$ is small enough, then $\lim_{\delta\to 0}\frac{\partial W_a}{\partial\delta} < 0$.

Given the strictly positive (negative) limits of $\frac{\partial p}{\partial\delta}$ and $\frac{\partial W}{\partial\delta}$ as $\delta\to 1$ ($\delta\to 0$) for any equilibrium as long as it exists, it follows that under the conditions of the proposition, there exist $\underline{\delta}$ and $\overline{\delta}$, $0 < \underline{\delta} < \overline{\delta} < 1$, such that both the share price and welfare in any equilibrium that exists increase (decrease) in δ for $\delta > \overline{\delta}$ ($\delta < \underline{\delta}$), as required. ■

Proof of Proposition 7. We start by noting that if $q^* = H^{-1}\left(\phi\right)$, then all shareholders are indifferent between buying and selling, and the tie-breaking rule we adopt implies that in equilibrium, no shareholder trades. While this tie-breaking rule implies that the trading strategies of shareholders in the delegation equilibrium are not continuous in q^* as $q^*\to H^{-1}\left(\phi\right)$, the expected welfare of shareholders in any equilibrium continuously converges to welfare in the equilibrium with $q^* = H^{-1}(\phi)$. Indeed, shareholder welfare in the equilibrium in which $q^* = H^{-1}(\phi)$ and shareholders thus do not trade is

$$e \cdot \mathbb{E}\left[v\left(b, H^{-1}(\phi)\right)\right] = e \cdot v\left(\mathbb{E}[b], H^{-1}(\phi)\right) = e \cdot \left(v_0 + \phi \mathbb{E}\left[\theta | q > H^{-1}(\phi)\right]\right). \tag{27}$$

Using (16) and (7), it is easy to see that the limit of shareholder welfare in both the conservative equilibrium ($e \cdot \lim_{q^* \searrow H^{-1}(\phi)} v(\beta_c, q^*)$) and in the activist equilibrium ($e \cdot \lim_{q^* \nearrow H^{-1}(\phi)} v(\beta_a, q^*)$) is the same and equals (27), as required.

Proof of the expressions for b_m^* and W_m^* in (19). The choice of the optimal board is equivalent to choosing the cutoff q^* that maximizes expected shareholder welfare. Recall from Section 5 and (17) that $v(b, q^*)$ is a hump-shaped function in q^* with a maximum at $q^* = -b$. Thus, within the range of q^* that generates a conservative equilibrium or the equilibrium where shareholders are indifferent and do not trade ($H(q^*) \leq \phi \Leftrightarrow q^* \geq H^{-1}(\phi)$), (16) implies that the optimal cutoff q^* is the point closest to $-\beta_c$ in this range, i.e., $\max\{-\beta_c, H^{-1}(\phi)\}$. Similarly, within the range of q^* that generates an activist equilibrium or the equilibrium where shareholders are indifferent and do not trade ($H(q^*) \geq \phi \Leftrightarrow q^* \leq H^{-1}(\phi)$), the optimal cutoff q^* is the point closest to $-\beta_a$ in this range, i.e., $\min\{-\beta_a, H^{-1}(\phi)\}$. Since $\beta_c < \beta_a$, there are three cases to consider.

1. If $H^{-1}(\phi) \leq -\beta_a$, then any $q^* < H^{-1}(\phi)$ generates an activist equilibrium, and it is welfare inferior to the equilibrium with $q^* = H^{-1}(\phi)$. At the same time, setting $q^* = -\beta_c$ would generate a conservative equilibrium that is superior to an equilibrium with $q^* = H^{-1}(\phi)$ because $-\beta_c > -\beta_a \geq H^{-1}(\phi)$. Therefore, in this case $b_m^* = \beta_c$.

2. If $-\beta_c \leq H^{-1}(\phi)$, then any $q^* > H^{-1}(\phi)$ generates a conservative equilibrium, and it is welfare inferior to an equilibrium with $q^* = H^{-1}(\phi)$. At the same time, setting $q^* = -\beta_a$ would generate an activist equilibrium that is superior to an equilibrium with $q^* = H^{-1}(\phi)$ because $-\beta_a < -\beta_c \leq H^{-1}(\phi)$. Therefore, in this case $b_m^* = \beta_a$.

3. If $-\beta_a < H^{-1}(\phi) < -\beta_c$, then the optimal cutoff among those that generate a conservative equilibrium is $-\beta_c$, and the optimal cutoff among those that generate an activist equilibrium is $-\beta_a$, and both generate higher welfare than $q^* = H^{-1}(\phi)$. Then, $b_m^* = \beta_a$

if $v(\beta_a, -\beta_a) > v(\beta_c, -\beta_c)$, and $b_m^* = \beta_c$ otherwise. Notice that

$$v(\beta_a, -\beta_a) > v(\beta_c, -\beta_c) \Leftrightarrow H^{-1}(\phi) > H^{-1}(\Phi) \Leftrightarrow \phi < \Phi, \tag{28}$$

where

$$\begin{aligned} \Phi &\equiv H(-\beta_c) + \mathbb{E}\left[\beta_a + q \mid -\beta_a < q < -\beta_c\right] \frac{H(-\beta_a) - H(-\beta_c)}{\beta_a - \beta_c} \\ &= H(-\beta_a) + \mathbb{E}\left[\beta_c + q \mid -\beta_a < q < -\beta_c\right] \frac{H(-\beta_a) - H(-\beta_c)}{\beta_a - \beta_c}. \end{aligned} \tag{29}$$

Thus, $b_m^* = \beta_a$ if $\phi < \Phi \Leftrightarrow H^{-1}(\phi) > H^{-1}(\Phi)$ and $b_m^* = \beta_c$ if $\phi > \Phi \Leftrightarrow H^{-1}(\phi) < H^{-1}(\Phi)$. Also notice that $H(-\beta_a) > \Phi > H(-\beta_c)$, which implies $-\beta_a < H^{-1}(\Phi) < -\beta_c$.

Taken together, the three cases above imply that $b_m^* = \beta_c$ if either $H^{-1}(\phi) \le -\beta_a$ or $-\beta_a < H^{-1}(\phi)$ and $H^{-1}(\phi) < H^{-1}(\Phi)$. Since $-\beta_a < H^{-1}(\Phi)$, these two conditions together imply that $b_m^* = \beta_c$ if $H^{-1}(\phi) < H^{-1}(\Phi) \Leftrightarrow \phi > \Phi$. And, the three cases above imply that $b_m^* = \beta_a$ if either $-\beta_c \le H^{-1}(\phi)$ or $H^{-1}(\phi) < -\beta_c$ and $H^{-1}(\Phi) < H^{-1}(\phi)$. Since $H^{-1}(\Phi) < -\beta_c$, these two conditions together imply that $b_m^* = \beta_a$ if $H^{-1}(\phi) > H^{-1}(\Phi) \Leftrightarrow \phi < \Phi$. If $\phi = \Phi$, both β_a and β_c give the highest possible shareholder welfare.

We conclude that $b_m^* = \beta_a$ if $\phi < \Phi \Leftrightarrow v(\beta_a, -\beta_a) > v(\beta_c, -\beta_c)$ and $b_m^* = \beta_c$ otherwise. This proves (19).

Proof of (i)**.** It automatically follows from (19) and the fact that $\beta_a = \mathbb{E}[b|b > b_a] > \mathbb{E}[b]$ and $\beta_c = \mathbb{E}[b|b < b_c] < \mathbb{E}[b]$.

Proof of (ii)**.** According to (19), $W_m^* = e \cdot v(B, -B)$, where B is either β_c or β_a. We first prove that $v(B, -B)$ increases in B if and only if $H(-B) > \phi$. Indeed, based on (7),

$$\begin{aligned} v(B, -B) &= v_0 + B\left(H(-B) - \phi\right) + H(-B)\,\mathbb{E}\left[\theta | q > -B\right] \\ &= v_0 - \phi B + \int_{-B} (q + B)\, dF(q) \end{aligned} \tag{30}$$

if $-B \in (-\Delta, \Delta)$. If $-B > \Delta$, the last term in (30) is zero, and if $-B < -\Delta$, the last term is B. Therefore, $\frac{\partial v(B,-B)}{\partial B} = H(-B) - \phi$ for all $-B$, as required.

From (19), W_m^* depends on δ only through its effect on b_a and b_c. First, suppose $\phi > \Phi$. Then $b_m^* = \beta_c$, and the equilibrium under the optimal board is conservative in the sense that

$H(-\beta_c) < \phi$. Then, $W_m^* = e \cdot v(B, -B)|_{B=\beta_c}$ and $\frac{\partial v(B,-B)}{\partial B}|_{B=\beta_c} < 0$. Since β_c decreases in δ, it follows that W_m^* increases in δ. Second, suppose $\phi < \Phi$. Then $b_m^* = \beta_a$, and the equilibrium under the optimal board is activist in the sense that $H(-\beta_a) > \phi$. Then, $W_m^* = e \cdot v(B, -B)|_{B=\beta_a}$ and $\frac{\partial v(B,-B)}{\partial B}|_{B=\beta_a} > 0$. Since β_a increases in δ, it follows that W_m^* increases in δ. Thus, if $\phi \neq \Phi$, then W_m^* increases in δ. If $\phi = \Phi$, then (28) implies $W_m^* = e \cdot v(\beta_c, -\beta_c) = e \cdot v(\beta_a, -\beta_a)$, and since both terms increase in δ, so does W_m^*.

Proof of (iii)**.** Notice that the delegation equilibrium can replicate any conservative (activist) voting equilibrium if we set $b_m = -q_c$ ($b_m = -q_a$). Therefore, delegation to the optimal board always weakly dominates the voting equilibrium and strictly dominates it except the knife-edge cases when the voting equilibrium is already efficient, i.e., $q_c = -b_m^*$ or $q_a = -b_m^*$. Moreover, except for these knife-edge cases, given the continuity of the expected welfare function around b_m^* and a strictly possible benefit of delegation at b_m^*, it follows that there is a neighborhood around b_m^* such that if the manager's bias is in that neighborhood, then the delegation equilibrium is strictly more efficient than the voting equilibrium. ■

Proof of Proposition 8. We first compute the expected shareholder payoff in each type of equilibrium. If shareholder b expects the voting equilibrium to be conservative, his expected payoff is $V_c(b, q_c)$, where

$$V_c(b, q^*) = \begin{cases} (e+x)\, v(b, q^*) - xv(b_c, q^*) & \text{if } b < b_c \\ ev(b_c, q^*) & \text{if } b \geq b_c. \end{cases} \tag{31}$$

Similarly, if shareholder b expects the delegation (to a board with bias $b_m = -q_m$) equilibrium to be conservative, his expected payoff is $V_c(b, q_m)$. Recall that the delegation equilibrium is conservative if and only if $H(q_m) < \phi \Leftrightarrow -q_m < -H^{-1}(\phi)$.

If shareholder b expects the voting equilibrium to be activist, his expected payoff is $V_a(b, q_a)$, where

$$V_a(b, q^*) = \begin{cases} (e+x)\, v(b, q^*) - xv(b_a, q^*) & \text{if } b > b_a \\ ev(b_a, q^*) & \text{if } b \leq b_a. \end{cases} \tag{32}$$

Similarly, if shareholder b expects the delegation (to a board with bias $b_m = -q_m$) equilibrium to be activist, his expected payoff is $V_a(b, q_m)$. Recall that the delegation equilibrium is activist if and only if $H(q_m) > \phi \Leftrightarrow -q_m > -H^{-1}(\phi)$.

First, suppose shareholders expect the voting equilibrium to be conservative. Consider as an alternative a conservative board with bias $b_m = -q_m < -H^{-1}(\phi)$. Shareholder b prefers delegation to such a board over the conservative voting equilibrium if and only if $V_c(b, q_c) < V_c(b, q_m)$. We consider several cases:

1. If $b \geq b_c$, then

$$\begin{aligned} V_c(b, q_c) &< V_c(b, q_m) \Leftrightarrow v(b_c, q_c) < v(b_c, q_m) \Leftrightarrow \\ b_c(H(q_c) - H(q_m)) &< H(q_m)\,\mathbb{E}[\theta|q > q_m] - H(q_c)\,\mathbb{E}[\theta|q > q_c]. \end{aligned}$$

- If in addition $q_c < q_m$, then $H(q_c) - H(q_m) > 0$, so

$$V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow b_c < \mathbb{E}[-q| - q_m < -q < -q_c],$$

which never holds since $b_c > -q_c$. Thus, shareholders with $b \geq b_c$ never support delegation to a board who is more extreme than the marginal voter, i.e., $q_c < q_m \Leftrightarrow b_m < -q_c$.

- If instead $q_c > q_m$, then $H(q_c) - H(q_m) < 0$, so

$$V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow b_c > \mathbb{E}[-q| - q_c < -q < -q_m].$$

Since $b_c > -q_c$, this always holds if $b_c \geq -q_m$ and might even hold if $b_c < -q_m$. Thus, shareholders with $b \geq b_c$ support delegation to a board whenever $-q_m \in (-q_c, b_c]$, and might even do so if $-q_m > b_c$.

2. If $b < b_c$, then (2) and (31) imply

$$\begin{gathered} V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow v(b, q_c) - \delta v(b_c, q_c) < v(b, q_m) - \delta v(b_c, q_m) \Leftrightarrow \\ v(b, q_c) - v(b, q_m) < \delta[v(b_c, q_c) - v(b_c, q_m)] \Leftrightarrow \\ b(H(q_c) - H(q_m)) + H(q_c)\,\mathbb{E}[\theta|q > q_c] - H(q_m)\,\mathbb{E}[\theta|q > q_m] \\ < \delta[b_c(H(q_c) - H(q_m)) + H(q_c)\,\mathbb{E}[\theta|q > q_c] - H(q_m)\,\mathbb{E}[\theta|q > q_m]\ ]. \end{gathered}$$

- If in addition $q_c < q_m$, then

$$V_c(b, q_c) < V_c(b, q_m) \Leftrightarrow b < \delta b_c + (1 - \delta)\,\mathbb{E}[-q| - q_m < -q < -q_c],$$

and notice that since $-q_c < b_c$, then $\delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_m < -q < -q_c] < b_c$.

- If instead $q_c > q_m$, then

$$V_c\left(b, q_c\right) < V_c\left(b, q_m\right) \Leftrightarrow b > \delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_c < -q < -q_m] \,.$$

The overall support for delegation to the board is the combined support of shareholders with $b < b_c$ and $b > b_c$. Then:
(i) First, consider a board with $-q_m < -q_c \Leftrightarrow q_m > q_c$. Then only shareholders with $b < \delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_m < -q < -q_c] < b_c$ support delegation to the board. It follows that if $G\left(b_c\right) < \tau \Leftrightarrow 1-\delta < \tau$, then this type of board does not obtain τ-support.
(ii) Second, consider a board with $-q_m > -q_c \Leftrightarrow q_m < q_c$. Such a board obtains support from $b \geq b_c$ if $b_c > \mathbb{E}\left[-q\right| - q_c < -q < -q_m]$ and from $b < b_c$ that satisfy $b > \delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_c < -q < -q_m]$. There are two cases:

- If $b_c < \mathbb{E}\left[-q\right| - q_c < -q < -q_m]$, then $\delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_c < -q < -q_m] > b_c$. Thus, in this case, there is no support for delegation from either shareholders with $b \geq b_c$ or from those with $b < b_c$.

- If $b_c > \mathbb{E}\left[-q\right| - q_c < -q < -q_m]$, then $\delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_c < -q < -q_m] < b_c$. Thus, both shareholders with $b \geq b_c$ and with $b \in \left(\delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_c < -q < -q_m], b_c\right)$ support delegation. So overall, delegation receives support from shareholders with $b > \delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_c < -q < -q_m]$. Notice that $\mathbb{E}\left[-q\right| - q_c < -q < -q_m] > -q_c$, and hence the fraction of initial shareholders supporting delegation is

$$1 - G\left(\delta b_c + (1-\delta)\,\mathbb{E}\left[-q\right| - q_c < -q < -q_m]\right) < 1 - G\left(\delta b_c - (1-\delta)\, q_c\right).$$

  Since $\lim_{\tau\to 1} q_c = \bar{b}$, we have $\lim_{\tau\to 1} 1 - G\left(\delta b_c - (1-\delta)\, q_c\right) = 1 - G\left(\delta b_c - (1-\delta)\,\bar{b}\right) < 1$.

Combining (i) and (ii), we conclude that as $\tau \to 1$, no conservative board gains τ-support from shareholders if they expect the conservative voting equilibrium.

Next, suppose shareholders expect the voting equilibrium to be activist. Consider as an alternative an activist board with bias $b_m = -q_m > -H^{-1}(\phi)$. Shareholder b prefers delegation to such a board over the activist voting equilibrium if and only if $V_a\left(b, q_a\right) < V_a\left(b, q_m\right)$. We consider several cases:

1. If $b \leq b_a$, then

$$\begin{aligned} V_a(b,q_a) &< V_a(b,q_m) \Leftrightarrow v(b_a,q_a) < v(b_a,q_m) \Leftrightarrow \\ b_a(H(q_a)-H(q_m)) &< H(q_m)\mathbb{E}[\theta|q>q_m] - H(q_a)\mathbb{E}[\theta|q>q_a]. \end{aligned}$$

- If in addition $q_a > q_m$, then $H(q_a) - H(q_m) < 0$, so

$$V_a(b,q_a) < V_a(b,q_m) \Leftrightarrow b_a > \mathbb{E}[-q| - q_a < -q < -q_m],$$

which never holds given that $-q_a > b_a$. Thus, shareholders $b \leq b_a$ never support delegation to a board who is more extreme than the marginal voter, i.e., $q_m < q_a \Leftrightarrow b_m > -q_a$.

- If instead $q_a < q_m$, then $H(q_a) - H(q_m) < 0$, so

$$V_a(b,q_a) < V_a(b,q_m) \Leftrightarrow b_a < \mathbb{E}[-q| - q_m < -q < -q_a].$$

Since $b_a < -q_a$, this always holds if $b_a \leq -q_m$ and might even hold if $b_a > -q_m$. Thus, shareholders with $b \leq b_a$ support delegation to a board whenever $-q_m \in [b_a, -q_a)$, and might even do so if $-q_m < b_a$.

2. If $b > b_a$, then (2) and (32) imply

$$\begin{gathered} V_a(b,q_a) < V_a(b,q_m) \Leftrightarrow v(b,q_a) - \delta v(b_a,q_a) < v(b,q_m) - \delta v(b_a,q_m) \Leftrightarrow \\ v(b,q_a) - v(b,q_m) < \delta[v(b_a,q_a) - v(b_a,q_m)] \Leftrightarrow \\ b(H(q_a)-H(q_m)) + H(q_a)\mathbb{E}[\theta|q>q_a] - H(q_m)\mathbb{E}[\theta|q>q_m] \\ < \delta[b_a(H(q_a)-H(q_m)) + H(q_a)\mathbb{E}[\theta|q>q_a] - H(q_m)\mathbb{E}[\theta|q>q_m]\ ]. \end{gathered}$$

- If in addition $q_a > q_m$, then $H(q_a) < H(q_m)$, so

$$V_a(b,q_a) < V_a(b,q_m) \Leftrightarrow b > \delta b_a + (1-\delta)\mathbb{E}[-q| - q_a < -q < -q_m],$$

and notice that since $-q_a > b_a$, then $\delta b_a + (1-\delta)\mathbb{E}[-q| - q_a < -q < -q_m] > b_a$.

- If instead $q_a < q_m$, then $H(q_a) > H(q_m)$, so

$$V_a(b, q_a) < V_a(b, q_m) \Leftrightarrow b < \delta b_a + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_a].$$

The overall support for delegation to the board is the combined support of shareholders with $b \le b_a$ and $b > b_a$. Then:
(i) First, consider a board with $-q_m > -q_a \Leftrightarrow q_m < q_a$. Then only shareholders with $b > \delta b_a + (1-\delta)\mathbb{E}[-q| - q_a < -q < -q_m] > b_a$ support delegation to the board. It follows that if $1 - G(b_a) < \tau \Leftrightarrow 1 - \delta < \tau$, then this type of board does not obtain τ-support.
(ii) Second, consider a board with $-q_m < -q_a \Leftrightarrow q_m > q_a$. Such a board obtains support from $b \le b_a$ if $b_a < \mathbb{E}[-q| - q_m < -q < -q_a]$ and from $b > b_a$ that satisfy $b < \delta b_a + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_a]$. There are two cases:

- If $b_a > \mathbb{E}[-q| - q_m < -q < -q_a]$, then $\delta b_a + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_a] < b_a$. Thus, in this case, there is no support for delegation from either shareholders with $b \le b_a$ or from those with $b > b_a$.

- If $b_a < \mathbb{E}[-q| - q_m < -q < -q_a]$, then $\delta b_a + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_a] > b_a$. Thus, both shareholders with $b \le b_a$ and with $b \in (b_a, \delta b_a + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_a])$ support delegation. So overall, delegation receives support from shareholders with $b < \delta b_a + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_a]$. Notice that $\mathbb{E}[-q| - q_m < -q < -q_a] < -q_a$, and hence the fraction of initial shareholders supporting delegation is

$$G(\delta b_a + (1-\delta)\mathbb{E}[-q| - q_m < -q < -q_a]) < G(\delta b_a - (1-\delta) q_a).$$

  Note that $\lim_{\tau\to 1} q_a = -b_a$. Thus, $\lim_{\tau\to 1} G(\delta b_a - (1-\delta) q_a) = G(b_a) < 1$.

Combining (i) and (ii), we conclude that as $\tau \to 1$, no activist board gains τ-support from shareholders if they expect the activist voting equilibrium. ■

Online Appendix for "Trading and Shareholder Voting"

by Doron Levit[25], Nadya Malenko[26], and Ernst Maug[27]

A Extensions of the baseline model

A.1 Social concerns

In this section, we extend the model to study situations in which investors care about the decision on the proposal even if they do not own shares of the company. This may be the case if the firm's decision on the proposal has a social or environmental impact that would affect the investor's utility beyond his share ownership in the firm, e.g., a gun-control issue, pollution, etc. Specifically, we assume that if a shareholder owns $\alpha > 0$ shares in the firm, his utility is given by

$$\begin{aligned} u(d, \theta, b, \alpha) &= \alpha u(d, \theta, b) + \gamma bd & (33) \\ &= \alpha [v_0 + (\theta + b)(d - \phi)] + \gamma bd, & (34) \end{aligned}$$

where $\gamma \geq 0$ captures the sensitivity of the investor's utility to the proposal beyond his ownership in the firm. The baseline model assumes $\gamma = 0$.

Consider the trading stage, and suppose investors expect the proposal to be accepted if and only if $q > q^*$. Given price p, the shareholder chooses the amount of shares to trade t to solve

$$\max_{t \in [-e, x]} \{(e + t) v(b, q^*) + \gamma b H(q^*) - tp\},$$

where $v(b, q^*)$ is given by (7). It follows that the shareholder chooses to buy as many shares as he can if $v(b, q^*) > p$, and sell as many shares as he can if $v(b, q^*) < p$. Recall that $v(b, q^*)$ increases b if and only if $H(q^*) > \phi$. Thus, as in the baseline model, if $H(q^*) > \phi$, then the equilibrium is activist, the shareholder buys x shares if $b > b_a$ and sells e shares if $b < b_a$, where $b_a = G^{-1}(\delta)$. The shareholder with bias b_a is the marginal trader, who is indifferent between

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buying and selling shares, and the share price is $p_a = v(b_a, q^*)$. Similarly, if $H(q^*) < \phi$, then the equilibrium is conservative. In both cases, since investors are not pivotal for the outcome of the proposal, the trading stage is not directly affected by the proposal's impact on their social attitude.

Consider the voting stage. The buying investors account for the effect of the proposal on their utility beyond its direct impact on the share value. Thus, their as-if-pivotal behavior implies that they vote for the proposal if and only if

$$\begin{aligned} \mathbb{E}\left[u(1,\theta,b)\,|\text{public signal}\right] &> \mathbb{E}\left[u(0,\theta,b)\,|\text{public signal}\right] \Leftrightarrow \\ q + b\left(1+\frac{\gamma}{e+x}\right) &> 0 \Leftrightarrow q + b\left(1+(\gamma/e)(1-\delta)\right) > 0. \end{aligned}$$

Thus, the proposal is accepted if and only if $q > -q_a(\gamma)$, where

$$q_a(\gamma) \equiv -(1+(\gamma/e)(1-\delta))G^{-1}(1-\tau(1-\delta)). \tag{35}$$

The identity of the marginal voter is the same as in the baseline model and is given by $-q_a(0)$. Recall that $-q_a(0) > b_a$, and notice that $-q_a(\gamma)$ increases in γ if and only if $-q_a(0) > 0$. This implies that $\gamma > 0$ amplifies the incentives of the marginal voter: If $-q_a(0) > 0$, the marginal voter becomes more activist as γ increases, and if $-q_a(0) < 0$, he becomes more conservative, less extreme, and possibly even less activist than the marginal trader if γ is sufficiently large.

Overall, and similar to the baseline model, the equilibrium is the following.

Proposition 9. *An equilibrium of the game with trading and voting always exists.*

(*i*) *An* ***activist*** *equilibrium exists if and only if* $H(q_a(\gamma)) > \phi$, *where*

$$q_a(\gamma) \equiv -(1+(\gamma/e)(1-\delta))G^{-1}(1-\tau(1-\delta)). \tag{36}$$

In this equilibrium, a shareholder with bias b *buys* x *shares if* $b > b_a$ *and sells his entire endowment* e *if* $b < b_a$, *where* $b_a \equiv G^{-1}(\delta)$. *The proposal is accepted if and only if* $q > q_a(\gamma)$, *and the share price is given by* $p_a = v(b_a, q_a(\gamma))$.

(*ii*) *A* ***conservative*** *equilibrium exists if and only if* $H(q_c(\gamma)) < \phi$, *where*

$$q_c(\gamma) \equiv -(1+(\gamma/e)(1-\delta))G^{-1}((1-\delta)(1-\tau)). \tag{37}$$

In this equilibrium, a shareholder with bias b buys x shares if $b < b_c$ and sells his entire endowment e if $b > b_c$, where $b_c = G^{-1}(1-\delta)$. The proposal is accepted if and only if $q > q_c(\gamma)$, and the share price is given by $p_c = v(b_c, q_c(\gamma))$.

(iii) *Other equilibria do not exist.*

Since in the activist equilibrium $-q_a(0) > b_a$, larger γ increases the share price in the neighborhood of $\gamma = 0$ if and only if $-q_a(0) < 0$. Indeed, only in those circumstances the distance between $-q_a(\gamma)$ and b_a shrinks. Thus, while the trading phase is not directly affected by γ, it is affected indirectly through it effect on the likelihood of the proposal being accepted by the post-trade shareholder base. Intuitively, although atomistic shareholders do not internalize the effect of their own individual trading decisions on the firm's decision-making, they do take into account how social concerns will affect the marginal voter's behavior, and price it accordingly when trading.

Next, we calculate the expected welfare of the initial shareholder base in the activist equilibrium.

$$\begin{aligned}
W_a &= \Pr[b < b_a]\,\mathbb{E}[ep_a + \gamma bH(q_a(\gamma))\,|b < b_a] \\
&\quad + \Pr[b > b_a]\,\mathbb{E}[(e+x)\,v(b, q_a(\gamma)) - xp_a + \gamma bH(q_a(\gamma))\,|b > b_a] \\
&= \Pr[b < b_a]\,ep_a + \Pr[b > b_a]\,\mathbb{E}[(e+x)\,v(b, q_a(\gamma)) - xp_a|b > b_a] + \gamma\mathbb{E}[b]\,H(q_a(\gamma)) \\
&= e \cdot v(\beta_a, q_a(\gamma)) + \gamma\mathbb{E}[b]\,H(q_a(\gamma)) \\
&= e \cdot (v_0 + \beta_a(H(q_a(\gamma)) - \phi) + H(q_a(\gamma))\,\mathbb{E}[\theta|q > q_a(\gamma)]) + \gamma\mathbb{E}[b]\,H(q_a(\gamma)) \\
&= e \cdot (v_0 + (\beta_a + (\gamma/e)\,\mathbb{E}[b])\,(H(q_a(\gamma)) - \phi) + H(q_a(\gamma))\,\mathbb{E}[\theta|q > q_a(\gamma)]) + \phi\gamma\mathbb{E}[b] \\
&= e \cdot v(\beta_a + (\gamma/e)\,\mathbb{E}[b]\,, q_a(\gamma)) + \phi\gamma\mathbb{E}[b]
\end{aligned}$$

Similarly, one can show that the expected welfare of the initial shareholder base in the conservative equilibrium is

$$W_c = e \cdot v(\beta_c + (\gamma/e)\,\mathbb{E}[b]\,, q_c(\gamma)) + \phi\gamma\mathbb{E}[b]\,.$$

In both cases, we observe that as γ increases, the shareholder welfare function puts more weight on the unconditional expected bias of the shareholder base $\mathbb{E}[b]$.

For simplicity, we focus on the activist equilibrium. We make several observations. First, if

$$\beta_a + (\gamma/e)\,\mathbb{E}[b] < -q_a(0)\,(1 + (\gamma/e)(1-\delta)) < b_a$$

or

$$\beta_a + (\gamma/e)\,\mathbb{E}[b] > -q_a(0)\,(1 + (\gamma/e)(1-\delta)) > b_a,$$

then we have opposing effects on prices and welfare (the analog of Proposition 5).

Second, notice that in general, $\beta_a + (\gamma/e)\,\mathbb{E}[b] \neq \mathbb{E}[b]$ and $\beta_c + (\gamma/e)\,\mathbb{E}[b] \neq \mathbb{E}[b]$. Thus, the optimal board is biased in this context as well, for the same reasons as in the baseline model. (This is true regardless of whether or not the board's preferences are represented by $v(d, \theta, b_m)$ or $u(d, \theta, b_m)$.)

Last, we consider the effect of liquidity. Notice that δ has the same effect on β_a, b_a, and $-q_a(0)$ as in the baseline model: all of them increase, i.e., become more extreme. However, a larger δ can attenuate the incentives of the marginal voter. Indeed, the term $(1 + (\gamma/e)(1-\delta))$ decreases in δ. Hence, if $-q_a(0) < 0$, then the marginal voter becomes more extreme as δ increases. However, if $-q_a(0) > 0$, then the overall effect of δ is ambiguous. Intuitively, larger liquidity implies that the marginal voter has a larger stake in the firm in equilibrium, and hence, puts relatively more weight on the proposal's impact on the share value. This implies that an activist marginal voter ($-q_a(0) > 0$) will behave more conservatively, and a conservative marginal voter ($-q_a(0) < 0$) will become more activist in relative terms.

A.2 Heterogeneous endowments and trading frictions

In this section, we extend the model to environments in which shareholders differ not only with respect to their attitude toward the proposal, but also with respect to their endowments and ability to trade. Specifically, we assume that a shareholder with bias b has endowment of $e(b) > 0$ and is able to buy not more than $x(b) > 0$ shares. We denote by $\mathbf{e} \equiv \int_{-\bar{b}}^{\bar{b}} e(b)\, dG(b)$ the total endowment. Importantly, we do not impose any restrictions on the functions $e(\cdot)$ and $x(\cdot)$, and as such, we allow for various correlations between the shareholder's attitude toward the proposal, endowment, and ability to trade. In particular, our framework nests cases in which $e(\cdot)$ and $x(\cdot)$ increase in $|b|$, that is, more extreme shareholders have larger endowments and more ability to trade. We demonstrate that our main results continue to hold even when such arbitrary correlations hold.

To begin, note that the functional form of $v(b, q^*)$ does not change and is given by expression

(7). Therefore, as in the baseline model, shareholder b buys $x(b)$ shares if $v(b,q^*) > p$ and sells $e(b)$ shares if $v(b,q^*) < p$.

If $H(q^*) > \phi$, then $v(b,q^*)$ increases in b, there exists b_a such that $v(b,q^*) > p \Leftrightarrow b > b_a$, and the equilibrium is activist. The total demand for shares is $D(p) = \int_{b_a}^{\bar{b}} x(b)\,dG(b)$, and the total supply of shares is $S(p) = \int_{-\bar{b}}^{b_a} e(b)\,dG(b)$. The market clears if and only if

$$\int_{b_a}^{\bar{b}} x(b)\,dG(b) = \int_{-\bar{b}}^{b_a} e(b)\,dG(b). \tag{38}$$

It is straightforward to verify that there exists a unique $b_a \in \left(-\bar{b}, \bar{b}\right)$ that solves (38). Similarly, if $H(q^*) < \phi$, then $v(b,q^*)$ decreases in b, there exists b_c such that $v(b,q^*) > p \Leftrightarrow b < b_c$, and the equilibrium is conservative. The total demand for shares is $D(p) = \int_{-\bar{b}}^{b_c} x(b)\,dG(b)$, and the total supply of shares is $S(p) = \int_{b_c}^{\bar{b}} e(b)\,dG(b)$. The market clears if and only if

$$\int_{-\bar{b}}^{b_c} x(b)\,dG(b) = \int_{b_c}^{\bar{b}} e(b)\,dG(b). \tag{39}$$

Again, it is straightforward to verify that there exists a unique $b_c \in \left(-\bar{b}, \bar{b}\right)$ that solves (39).

Notice that if an exogenous shock (weakly) increases $x(b)$ for all b but leaves $e(b)$ unchanged, then the demand for shares increases, the supply does not change, and thus the market can clear only if b_a increases in the activist equilibrium and b_c decreases in the conservative equilibrium. In other words, when trading frictions are relaxed, the marginal trader become more extreme in equilibrium, as in the baseline model.

Next, we analyze the identity of the marginal voter. Notice that Lemma 1 continues to holds in this setup since, as before, each shareholder is more likely to vote for the proposal when q is larger. First, consider the activist equilibrium. Post trade, the shareholder base consists of shareholders with a bias larger than b_a. For any given $b > b_a$, shareholders with bias b collectively hold a fraction

$$g_a(b) \equiv g(b)\,\frac{x(b)+e(b)}{\mathbf{e}} \tag{40}$$

of all shares. Notice that

$$\begin{aligned}\int_{b_a}^{\bar{b}} g_a(b)\,db &= \frac{1}{\mathbf{e}}\int_{b_a}^{\bar{b}} x(b)\,g(b)\,db + \frac{1}{\mathbf{e}}\int_{b_a}^{\bar{b}} e(b)\,g(b)\,db \\ &= \frac{1}{\mathbf{e}}\int_{-\bar{b}}^{b_a} e(b)\,g(b)\,db + \frac{1}{\mathbf{e}}\int_{b_a}^{\bar{b}} e(b)\,g(b)\,db \\ &= \frac{1}{\mathbf{e}}\int_{-\bar{b}}^{\bar{b}} e(b)\,g(b)\,db = \frac{1}{\mathbf{e}}\mathbf{e} = 1,\end{aligned}$$

where the second equality follows from market clearing. Thus, we can view $g_a(b)$ as a density function with full support on $\left[b_a, \bar{b}\right]$. The corresponding cdf is given by

$$G_a(b) \equiv \int_{b_a}^{b} g_a(b)\,db. \tag{41}$$

Thus, the marginal voter is given by $-q_a \equiv G_a^{-1}(1-\tau)$. Notice that $G_a^{-1}(1-\tau) > b_a$, i.e., the marginal voter is more extreme than the marginal trader, as in the baseline model.

Similarly, in conservative equilibrium, the post-trade shareholder base consists of shareholders with a bias smaller than b_c. For any given $b < b_c$, shareholders with bias b collectively hold a fraction

$$g_c(b) \equiv g(b)\,\frac{x(b)+e(b)}{\mathbf{e}} \tag{42}$$

of all shares. Similar to the activist equilibrium, it can be shown that $\int_{-\bar{b}}^{b_c} g_c(b)\,db = 1$. Thus, we can view $g_c(b)$ as a density function with full support on $\left[-\bar{b}, b_c\right]$, and the corresponding cdf is given by

$$G_c(b) \equiv \int_{-\bar{b}}^{b} g_c(b)\,db. \tag{43}$$

Thus, the marginal voter is given by $-q_c \equiv G_c^{-1}(1-\tau)$, and notice again that $G_c^{-1}(1-\tau) < b_c$, that is, the marginal voter is more extreme than the marginal trader.

Given all of the above, the analysis of Section 4.3 is easily extended to a setup with heterogeneous endowment and trading frictions. In particular, the following result holds.

Proposition 10. *An equilibrium of the game with trading and voting always exists. Let b_a and b_c be the unique solutions of (38) and (39), respectively, and $G_a(b)$ and $G_c(b)$ be the distribution functions as defined by (41) and (43), respectively. Then:*

*(i) An **activist** equilibrium exists if and only if* $H(q_a) > \phi$, *where*

$$q_a \equiv -G_a^{-1}(1-\tau). \tag{44}$$

In this equilibrium, a shareholder with bias b buys $x(b)$ shares if $b > b_a$ and sells his entire endowment $e(b)$ if $b < b_a$. The proposal is accepted if and only if $q > q_a$, and the share price is given by $p_a = v(b_a, q_a)$.

*(ii) A **conservative** equilibrium exists if and only if* $H(q_c) < \phi$, *where*

$$q_c \equiv -G_c^{-1}(1-\tau). \tag{45}$$

In this equilibrium, a shareholder with bias b buys $x(b)$ shares if $b < b_c$ and sells his entire endowment $e(b)$ if $b > b_c$. The proposal is accepted if and only if $q > q_c$, and the share price is given by $p_c = v(b_c, q_c)$.

(iii) Other equilibria do not exist.

Next, as in the baseline model, we define $\beta_a = \int_{b_a}^{\bar{b}} b g_a(b)\, db$ and $\beta_c = \int_{-\bar{b}}^{b_c} b g_a(b)\, db$ as the average bias of the post-trade shareholder base. Notice that $\beta_a > b_a$ and $\beta_c < b_c$. The expected welfare of the initial shareholder base is

$$\begin{aligned}
W_a &= p_a \int_{-\bar{b}}^{b_a} e(b)\, g(b)\, db + \int_{b_a}^{\bar{b}} \left[(e(b)+x(b))\, v(b,q_a) - x(b)\, p_a \right] g(b)\, db \\
&= p_a \left[\int_{-\bar{b}}^{b_a} e(b)\, g(b)\, db - \int_{b_a}^{\bar{b}} x(b)\, g(b)\, db \right] + \int_{b_a}^{\bar{b}} (e(b)+x(b))\, v(b,q_a)\, g(b)\, db \\
&= p_a \cdot 0 + \int_{b_a}^{\bar{b}} (e(b)+x(b))\, v(b,q_a)\, g(b)\, db = \mathbf{e} \int_{b_a}^{\bar{b}} v(b,q_a)\, g_a(b)\, db \\
&= \mathbf{e} \cdot v\left(\int_{b_a}^{\bar{b}} b g_a(b)\, db, q_a \right) = \mathbf{e} \cdot v(\beta_a, q_a),
\end{aligned}$$

which is the valuation of the average post-trade shareholder. Indeed, the third equality follows from the market-clearing condition, the fourth equality follows from the definition of $g_a(b) = \frac{e(b)+x(b)}{\mathbf{e}} g(b)$, the fifth equality follows from the linearity of $v(b, q^*)$ in b, and the last equality follows from the definition of β_a. Similar analysis shows that in the conservative equilibrium, $W_c = \mathbf{e} \cdot v(\beta_c, q_c)$.

Since in the activist equilibrium $p_a = v(b_a, q_a)$ and $W_a = \mathbf{e} \cdot v(\beta_a, q_a)$, and in the conservative equilibrium $p_c = v(b_c, q_c)$ and $W_c = \mathbf{e} \cdot v(\beta_c, q_c)$, the analysis of Sections 5 and 6 is extended to the setup with heterogeneous endowment and trading frictions, with the exception of the comparative statics with respect to δ. The analogous analysis about the effect of δ can be obtained if instead one considers an exogenous shock that (weakly) increases $x(b)$ for all b but leaves $e(b)$ unchanged. We omit this analysis for brevity.

A.3 Trading after voting

In this section, we extend the baseline model by allowing for another round of trade after shareholders vote. The purpose of this extension is to explicitly analyze the effect of the voting outcome on the share price and shareholders' welfare, and to demonstrate that prices and welfare can move in opposite directions. In addition, the analysis shows the robustness of our baseline model to a dynamic trading environment.

Suppose that with probability $\varrho \in (0,1)$ there is another round of trade after the voting outcome is determined, but before fundamentals θ are realized. The post-vote trading stage features the same frictions as pre-vote trading, namely, investors can buy at most x shares and cannot sell more than the shares they own at that point (which would be at most $e+x$ if they bought shares at the pre-vote trading stage). With probability $1-\varrho$ trade is not feasible at the post-vote stage, and the game ends. We will focus on the case $\varrho \to 1$. (We introduce $\varrho < 1$ to break a shareholder's indifference in the pre-vote trading stage when such indifference occurs due to the introduction of a second round of trade.)

We denote the price at the second, post-vote, round of trade by $p_2(d,q)$, and the price at the first, pre-vote, round of trade by p_1. Note that the post-vote price depends on the voting outcome and the realization of q.

As in the baseline model, we focus on equilibria in which the proposal is accepted if and only if $q > q^*$. Given q^*, the expected shareholder value at the pre-vote stage is

$$\begin{aligned} \pi(b, q^*) &= \varrho \mathbb{E}\left[\max\left\{p_2\left(\mathbf{1}_{q>q^*}, q\right), v\left(\mathbf{1}_{q>q^*}, q, b\right)\right\}\right] + (1-\varrho)\,\mathbb{E}\left[v\left(\mathbf{1}_{q>q^*}, q, b\right)\right] \\ &= \varrho \left(\begin{array}{c} H(q^*)\,\mathbb{E}\left[\max\left\{p_2(1,q), v(1,q,b)\right\} | q > q^*\right] \\ +\left(1-H(q^*)\right)\mathbb{E}\left[\max\left\{p_2(0,q), v(0,q,b)\right\} | q < q^*\right] \end{array} \right) + (1-\varrho)\, v(b, q^*), \end{aligned}$$

where $v(d,q,b)$ is given by (1) and $v(b,q^*)$ is given by (7).

Consider the post-vote round of trade, which occurs after the public signal q and the voting outcome d are revealed. Note that

$$v(1,q,b) \geq p_2(1,q) \Leftrightarrow b > b_1(q) \equiv \frac{p_2(1,q) - v_0}{1-\phi} - q$$

and

$$v(0,q,b) \geq p_2(0,q) \Leftrightarrow b < b_0(q) \equiv \frac{p_2(0,q) - v_0}{-\phi} - q.$$

In words, buyers upon proposal approval are investors with a large b, and buyers upon proposal rejection are investors with a small b. Moreover, conditional on q and d, prices are determined by the valuation of the marginal trader: $p_2(1,q) = v(1,q,b_1(q))$ and $p_2(0,q) = v(0,q,b_0(q))$.

Note that b_1 and b_0 do not depend on q. Indeed, suppose $q > q^*$, and let $e(b) \geq 0$ be the number of shares shareholder b owns at the beginning of the second round of trade. Then, the demand is $x(1 - G(b_1(q)))$ and the supply is $\int_{-\bar{b}}^{b_1(q)} e(b)\, g(b)\, db$. Notice that the demand and supply depend on q only through $b_1(q)$, and thus, market clearing which uniquely pins down $b_1(q)$, implies that $b_1(q)$ does not depend on q. Similarly, $b_0(q)$ does not depend on q.

Given the observations above, we have

$$\begin{aligned}
\pi(b,q^*) &= \varrho \begin{pmatrix} H(q^*)\,\mathbb{E}\left[\max\{v(1,q,b_1), v(1,q,b)\} | q > q^*\right] \\ +(1-H(q^*))\,\mathbb{E}\left[\max\{v(0,q,b_0), v(0,q,b)\} | q < q^*\right] \end{pmatrix} + (1-\varrho)\, v(b,q^*) \\
&= \varrho \left(v_0 + \begin{matrix} (1-\phi)\, H(q^*)\,\mathbb{E}\left[\max\{q+b_1, q+b\} | q > q^*\right] \\ -\phi(1-H(q^*))\,\mathbb{E}\left[\min\{q+b_0, q+b\} | q < q^*\right] \end{matrix} \right) + (1-\varrho)\, v(b,q^*) \\
&= v_0 + H(q^*)\,\mathbb{E}\left[q | q > q^*\right] + \varrho \begin{pmatrix} (1-\phi)\, H(q^*) \max\{b_1, b\} \\ -\phi(1-H(q^*)) \min\{b_0, b\} \end{pmatrix} + (1-\varrho)\, b\,(H(q^*) - \phi)\,.
\end{aligned}$$

Recall that we focus on the case $\varrho \to 1$. Therefore, (1) if $b < \min\{b_0, b_1\}$ or $\phi \geq 1$, then $\pi(b,q^*)$ decreases in b; (2) if $b > \max\{b_0, b_1\}$ or $\phi \leq 0$, then $\pi(b,q^*)$ increases in b; and (3) if $\min\{b_0, b_1\} < b < \max\{b_0, b_1\}$ and $\phi \in (0,1)$, then $\pi(b,q^*)$ increases in b if and only if $H(q^*) - \phi > 0$. We conclude that in equilibrium of the first round of trade, there exist b_L^* and b_H^* , $-\bar{b} \leq b_L^* < b_H^* \leq \bar{b}$, such that the investor buys shares if and only if $b \leq b_L^*$ or $b \geq b_H^*$. Thus, market clearing in the first round of trade implies that

$$[G(b_L^*) + 1 - G(b_H^*)]\, x = [G(b_H^*) - G(b_L^*)]\, e \Leftrightarrow G(b_H^*) - G(b_L^*) = \delta, \tag{46}$$

where

$$p_1 = \pi\left(b_H^*, q^*\right) = \pi\left(b_L^*, q^*\right). \tag{47}$$

Since $\pi\left(\cdot, q^*\right)$ has a unique minimum and is convex, there are unique b_L^* and b_H^* , $-\bar{b} \leq b_L^* < b_H^* \leq \bar{b}$, that solve (46) and (47). Notice that b_L^* and b_H^* depend on b_0 and b_1, where either $b_L^* > -\bar{b}$ or $b_H^* < \bar{b}$, but not necessarily both.

As in the baseline model, if the cutoff q^* is taken as given, we have two separate cases to analyze:

1. Suppose $H\left(q^*\right) > \phi$. Then, the minimum of $\pi\left(b, q^*\right)$ is obtained at $\min\left\{b_0, b_1\right\}$ and thus, $b_L^* < \min\left\{b_0, b_1\right\} < b_H^*$. Thus, if $q > q^*$, the demand is $\left(1 - G\left(b_1\right)\right)x$, the supply is $\left[G\left(b_L^*\right) + \max\{0, G\left(b_1\right) - G\left(b_H^*\right)\}\right]\left(e + x\right)$, and the market clears whenever

$$\left(1 - G\left(b_1\right)\right)\delta = G\left(b_L^*\right) + \max\{0, G\left(b_1\right) - G\left(b_H^*\right)\}. \tag{48}$$

   If $q < q^*$, the demand is $G\left(b_0\right)x$, the supply is $\min\{1 - G\left(b_0\right), 1 - G\left(b_H^*\right)\}\left(e + x\right)$, and the market clears whenever

$$G\left(b_0\right)\delta = \min\{1 - G\left(b_0\right), 1 - G\left(b_H^*\right)\}. \tag{49}$$

   Combined with (46) and (47), equations (48) and (49) allow us to pin down b_0 and b_1 in the case where $H\left(q^*\right) > \phi$.

2. Suppose $H\left(q^*\right) < \phi$. Then, the minimum of $\pi\left(b, q^*\right)$ is obtained at $\max\left\{b_0, b_1\right\}$ and thus, $b_L^* < \max\left\{b_0, b_1\right\} < b_H^*$. Thus, if $q > q^*$, the demand is $\left(1 - G\left(b_1\right)\right)x$, the supply is $\min\{G\left(b_1\right), G\left(b_L^*\right)\}\left(e + x\right)$, and the market clears whenever

$$\left(1 - G\left(b_1\right)\right)\delta = \min\{G\left(b_1\right), G\left(b_L^*\right)\}. \tag{50}$$

   If $q < q^*$, the demand is $G\left(b_0\right)x$, the supply is $\left[1 - G\left(b_H^*\right) + \max\{0, G\left(b_L^*\right) - G\left(b_0\right)\}\right]\left(e + x\right)$, and the market clears whenever

$$G\left(b_0\right)\delta = 1 - G\left(b_H^*\right) + \max\{0, G\left(b_L^*\right) - G\left(b_0\right)\}. \tag{51}$$

   Combined with (46) and (47), equations (50) and (51) allow us to pin down b_0 and b_1 in the case where $H(q^*) < \phi$.

Essentially, unlike the baseline model, it is now possible that both very conservative and very activist investors buy shares at the first-round of stage, while moderate investors sell. Then, at the post-vote trading stage, the activist shareholders buy shares from (sell shares to) the conservative shareholders if the proposal is accepted (rejected). Intuitively, even if the proposal is likely to be accepted, the option to trade after the vote gives incentives to conservative shareholders to buy shares with anticipation that they can sell it to activists if the expected outcome materializes; however, they can also enjoy the upside of owning many shares in the firm if the proposal is unexpectedly rejected.

The analysis so far has taken q^* as given. Below, we endogenize the voting stage and analyze the extended game with post-vote trading for the case $\phi = 0$. We fully characterize the equilibrium and illustrate the opposing welfare and price reactions to the voting outcome.

A.3.1 Complete analysis for $\phi = 0$

If $\phi = 0$, then $v(d, \theta, b) = v_0 + (\theta + b) d$ and

$$\begin{aligned}
\pi(b, q^*) &= v_0 + H(q^*) E[q|q > q^*] + [\varrho \max\{b_1, b\} + (1-\varrho) b] H(q^*) \\
&= v(\varrho \max\{b_1, b\} + (1-\varrho) b, q^*)
\end{aligned}$$

is strictly increasing in b. Therefore, $b_L^* = -\bar{b}$, and shareholders buy shares at the pre-vote trading stage if and only if $b > b_H^* = b_a = G^{-1}(\delta)$, just as in the baseline model. Suppose a second round of trade after the voting stage occurs. If $q < q^*$, then the proposal is rejected, the share value to all shareholders is v_0, and no trade occurs. Suppose $q > q^*$. Equation (48) implies that b_1 solves

$$(1 - G(b_1))\delta = \max\{0, G(b_1) - \delta\}.$$

If $b_1 \leq G^{-1}(\delta)$, then the supply is zero, and hence, the market cannot clear. Thus, it must be that $b_1 > G^{-1}(\delta)$, and in particular,

$$b_1 = G^{-1}\left(\frac{2\delta}{1+\delta}\right),$$

which is indeed larger than $G^{-1}(\delta)$. In equilibrium: (1) shareholders with $b < b_a = G^{-1}(\delta)$ sell; (2) shareholders with $b \in (b_a, b_1) = (G^{-1}(\delta), G^{-1}(\frac{2\delta}{1+\delta}))$ buy at the first stage and sell at the second stage if the proposal is approved and there is trade; and (3) shareholders with $b > b_1 = G^{-1}(\frac{2\delta}{1+\delta})$ buy at the first stage and buy more shares at the second stage if the proposal is approved and there is trade. The share price in the first round of trade (assuming $\varrho \to 1$) is the valuation of the marginal trader b_a, which is given by $\pi(b_a, q^*)$. Therefore,

$$p_1 = \pi(b_a, q^*) = v(\max\{b_1, b_a\}, q^*) = v(b_1, q^*) = v_0 + H(q^*)(b_1 + E[q|q > q^*]).$$

In the second round of trade, the share price upon rejection of the proposal is always v_0. The share price upon approval of the proposal and realization of q is the valuation of the marginal trader b_1, and hence, $p_2(1, q) = v(1, q, b_1)$. Thus, $p_2(d, q) = v(d, q, b_1)$.

Last, we consider the identity of the marginal voter. At the voting stage, the shareholders of the firm are those with $b > b_a$, and each owns $x + e$ shares. A shareholder with $b \in [b_a, b_1]$ expects to get v_0 if the proposal is rejected and $v_0 + q + b_1$ if the proposal is accepted. Thus, this shareholder votes for the proposal if and only if $q > -b_1$. A shareholder with $b > b_1$ expects to get $(e + x) v_0$ if the proposal is rejected and $(e + x)(v_0 + b + q) + x(b - b_1)$ if the proposal is accepted. Thus, this shareholder votes for the proposal if and only if $q > -[b(1 + \delta) - \delta b_1]$. Combined, shareholders vote for the proposal if and only if

$$q > -[\max\{b, b_1\}(1 + \delta) - \delta b_1]. \tag{52}$$

Notice that $b(1 + \delta) - \delta b_1 > b \Leftrightarrow b > b_1$, and thus the introduction of the post-vote trading stage implies that shareholders vote as if their bias is higher (whether or not they intend to sell or buy their shares). Intuitively, shareholders who expect to sell have stronger incentives to approve the proposal because they internalize the positive effect of approving the proposal on the valuation of the marginal trader at the post-vote stage, b_1, and therefore, on the price they expect to receive for their shares. The buying shareholders also have stronger incentives to approve the proposal since they benefit when the proposal is approved.

As in the baseline model, the marginal trader is still given by the shareholder with bias $G^{-1}(1 - (1 - \delta)\tau)$. However, the proposal is approved if and only if

$$q > q_a = -\left[\max\left\{G^{-1}(1 - (1 - \delta)\tau), b_1\right\}(1 + \delta) - \delta b_1\right].$$

Notice that $G^{-1}(1 - (1 - \delta)\tau) < b_1 \Leftrightarrow \tau \geq \frac{1}{1+\delta}$. Thus, if $\tau \geq \frac{1}{1+\delta}$, then the marginal voter is among shareholders with $b \in [b_a, b_1]$, and the proposal is approved if and only if $q > -b_1$. If $\tau < \frac{1}{1+\delta}$, then the marginal voter is among shareholders with $b > b_1$, and the proposal is approved if and only if $q > -[G^{-1}(1-(1-\delta)\tau)(1+\delta) - \delta b_1]$. Either way, the proposal is approved only if $q > -b_1$.

The next result summarizes the observations above.

Proposition 11. *Consider the game with two rounds of trade, one before the vote and one after, and suppose $\phi = 0$. Define*

$$q_a \equiv -\left[\max\left\{G^{-1}(1-(1-\delta)\tau), b_1\right\}(1+\delta) - \delta b_1\right], \tag{53}$$

$$b_a \equiv G^{-1}(\delta), \text{ and } b_1 \equiv G^{-1}\left(\frac{2\delta}{1+\delta}\right). \tag{54}$$

Then, the unique equilibrium is activist and has the following properties:

(*i*) *In the first round of trade, a shareholder with bias b buys x shares if $b > b_a$ and sells his entire endowment e if $b < b_a$. The share price at the first round of trade is given by $p_1 = v(b_1, q_a)$.*

(*ii*) *At the voting stage, a shareholder votes for the proposal if and only if*

$$q > -[\max\{b, b_1\}(1+\delta) - \delta b_1], \tag{55}$$

and the proposal is accepted if and only if $q > q_a$.

(*iii*) *In the second round of trade, no trade occurs if the proposal is rejected. If the proposal is accepted, a shareholder with bias b buys x shares if $b > b_1$ and sells his entire holdings $e+x$ if $b \in [b_a, b_1]$. The share price at the second round of trade conditional on realization q and decision d is given by $p_2(d, q) = v(d, q, b_1)$.*

Price and welfare reactions to voting outcomes. Given the result above, we can analyze the average price reaction to the voting outcome, defined as

$$\Delta P(d) \equiv \mathbb{E}[p_2(d, q) - p_1 | d].$$

Since prices are martingales, $\Pr[d=1]\,\Delta P(d) + \Pr[d=0]\,\Delta P(0) = 0$, and thus, $\Delta P(1) > 0 \Leftrightarrow \Delta P(0) < 0$. Also notice that

$$\begin{aligned}
\Delta P(1) &= \mathbb{E}\left[p_2(1,q) - p_1 | q > q_a\right] = \mathbb{E}\left[v(1,q,b_1) | q > q_a\right] - v(b_1, q_a) \\
&= \mathbb{E}\left[v_0 + q + b_1 | q > q_a\right] - v_0 - b_1 H(q_a) - H(q_a)\,\mathbb{E}\left[\theta | q > q_a\right] \\
&= (1 - H(q_a))\left(\mathbb{E}\left[\theta | q > q_a\right] + b_1\right).
\end{aligned}$$

Thus,

$$\Delta P(1) < 0 \Leftrightarrow -\mathbb{E}\left[\theta | q > q_a\right] > b_1.$$

Intuitively, the average price reaction depends on how the proposal approval, on average, affects the payoff of the marginal trader b_1 at the post-vote trading stage.

Next, we turn to calculate the reaction of shareholder welfare to the voting outcome (when $\varrho \to 1$). Recall that shareholders with $b_a < b < b_1$ buy x shares in the first round of trade, and then sell their entire stake for $p_2(1,q)$ after the vote if the proposal is approved, and get v_0 if it is rejected. Thus, their valuation is the same as that of a shareholder with bias b_1. Given this observation, the expected welfare of the initial shareholder base (prior to the first round of trade) is given by

$$\begin{aligned}
W &= \Pr\left[b < b_a\right] e p_1 + \Pr\left[b_a < b < b_1\right] \mathbb{E}\left[(e+x)\, v(b_1, q_a) - x p_1 | b_a < b < b_1\right] \\
&\quad + \Pr\left[b > b_1\right] \mathbb{E}\left[(e+x)\, v(b, q_a) - x p_1 + x\left(v(b, q_a) - v(b_1, q_a)\right) | b > b_1\right] \\
&= \Pr\left[b_a < b < b_1\right](e+x)\, v(b_1, q_a) \\
&\quad + \Pr\left[b > b_1\right] \mathbb{E}\left[(e+x)\, v(b, q_a) + x\left(v(b, q_a) - v(b_1, q_a)\right) | b > b_1\right] \\
&= \Pr\left[b > b_1\right](e + 2x)\, \mathbb{E}\left[v(b, q_a) | b > b_1\right] = e \cdot \mathbb{E}\left[v(b, q_a) | b > b_1\right] \\
&= e \cdot v\left(\mathbb{E}\left[b | b > b_1\right], q_a\right).
\end{aligned}$$

The second equality follows from market clearing at the pre-vote stage, the third equality follows from market clearing at the post-vote stage, the fourth equality follows from the definition of b_1, and the last equality follows from the linearity of $v(\cdot, q_a)$. Thus, as in the baseline model, the expected welfare is given by the valuation of the average post-trade shareholder, but now it accounts for both the pre-vote and the post-vote trading stages.

The expected average welfare of shareholders conditional on voting outcome d is given by

$e \cdot v_0$ if $d = 0$ and by

$$W(1) = e \cdot (v_0 + \mathbb{E}[b|b > b_1] + \mathbb{E}[\theta|q > q_a])$$

if $d = 1$. As with prices, we can define the average welfare reaction to the voting outcome as

$$\Delta W(d) \equiv \mathbb{E}[W(d) - W|d].$$

Notice that $\Delta W(1) > 0 \Leftrightarrow \Delta W(0) < 0$. Moreover,

$$\begin{aligned}
\Delta W(1) &= e \cdot (v_0 + \mathbb{E}[b|b > b_1] + \mathbb{E}[\theta|q > q_a]) - e \cdot v(\mathbb{E}[b|b > b_1], q_a) \\
&= e(1 - H(q_a)) \cdot (\mathbb{E}[b|b > b_1] + \mathbb{E}[\theta|q > q_a]),
\end{aligned}$$

and thus,

$$\Delta W(1) > 0 \Leftrightarrow \mathbb{E}[b|b > b_1] > -\mathbb{E}[\theta|q > q_a].$$

Since $\mathbb{E}[b|b > b_1] > b_1$, the next proposition follows.

Proposition 12. *The average welfare and price reactions to voting outcomes have opposite signs if and only if*

$$\mathbb{E}[b|b > b_1] > -\mathbb{E}[\theta|q > q_a] > b_1. \tag{56}$$

In those cases, the average welfare reaction to proposal approval (rejection) is positive (negative), while the average price reaction is negative (positive).

When is condition (56) satisfied? Recall that $q_a \leq -b_1$, where the inequality is strict if and only if $\tau < \frac{1}{1+\delta}$. Thus, if $\tau \geq \frac{1}{1+\delta}$, then $q_a = -b_1$, $-\mathbb{E}[\theta|q > q_a] = -\mathbb{E}[\theta|q > -b_1] < b_1$, and condition (56) does not hold. Intuitively, if the marginal voter behaves the same way as the marginal trader in post-vote trading, then the price reaction to proposal approval must be positive since the approval always benefits the marginal trader who sets the post-vote price. Since shareholder welfare is determined by the post-trade shareholder, who is even more activist than the marginal trader and hence more biased toward the proposal, the average welfare reaction is also positive. In this case, both price and welfare reactions to proposal approval are positive.

In contrast, if $\tau < \frac{1}{1+\delta}$, then $q_a < -b_1$ and the marginal voter is more activist than the marginal trader. Therefore, whenever $q \in [q_a, -b_1]$, the proposal is accepted although it reduces the value of the marginal trader. In those cases, the price reaction to proposal approval is negative. At the same time, $\mathbb{E}[b|b > b_1] > b_1$ implies that it is possible that the average post-trade shareholder is more activist than the marginal voter, i.e., $\mathbb{E}[b|b > b_1] > -q_a$. If in addition $q \in [q_a, -b_1]$, then approval of the proposal benefits the average post-trade shareholder, and as a result the realized welfare reaction is positive. If the weight on realizations of $q \in [q_a, -b_1]$ is sufficiently high, which certain distributions can guarantee, then the welfare and price reaction to voting outcomes have opposite signs.

A.4 Trading with partial sales of endowments

Our basic model assumes that shareholders can sell their entire endowment. In this section, we relax this assumption to incorporate scenarios where trading frictions are particularly high and do not allow initial shareholders to exit the firm completely. Specifically, we assume that when trading, shareholders can buy up to x shares or sell up to $y \in (0, e)$ shares, while retaining the remaining $e - y$ shares. All other assumptions in the baseline model remain unchanged. We provide a general discussion of the model in which we allow for $y < e$ in Section A.4.1, offer a more rigorous analysis in Section A.4.2, and gather the formal proofs in Section A.4.3.

A.4.1 Discussion of the model with $y < e$

Note that this extension allows us to separate the effect of market depth (captured by x and y, the amounts that shareholders can trade) from the effects of $\frac{x}{y}$, which captures the asymmetry between trading frictions on the buy-side and those on the sell-side. For simplicity, in what follows, we set $e = 1$. The formal analysis of this extension is presented in Section A.4.2, and we only summarize the key steps and conclusions here. As the following discussion demonstrates, our main results continue to hold in this extension.

When shareholders cannot exit their entire position in the firm, the post-trade shareholder base is composed of the buying shareholders, who hold $1 + x$ shares each, and the selling shareholders, who hold $1 - y$ shares each. This change does not materially affect the characterization of the equilibrium as given in Proposition 3. In particular, any equilibrium is either conservative or activist. For example, if the equilibrium is conservative, the marginal voter is given by

$$-q_c = \begin{cases} G^{-1}\left(\frac{1-\tau}{1+\frac{\delta}{1-\delta}y}\right) & \text{if } \frac{x(1-y)}{x+y} \leq \tau \\ G^{-1}(1-\frac{\tau}{1-y}) & \text{if } \frac{x(1-y)}{x+y} > \tau \end{cases} \tag{57}$$

and the marginal trader is given by

$$b_c = G^{-1}(1 - \frac{x}{x+y}). \tag{58}$$

In this equilibrium, which exists if and only if $q_c > F^{-1}(1-\phi)$, the shareholder buys x shares if $b < b_c$ and sells y shares otherwise. The proposal is accepted if and only if $q > q_c$, and the share price is given by $p_c = v(b_c, q_c)$.

As $y \to 1$, this setting converges to our baseline model and the activist and conservative equilibria can co-exist. As $y \to 0$, the equilibrium becomes unique and converges to the no-trade benchmark.

The key difference that distinguishes the analysis with $y < 1$ from the baseline model is that the marginal voter can now be less extreme than the marginal trader if y is sufficiently close to zero. Intuitively, when y is very small, the supply of shares is very low, and only the most extreme shareholders, those with the highest willingness to pay, buy shares in equilibrium. In other words, the marginal trader is very extreme and, as $y \to 0$, converges to $-\bar{b}$ in the conservative equilibrium and to $\bar{b}$ in the activist equilibrium. By contrast, the post-trade shareholder base is very similar to the initial shareholder base because the volume of trade is low due to small y. As such, the marginal voter is relatively moderate and, as $y \to 0$, the marginal voter converges to $q_{NoTrade} \in \left(-\bar{b}, \bar{b}\right)$.

As in the baseline model, the expected welfare of the pre-trade shareholder base is equal to the expected welfare of the post-trade shareholder base, because prices are just transfers from buying to selling shareholders. However, different from the baseline model, since selling shareholders cannot exit their entire position in the firm, the expected welfare of the post-trade shareholder base is now a weighted average of the buying shareholders' expected welfare and the selling shareholders' expected welfare, where the weight on the former is always larger than the weight on the latter. To see this explicitly, consider, for example, a conservative equilibrium. Then, the expected shareholder welfare is

$$W_c = (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b, q_c)\,|b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b, q_c)\,|b < b_c]. \tag{59}$$

Market clearing implies that $(1-y)\Pr[b>b_c]+(1+x)\Pr[b<b_c]=1$, and hence, indeed, W_c is a weighted average of $\mathbb{E}[v(b,q_c)|b>b_c]$ and $\mathbb{E}[v(b,q_c)|b<b_c]$, the welfare of selling and buying shareholders, respectively. Since $(1-y)\Pr[b>b_c]$ decreases in y, the weight that is put on the selling shareholders is decreasing in y, as they hold a smaller and smaller fraction of the firm post-trade. As $y\to 0$, $W_c\to\mathbb{E}[v(b,q_{NoTrade})]$, and as $y\to 1$, $W_c\to\mathbb{E}[v(b,q_c)|b<b_c]$, just as in the baseline model.

As before, the expected shareholder welfare obtains its maximum exactly when the bias of the marginal voter is equal to the average bias of the post-trade shareholder. Thus, our results on the optimal majority rule and the optimal board naturally extend to this setup. For example, in the conservative equilibrium, the average bias of the post-trade shareholder base is $(1-y)\Pr[b>b_c]\mathbb{E}[b|b>b_c]+(1+x)\Pr[b<b_c]\mathbb{E}[b|b<b_c]$, which includes the biases of both buying and selling shareholders. Note that this bias is always strictly smaller than $\mathbb{E}[b]$, and similarly, the average bias of the post-trade shareholder base in the activist equilibrium is strictly larger than $\mathbb{E}[b]$. These observations imply that the bias of the optimal board is different from $\mathbb{E}[b]$ in this setup as well.

Finally, the extension to $y<1$ allows us to revisit our results on the effect of liquidity on welfare when trading frictions have a symmetric effect on buy and sell orders. For this purpose we impose $x=y$ and consider the effect of increasing x and y by the same amount, which can be interpreted as an increase in market depth. We show that the expected shareholder welfare decreases in market depth under similar conditions to those specified in Proposition 6 part (ii). For example, the expected welfare in the conservative equilibrium W_c decreases in market depth whenever $|1-F(q_c)-\phi|$ is relatively small and the marginal voter in this equilibrium is more conservative than the average post-trade shareholder. The intuition is the same as in the baseline model.

A.4.2 Analysis

Define

$$\delta(y)\equiv\frac{x}{y+x}. \tag{60}$$

We prove the following results. The first result is the analog of Proposition 3.

Proposition 13. *Consider the setup of the baseline model where shareholders can only sell $y<1$ of their shares. An equilibrium of the game with trading and voting always exists.*

(i) A ***conservative*** *equilibrium exists if and only if* $q_c > F^{-1}(1-\phi)$*, where*

$$q_c = \begin{cases} -G^{-1}\left((1-\delta(1))(1-\tau)\right) & \text{if} \quad \delta(y)(1-y) \leq \tau \\ -G^{-1}(\frac{1-\tau-y}{1-y}) & \text{if} \quad \delta(y)(1-y) > \tau \end{cases}. \tag{61}$$

In this equilibrium, the shareholder buys x *shares if* $b < b_c$ *and sells* y *shares if* $b > b_c$*, where*

$$b_c = G^{-1}(1-\delta(y)). \tag{62}$$

The proposal is accepted if and only if $q > q_c$*, and the share price is given by* $p_c = v(b_c, q_c)$*.*

(ii) An ***activist*** *equilibrium exists if and only if* $q_a < F^{-1}(1-\phi)$*, where*

$$q_a = \begin{cases} -G^{-1}\left(1-(1-\delta(1))\tau\right) & \text{if} \quad \delta(y)(1-y) \leq 1-\tau \\ -G^{-1}(\frac{1-\tau}{1-y}) & \text{if} \quad \delta(y)(1-y) > 1-\tau \end{cases}. \tag{63}$$

In this equilibrium, the shareholder buys x *shares if* $b > b_a$ *and sells* y *shares if* $b < b_a$*, where*

$$b_a \equiv G^{-1}(\delta(y)). \tag{64}$$

The proposal is accepted if and only if $q > q_a$*, and the share price is given by* $p_a = v(b_a, q_a)$*.*

(iii) Other equilibria do not exist.

The second result is the analog of Lemma 2.

Lemma 5. *In any equilibrium, the expected welfare of the shareholder base pre-trade is equal to the expected welfare of the shareholder base post-trade. In particular,*

$$W_c = (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b, q_c)\,|b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b, q_c)\,|b < b_c] \tag{65}$$

and

$$W_a = (1-y)\Pr[b < b_a]\,\mathbb{E}[v(b, q_a)\,|b < b_a] + (1+x)\Pr[b > b_a]\,\mathbb{E}[v(b, q_a)\,|b > b_a]. \tag{66}$$

The third result is the analog of Lemma 3.

Lemma 6. *The expected welfare obtains its maximum exactly when the bias of the marginal voter is equal to the average bias of the post-trade shareholder, which is given by* $(1-y)\,\delta(y)\,\mathbb{E}[b|b>b_c]+(1+x)(1-\delta(y))\,\mathbb{E}[b|b<b_c]$ *in the conservative equilibrium and by* $(1-y)\,\delta(y)\,\mathbb{E}[b|b<b_a]+(1+x)(1-\delta(y))\,\mathbb{E}[b|b>b_a]$ *in the activist equilibrium. Moreover, the average bias of the post-trade shareholder in the conservative (activist) equilibrium is strictly smaller (larger) than* $\mathbb{E}[b]$*.*

Finally, the last result is the analog of Proposition 6 part (ii).

Proposition 14. *Suppose* $x=y$*. Then:*

(i) If a conservative equilibrium exists (i.e., $q_c > F^{-1}(1-\phi)$*) then here exists* $\phi_c > 1-F(q_c)$ *such that the expected shareholder welfare in the conservative equilibrium* W_c *decreases in market depth (i.e., a change in* y *and in* x *by the same amount) if and only if* $\phi \in (1-F(q_c), \phi_c)$ *and the marginal voter in this equilibrium is more conservative than the average post-trade shareholder (i.e.,* $-q_c < (1-y)\,0.5\mathbb{E}[b|b>b_c]+(1+y)\,0.5\mathbb{E}[b|b<b_c]$*).*

(ii) If an activist equilibrium exists (i.e., $q_a < F^{-1}(1-\phi)$*) then there exists* $\phi_a < 1-F(q_a)$ *such that the expected shareholder welfare in the activist equilibrium* W_a *decreases in market depth (i.e., a change in* y *and in* x *by the same amount) if and only if* $\phi \in (\phi_a, 1-F(q_a))$ *and the marginal voter in this equilibrium is less conservative than the average post-trade shareholder (i.e.,* $-q_a > (1-y)\,0.5\mathbb{E}[b|b<b_a]+(1+y)\,0.5\mathbb{E}[b|b>b_a]$*).*

A.4.3 Proofs

Proof of Proposition 13. Notice that Lemma 1 continues to hold in this setup and the expected value of shareholder b is given by (7). We consider three cases. First, suppose that $q^* > F^{-1}(1-\phi)$ (conservative equilibrium). The proof of Proposition 2 can be repeated in a setup with $y<1$ to show that if $q^* > F^{-1}(1-\phi)$ then $v(b,q^*)$ decreases in b and therefore there exists b_c such that $v(b,q^*) > p \Leftrightarrow b < b_c$. The key difference is that the market clears if and only if

$$xG(b_c) = y(1-G(b_c)) \Leftrightarrow G(b_c) = 1-\delta(y)\,. \tag{67}$$

After the trading stage, shareholders with $b < b_c$, of which there $1-\delta(y)$, hold $1+x$ shares. Shareholders with $b > b_c$, of which there are $\delta(y)$, hold $1-y$ shares. Recall shareholder b votes his shares for the proposal if and only if $q > -b$. Therefore, if $\delta(y)(1-y) \le \tau$ then the marginal voter is among the buying shareholders, those with $b < b_c$, and if $\delta(y)(1-y) > \tau$ then the marginal voter is among the selling shareholders, those with $b > b_c$. Let us write the identity of the marginal voter explicitly. If $\delta(y)(1-y) \le \tau$ then the marginal voter is determined by

$$\int_{-\bar{b}}^{-q_c} (1+x)\, dG(b) = 1-\tau \Leftrightarrow G(-q_c) = \frac{1-\tau}{1+x} \Leftrightarrow q_c = -G^{-1}\left((1-\delta(1))(1-\tau)\right),$$

just as in the baseline model (recall $\delta = \delta(1)$ in the baseline model). If $\delta(y)(1-y) > \tau$ then the marginal voter is determined by

$$\begin{aligned} \int_{-\bar{b}}^{b_c} (1+x)\, dG(b) + \int_{b_c}^{-q_c} (1-y)\, dG(b) &= 1-\tau \Leftrightarrow \\ G(b_c)(1+x) + (1-y)\left(G(-q_c) - G(b_c)\right) &= 1-\tau \Leftrightarrow \\ G(b_c)(y+x) + (1-y)\, G(-q_c) &= 1-\tau \Leftrightarrow \\ \delta(y)(y+x) + (1-y)\, G(-q_c) &= 1-\tau \Leftrightarrow y + (1-y)\, G(-q_c) = 1-\tau \Leftrightarrow \\ q_c &= -G^{-1}\left(\frac{1-\tau-y}{1-y}\right), \end{aligned}$$

as required. Hence, the cutoff in this "conservative" equilibrium is q_c as given by (61). Similarly to the proof of Proposition 2, the share price is $p_c = v(b_c, q_c)$.

Second, suppose that $q^* < F^{-1}(1-\phi)$ (activist equilibrium). The proof of Proposition 2 can be repeated in a setup with $y < 1$ to show that if $q^* < F^{-1}(1-\phi)$ then $v(b, q^*)$ increases in b and therefore there exists b_a such that $v(b, q^*) > p \Leftrightarrow b > b_c$. The key difference is that the market clears if and only if

$$x\left(1 - G(b_a)\right) = yG(b_a) \Leftrightarrow G(b_a) = \delta(y). \tag{68}$$

After the trading stage, shareholders with $b > b_a$, of which there $1-\delta(y)$, hold $1+x$ shares. Shareholders with $b < b_a$, of which there are $\delta(y)$, hold $1-y$ shares. Recall shareholder b votes his shares for the proposal if and only if $q > -b$. Therefore, if $\delta(y)(1-y) \le 1-\tau$ then the marginal voter is among the buying shareholders, those with $b > b_a$, and if $\delta(y)(1-y) > 1-\tau$

then the marginal voter is among the selling shareholders, those with $b < b_a$. Let us write the identity of the marginal voter explicitly. If $\delta(y)(1-y) \leq 1-\tau$ then the marginal voter is determined by

$$\begin{aligned}
\int_{q_a}^{\bar{b}} (1+x)\, dG(b) &= \tau \Leftrightarrow \\
1 - G(q_a) &= \frac{\tau}{1+x} \Leftrightarrow \\
q_a &= -G^{-1}(1 - (1-\delta(1))\tau),
\end{aligned}$$

just as in the baseline model. If $\delta(y)(1-y) > 1-\tau$ then the marginal voter is determined by

$$\begin{aligned}
\int_{q_a}^{b_a} (1-y)\, dG(b) + \int_{b_a}^{\bar{b}} (1+x)\, dG(b) = \tau \Leftrightarrow \\
(1-y)(G(b_a) - G(q_a)) + (1+x)(1 - G(b_a)) = \tau \Leftrightarrow \\
G(q_a) = \frac{1 + x - \tau - (y+x) G(b_a)}{1-y} \Leftrightarrow \\
G(q_a) = \frac{1-\tau}{1-y} q_a = -G^{-1}\left(\frac{1-\tau}{1-y}\right),
\end{aligned}$$

as required. Hence, the cutoff in this "conservative" equilibrium is q_a as given by (63). Similarly to the proof of Proposition 2, the share price is $p_a = v(b_a, q_a)$.

Third, the same arguments that are outlined in the proof of Proposition 2 to show that an equilibrium with $F(q^*) = 1-\phi$ does not exist, hold in this case as well.

Finally, notice that if $\delta(y)(1-y) \leq \min\{\tau, 1-\tau\}$ or $\delta(y)(1-y) \geq \max\{\tau, 1-\tau\}$ then $q_a < q_c$. Suppose $\tau < \delta(y)(1-y) < 1-\tau$ then $q_c = -G^{-1}(\frac{1-\tau-y}{1-y})$ and $q_a = -G^{-1}(1-(1-\delta(1))\tau)$, and $q_a < q_c$ if and only if $-y < x$ which always holds. If $1-\tau < \delta(y)(1-y) < \tau$ then $q_c = -G^{-1}((1-\delta(1))(1-\tau))$ and $q_a = -G^{-1}(\frac{1-\tau}{1-y})$, and $q_a < q_c$ if and only if $-y < x$ which always holds. Since $q_a < q_c$, in any event either $F(q_c) > 1-\phi$, $F(q_a) < 1-\phi$, or both. Therefore, an equilibrium always exists. ■

Proof of Lemma 5. The expected shareholder welfare in a conservative equilibrium is

$$\begin{aligned} W_c &= \Pr[b > b_c]\,\mathbb{E}[(1-y)\,v(b,q_c) + yp_c | b > b_c] + \Pr[b < b_c]\,\mathbb{E}[(1+x)\,v(b,q_c) - xp_c | b < b_c] \\ &= \Pr[b > b_c]\,yp_c - \Pr[b < b_c]\,xp_c \\ &\quad + (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b,q_c)\,|b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b,q_c)\,|b < b_c] \end{aligned}$$

Notice that $\Pr[b < b_c] = \frac{y}{y+x}$ and hence

$$\Pr[b > b_c]\,yp_c - \Pr[b < b_c]\,xp_c = \left(\frac{x}{y+x}y - \frac{y}{y+x}x\right)p_c = 0$$

Then

$$W_c = (1-y)\Pr[b > b_c]\,\mathbb{E}[v(b,q_c)\,|b > b_c] + (1+x)\Pr[b < b_c]\,\mathbb{E}[v(b,q_c)\,|b < b_c],$$

as required. Similarly, the expected shareholder welfare in an activist equilibrium is

$$\begin{aligned} W_a &= \Pr[b < b_a]\,\mathbb{E}[(1-y)\,v(b,q_a) + yp_a | b < b_a] + \Pr[b > b_a]\,\mathbb{E}[(1+x)\,v(b,q_a) - xp_a | b > b_a] \\ &= \Pr[b < b_a]\,yp_a - \Pr[b > b_a]\,xp_a \\ &\quad + (1-y)\Pr[b < b_a]\,\mathbb{E}[v(b,q_a)\,|b < b_a] + (1+x)\Pr[b > b_a]\,\mathbb{E}[v(b,q_a)\,|b > b_a] \end{aligned}$$

Notice that $\Pr[b > b_a] = \frac{y}{y+x}$ and hence

$$\Pr[b < b_a]\,yp_a - \Pr[b > b_c]\,xp_a = 0$$

Then

$$W_a = (1-y)\Pr[b < b_a]\,\mathbb{E}[v(b,q_a)\,|b < b_a] + (1+x)\Pr[b > b_a]\,\mathbb{E}[v(b,q_a)\,|b > b_a],$$

as required. ■

Proof of Lemma 6. Notice that

$$\begin{aligned}
\frac{\partial W_c}{\partial q^*} &= -f\left(q^*\right)\left[\left(1-y\right)\delta\left(y\right)\left(\mathbb{E}\left[b|b>b_c\right]+q^*\right)+\left(1+x\right)\left(1-\delta\left(y\right)\right)\left(\mathbb{E}\left[b|b<b_c\right]+q^*\right)\right] \\
&= -f\left(q^*\right)\left[\left(1-y\right)\delta\left(y\right)\mathbb{E}\left[b|b>b_c\right]+\left(1+x\right)\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b<b_c\right]+q^*\right]
\end{aligned}$$

Thus, the optimal cutoff satisfies

$$-q^* = \left(1-y\right)\delta\left(y\right)\mathbb{E}\left[b|b>b_c\right]+\left(1+x\right)\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b<b_c\right].$$

Since $\delta\left(y\right)\mathbb{E}\left[b|b>b_c\right]+\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b<b_c\right]=\mathbb{E}\left[b\right]$ and $x\left(1-\delta\left(y\right)\right)=y\delta\left(y\right)$, we have

$$\begin{aligned}
\left(1-y\right)\delta\left(y\right)\mathbb{E}\left[b|b>b_c\right]+\left(1+x\right)\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b<b_c\right] &< \mathbb{E}\left[b\right] \Leftrightarrow \\
-y\delta\left(y\right)\mathbb{E}\left[b|b>b_c\right]+x\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b<b_c\right] &< 0 \Leftrightarrow \\
x\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b<b_c\right] &< y\delta\left(y\right)\mathbb{E}\left[b|b>b_c\right] \Leftrightarrow \\
\mathbb{E}\left[b|b<b_c\right] &< \mathbb{E}\left[b|b>b_c\right],
\end{aligned}$$

which always holds.

Also notice that

$$\begin{aligned}
\frac{\partial W_a}{\partial q^*} &= -f\left(q^*\right)\left[\left(1-y\right)\delta\left(y\right)\left(\mathbb{E}\left[b|b<b_a\right]+q^*\right)+\left(1+x\right)\left(1-\delta\left(y\right)\right)\left(\mathbb{E}\left[b|b>b_a\right]+q^*\right)\right] \\
&= -f\left(q^*\right)\left[\left(1-y\right)\delta\left(y\right)\mathbb{E}\left[b|b<b_a\right]+\left(1+x\right)\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b>b_a\right]+q^*\right]
\end{aligned}$$

Thus, the optimal cutoff satisfies

$$-q^* = \left(1-y\right)\delta\left(y\right)\mathbb{E}\left[b|b<b_a\right]+\left(1+x\right)\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b>b_a\right].$$

Since $\delta\left(y\right)\mathbb{E}\left[b|b<b_a\right]+\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b>b_a\right]=\mathbb{E}\left[b\right]$ and $x\left(1-\delta\left(y\right)\right)=y\delta\left(y\right)$, we have

$$\begin{aligned}
\left(1-y\right)\delta\left(y\right)\mathbb{E}\left[b|b<b_a\right]+\left(1+x\right)\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b>b_a\right] &> \mathbb{E}\left[b\right] \Leftrightarrow \\
-y\delta\left(y\right)\mathbb{E}\left[b|b<b_a\right]+x\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b>b_a\right] &> 0 \Leftrightarrow \\
x\left(1-\delta\left(y\right)\right)\mathbb{E}\left[b|b>b_a\right] &> y\delta\left(y\right)\mathbb{E}\left[b|b<b_a\right] \Leftrightarrow \\
\mathbb{E}\left[b|b>b_a\right] &> \mathbb{E}\left[b|b<b_a\right],
\end{aligned}$$

as required. ■

Proof of Proposition 14. Notice that if $x = y$ then $\delta(y) = 0.5$, $b_c = b_a = G^{-1}(0.5)$, and from the expressions in Proposition 13 it can be verified that $\frac{\partial q_c(y)}{\partial y} > 0 > \frac{\partial q_a(y)}{\partial y}$.

Consider the conservative equilibrium first. In this case,

$$\begin{aligned} W_c &= (1-y)\,0.5\mathbb{E}\left[v(b,q_c)\,|b>b_c\right] + (1+y)\,0.5\mathbb{E}\left[v(b,q_c)\,|b<b_c\right] \\ &= (1-y)\,0.5\begin{pmatrix} \mathbb{E}[b|b>b_c]\,(1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} + (1+y)\,0.5\begin{pmatrix} \mathbb{E}[b|b<b_c]\,(1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} \frac{\partial W_c}{\partial y} &= -0.5\begin{pmatrix} \mathbb{E}[b|b>b_c]\,(1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} + 0.5\begin{pmatrix} \mathbb{E}[b|b<b_c]\,(1-F(q_c)-\phi) \\ +v_0+(1-F(q_c))\,\mathbb{E}[\theta|q>q_c] \end{pmatrix} \\ &\quad -f(q_c)\frac{\partial q_c}{\partial y}\left[(1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c] + q_c\right] \\ &= -0.5\left(\mathbb{E}[b|b>b_c] - \mathbb{E}[b|b<b_c]\right)(1-F(q_c)-\phi) \\ &\quad -f(q_c)\frac{\partial q_c}{\partial y}\left[(1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c] + q_c\right] \end{aligned}$$

Therefore, $\frac{\partial \hat{W}_c^*}{\partial y} < 0$ if and only if

$$\phi < \phi_c \equiv (1-F(q_c)) + f(q_c)\frac{\partial q_c}{\partial y}\frac{(1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c] + q_c}{0.5\left(\mathbb{E}[b|b>b_c] - \mathbb{E}[b|b<b_c]\right)}$$

Recall the conservative equilibrium exists if and only if $\phi > 1 - F(q_c)$. Thus the interval in which $\frac{\partial \hat{W}_c^*}{\partial y} < 0$ is non-empty if and only if $1 - F(q_c) < \phi_c$, which holds if and only if $-q_c < (1-y)\,0.5\mathbb{E}[b|b>b_c] + (1+y)\,0.5\mathbb{E}[b|b<b_c]$. This completes part (i).

Consider the activist equilibrium. If $x = y$ then

$$\begin{aligned} W_a &= (1-y)\,0.5\mathbb{E}\left[v(b,q_a)\,|b<b_a\right] + (1+y)\,0.5\mathbb{E}\left[v(b,q_a)\,|b>b_a\right] \\ &= (1-y)\,0.5\begin{pmatrix} \mathbb{E}[b|b<b_a]\,(1-F(q_a)-\phi) \\ +v_0+(1-F(q_a))\,\mathbb{E}[\theta|q>q_a] \end{pmatrix} + (1+y)\,0.5\begin{pmatrix} \mathbb{E}[b|b>b_a]\,(1-F(q_a)-\phi) \\ +v_0+(1-F(q_a))\,\mathbb{E}[\theta|q>q_a] \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned}
\frac{\partial W_a}{\partial y} &= -0.5\begin{pmatrix} \mathbb{E}\left[b|b<b_a\right]\left(1-F\left(q_a\right)-\phi\right) \\ +v_0+\left(1-F\left(q_a\right)\right)\mathbb{E}\left[\theta|q>q_a\right]\end{pmatrix} + 0.5\begin{pmatrix} \mathbb{E}\left[b|b>b_a\right]\left(1-F\left(q_a\right)-\phi\right) \\ +v_0+\left(1-F\left(q_a\right)\right)\mathbb{E}\left[\theta|q>q_a\right]\end{pmatrix} \\
&\quad -f\left(q_a\right)\frac{\partial q_a}{\partial y}\left(\left(1-y\right)0.5\mathbb{E}\left[b|b<b_a\right]+\left(1+y\right)0.5\mathbb{E}\left[b|b>b_a\right]+q_a\right) \\
&= 0.5\left(\mathbb{E}\left[b|b>b_a\right]-\mathbb{E}\left[b|b<b_a\right]\right)\left(1-F\left(q_a\right)-\phi\right) \\
&\quad -f\left(q_a\right)\frac{\partial q_a}{\partial y}\left(\left(1-y\right)0.5\mathbb{E}\left[b|b<b_a\right]+\left(1+y\right)0.5\mathbb{E}\left[b|b>b_a\right]+q_a\right)
\end{aligned}$$

Therefore, $\frac{\partial \hat{W}_a^*}{\partial y} < 0$ if and only if

$$\phi > \phi_a \equiv 1-F\left(q_a\right)-f\left(q_a\right)\frac{\partial q_a}{\partial y}\frac{\left(1-y\right)0.5\mathbb{E}\left[b|b<b_a\right]+\left(1+y\right)0.5\mathbb{E}\left[b|b>b_a\right]+q_a}{0.5\left(\mathbb{E}\left[b|b>b_a\right]-\mathbb{E}\left[b|b<b_a\right]\right)}$$

Recall the activist equilibrium exists if and only if $\phi < 1-F\left(q_a\right)$. Thus the interval in which $\frac{\partial \hat{W}_a^*}{\partial y} < 0$ is non-empty if and only if $1-F\left(q_a\right) > \phi_a$, which holds if and only if $-q_a > \left(1-y\right)0.5\mathbb{E}\left[b|b<b_a\right]+\left(1+y\right)0.5\mathbb{E}\left[b|b>b_a\right]$. This completes part (ii).
■

B Supplementary results for the baseline model

B.1 Optimal majority requirement

Proposition 15. *The optimal majority requirement in the conservative equilibrium, denoted by τ_c, satisfies*

$$-q_c\left(\tau_c\right) = \min\{\beta_c, -H^{-1}\left(\phi\right)\}, \tag{69}$$

and the optimal majority requirement in the activist equilibrium, denoted by τ_a, satisfies

$$-q_a\left(\tau_a\right) = \max\{\beta_a, -H^{-1}\left(\phi\right)\} \tag{70}$$

where $q_c\left(\tau\right)$ and $q_a\left(\tau\right)$ are given by (11) and (10).[28]

Proof. Consider first the conservative equilibrium, which exists if and only if $H\left(q_c\right) < \phi$. Re-

[28] We analyze the optimal threshold in a given equilibrium rather than across all equilibria, because when multiple equilibria exist, unless a selection is imposed, the optimal threshold is not well defined.

call $W_c = v(\beta_c, q_c)$, where $b_c = G^{-1}(1-\delta)$, $\beta_c = \mathbb{E}[b|b < b_c]$, and $q_c = -G^{-1}((1-\delta)(1-\tau))$. Using (7),

$$\frac{\partial W_c}{\partial \tau} = -(\beta_c + q_c)\frac{\partial q_c}{\partial \tau} f(q_c)$$

Using (11), we get $\frac{\partial q_c}{\partial \tau} = \frac{1-\delta}{g(-q_c)} > 0$. Plugging into $\frac{\partial W_c}{\partial \tau}$, we get

$$\frac{\partial W_c}{\partial \tau} = -(1-\delta)(\beta_c + q_c)\frac{f(q_c)}{g(-q_c)},$$

and hence, $\frac{\partial W_c}{\partial \tau} > 0 \Leftrightarrow -q_c > \beta_c$. Recall that the conservative equilibrium exists if and only if $H(q_c) < \phi \Leftrightarrow -q_c < -H^{-1}(\phi)$. Also notice that $-q_c(\tau)$ spans $\left[-\bar{b}, b_c\right]$ as a decreasing function of τ, and $\beta_c \in \left(-\bar{b}, b_c\right)$. Therefore, there is a unique $\hat{\tau}_c \in (0,1)$ such that $-q_c(\hat{\tau}_c) = \beta_c$. Thus, if $\beta_c < -H^{-1}(\phi)$ then $\tau_c = \hat{\tau}_c$, and if $\beta_c \geq -H^{-1}(\phi)$ then the closet marginal voter to β_c that implies a conservative equilibrium is $-q^* = -H^{-1}(\phi)$. Thus, $-q_c(\tau_c) = \min\{\beta_c, -H^{-1}(\phi)\}$ as required.[29]

Next, consider the activist equilibrium, which exists if and only if $H(q_a) > \phi$. Recall $W_a = v(\beta_a, q_a)$, where $b_a = G^{-1}(\delta)$, $\beta_a = \mathbb{E}[b|b > b_a]$, and $q_a = -G^{-1}(1-\tau(1-\delta))$. Using (7),

$$\frac{\partial W_a}{\partial \tau} = -(\beta_a + q_a)\frac{\partial q_a}{\partial \tau} f(q_a).$$

Using (11), we get $\frac{\partial q_a}{\partial \tau} = \frac{1-\delta}{g(-q_a)} > 0$. Plugging into $\frac{\partial W_a}{\partial \tau}$, we get

$$\frac{\partial W_a}{\partial \tau} = -(1-\delta)(\beta_a + q_a)\frac{f(q_a)}{g(-q_a)},$$

and hence, $\frac{\partial W_a}{\partial \tau} > 0 \Leftrightarrow -q_a > \beta_a$. Recall that the activist equilibrium exists if and only if $H(q_a) > \phi \Leftrightarrow -q_a > -H^{-1}(\phi)$. Also notice that $-q_a(\tau)$ spans $\left[b_a, \bar{b}\right]$ as a decreasing function of τ, and $\beta_a \in \left(b_a, \bar{b}\right)$. Therefore, there is a unique $\hat{\tau}_a \in (0,1)$ such that $-q_a(\hat{\tau}_a) = \beta_a$. Thus, if $\beta_a > -H^{-1}(\phi)$ then $\tau_a = \hat{\tau}_a$, and if $\beta_a \leq -H^{-1}(\phi)$ then the closet marginal voter to β_a that implies an activist equilibrium is $-q^* = -H^{-1}(\phi)$. Thus, $-q_a(\tau_a) = \max\{\beta_a, -H^{-1}(\phi)\}$ as required. ■

[29] As in the proof of Proposition 7, if $q^* = H^{-1}(\phi)$, the tie-breaking rule we adopt implies that no shareholder trades. While this tie-breaking rule implies that the trading strategies of shareholders in the delegation equilibrium are not continuous in q^* when $q^* = H^{-1}(\phi)$, the expected welfare of shareholders is nevertheless continuous in q^* when $q^* = H^{-1}(\phi)$, which is the only relevant consideration in the derivation of the optimal majority requirement.

B.2 Effect of δ on the benefits of delegation to the optimal board

In this section, we examine the effect of trading frictions on the benefit from delegation. Specifically, we ask how the comparison between delegation to an optimal board and decision-making via shareholder voting depends on liquidity δ. For this purpose, we assume that whenever multiple equilibria exist in the voting game, shareholders will coordinate on the equilibrium with the highest expected welfare. Hence, we are interested in the benefit from delegation $D(\delta) \equiv W_m^* - max\{W_a, W_c\}$, where W_a and W_c are given by (16). From part (iii) of Proposition 7, $D(\delta) \geq 0$ for all $\delta \in (0,1)$. A direct implication of part (ii) of Proposition 7 is that if expected shareholder welfare in the voting equilibrium decreases with δ, which happens under the conditions identified in Proposition 6 part (ii), then the benefit from delegation to the optimal board increases in δ:

Corollary 4. *If the expected welfare in the voting equilibrium decreases in δ, then the benefit from delegation to the optimal board, $D(\delta)$, increases in δ.*

Next, we show that generally, the effect of trading frictions on the benefit from delegation is ambiguous, and $D(\delta)$ may be increasing or decreasing. To see this, compare, for example, the conservative equilibrium in the voting game and delegation to an optimal conservative board. The welfare benefit of delegation is

$$D_c(\delta) \equiv v(\beta_c, -\beta_c) - W_c = \beta_c [F(q_c) - F(-\beta_c)] + \int_{-\beta_c} qdF(q) - \int_{q_c} qdF(q), \tag{71}$$

and hence

$$\frac{\partial D_c}{\partial \delta} = \frac{\partial q_c}{\partial \delta}(q_c + \beta_c) f(q_c) + \frac{\partial \beta_c}{\partial \delta}(F(q_c) - F(-\beta_c)). \tag{72}$$

Note that

$$q_c > -\beta_c \Leftrightarrow F(q_c) > F(-\beta_c).$$

Hence, the first and the second expression both change signs at $q_c = -\beta_c$. Since $\frac{\partial q_c}{\partial \delta} > 0$ and $\frac{\partial \beta_c}{\partial \delta} < 0$, the first expression is negative (positive) and the second expression is positive (negative) if $q_c + \beta_c < 0$ ($q_c + \beta_c > 0$). Therefore, $D_c(\delta)$ may be increasing or decreasing depending on the relative size of these expressions. Intuitively, if the marginal voter is more (less) conservative then the average post-trade shareholder (i.e., $-q_c < (>) \beta_c$), then an increase in liquidity, which makes the marginal voter in the voting game even more conservative ($\frac{\partial q_c}{\partial \delta} >$

0), increases (decreases) the benefit from delegation. On the other hand, an increase in liquidity also makes the average post-trade shareholder more conservative ($\frac{\partial \beta_c}{\partial \delta} < 0$), which increases the efficiency of the voting equilibrium and thus reduces (increases) the benefit from delegation.

The next result shows which of these effects dominates when liquidity is relatively high or low.

Proposition 16. *There exist* $0 < \underline{\delta} \leq \overline{\delta} < 1$ *such that:*

(i) If $G(\mathbb{E}[b]) \neq 1 - \tau$*, then* $\Delta(\delta') > \Delta(\delta'')$ *for all* $\delta' < \underline{\delta}$ *and* $\delta'' > \overline{\delta}$.

(ii) If $G(\mathbb{E}[b]) = 1 - \tau$*, then* $\Delta(\delta') > \Delta(\delta'')$ *for all* $\delta' \in (\underline{\delta}, \overline{\delta})$ *and* $\delta'' \notin (\underline{\delta}, \overline{\delta})$.

Intuitively, consider the generic case (i) in which the bias of the marginal voter in the no-trade equilibrium, $-q_{NoTrade}$, does not happen to be equal to the average bias of the pre-trade shareholder base. If liquidity is low and converges to zero ($\delta \to 0$), then both voting equilibria converge to the no-trade equilibrium, which is then strictly inferior to the case with optimal delegation in which marginal voter and the average shareholder are aligned. Conversely, as liquidity increases, extreme shareholders can build larger positions in the firm and tilt the voting outcome in their favor more often, which reduces the benefit of delegation to an optimal board. Indeed, in the limit, as liquidity becomes large ($\delta \to 1$), both the marginal trader and the marginal voter converge to the most extreme shareholder, so their preferences are fully aligned and delegation adds no value. Hence, the benefits of delegation are large if liquidity is low and small if liquidity is large. The case in which the no-trade equilibrium is efficient is different, because then the voting equilibria converge to an efficient no-trade equilibrium, delegation adds no value and the benefits from delegation arise only for intermediate values of liquidity (part (ii) of the proposition).

Proof of Proposition 16. Recall that $\lim_{\delta\to 1} b_c = -\overline{b}$ and $\lim_{\delta\to 1} b_a = \overline{b}$. Then we have $\lim_{\delta\to 1} \beta_c = -\overline{b}$ and $\lim_{\delta\to 1} \beta_a = \overline{b}$. Also recall that $\lim_{\delta\to 1}(-q_c) = -\overline{b}$ and $\lim_{\delta\to 1}(-q_a) = \overline{b}$. Therefore, in both the voting game and the delegation game, the marginal trader and the decision-maker (marginal voter or board, respectively) converge to the most extreme shareholder. Therefore, the expected welfare of shareholders in both cases is the same and equals the valuation of the most extreme shareholder. This means that $\lim_{\delta\to 1} D(\delta) = 0$.

Next, consider the limit $\delta \to 0$. Recall that $\lim_{\delta\to 0} q_c = \lim_{\delta\to 0} q_a = q_{NoTrade}$ and $\lim_{\delta\to 0} b_c = \overline{b}$ and $\lim_{\delta\to 0} b_a = -\overline{b}$, and hence, $\lim_{\delta\to 0} \beta_c = \lim_{\delta\to 0} \beta_a = \mathbb{E}[b]$.

First, consider the case where $-q_{NoTrade} \neq \mathbb{E}[b]$. Since $v(\mathbb{E}[b], q)$ achieves its maximum at $q = -\mathbb{E}[b]$, we have

$$\lim_{\delta\to 0} W_m^* = e \cdot v(\mathbb{E}[b], -\mathbb{E}[b]) > e \cdot v(\mathbb{E}[b], q_{NoTrade}) = W_{NoTrade}^* = \lim_{\delta\to 0} W_v^*.$$

Thus, in this case, $\lim_{\delta\to 0} D(\delta) > 0$. Combining it with $\lim_{\delta\to 1} D(\delta) = 0$ and using the continuity of W_m^* and W_v^* in δ, implies that $D(\delta') > D(\delta'')$ for all δ' sufficiently close to 0 and all δ'' sufficiently close to 1, which proves the statement in part (i).

Second, consider the case where $-q_{NoTrade} = \mathbb{E}[b]$. Then $\lim_{\delta\to 0} W_m^* = \lim_{\delta\to 0} W_v^*$, so $\lim_{\delta\to 0} D(\delta) = 0$. Since $\lim_{\delta\to 1} D(\delta) = 0$, to prove the statement in part (ii), it is sufficient to show that there exists $\delta \in (0,1)$ such that the benefit of delegation to an optimal board is strictly positive, i.e., the bias of the marginal voter in the voting game is different form the post-trade average shareholder. This follows immediately from the fact that q_c and q_a are equal to β_c or β_a only under knife-edge conditions on parameters. ■