Inequality Snowballing

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February 22, 2022

ABSTRACT: It has long been argued that efficient policies tend to provide larger legal entitlements to the rich than to the poor. This article shows how efficient legal rules can become even more skewed against the poor *over time* by sowing the seeds of their own vicious cycles. Repeated application over time of statically efficient legal rules can lead to rules that become increasingly adverse to the poor, which the article calls "policy snowballing."

Consider a set of polluters choosing between locating in places with rich versus poor people and facing a strict liability rule for damage to wages. Polluters will disproportionately locate in the poor area, where they face lower damages. That disproportionate share of polluters in the poor area drives down the wages of the poor further, making subsequent polluters locate yet more disproportionately in poor neighborhoods, driving down the poor's wages yet further.

We identify the conditions for snowballing and explore its dynamics. When compensation for the harm is incomplete, policy snowballing can lead to spiraling income inequality. As a result, if there are government taxes and transfers to the poor, those would be inadequate if calculated in a static way that ignores the snowballing. The article raises the intriguing prospect that efficient policymaking could be a contributing factor to increasing inequality over time.

Keywords: taxes and transfers; economic inequality; torts; dynamics; efficiency

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1 Introduction

The United States has long-term clusters of poverty and pollution. For example, Richmond, California, has had multiple rounds of polluters enter over a nearly 150-year-long history (Walker, 2001; Moore, 2000). Throughout, it has been one of the poorest places in the Bay Area (Allen and Li, 2016). Mining and chemical plants arrived in the late 19th century. Then Standard Oil and other petrochemical industries arrived around the turn of the century. Shipyards arrived with World War II. After the war, the existing industrial footprint continued to expand, especially the Chevron oil refinery. The last thirty years have seen a substantial number of high-profile explosions and mishaps at the industrial sites. These torts have settled, generally at a low per-person dollar amount. Of course, such poverty-pollution clusters have many causes. This article shows how a distinctive aspect of the law may contribute to such clusters—and other concentrations of poverty—and explores the dynamics of the phenomenon. In particular, this paper develops a model in which inequality "snowballs" due to repeated application of efficient legal rules, as these rules lay the seeds of their own vicious cycles.

To understand the intuition behind the paper's model, consider the long-run effects of adopting efficient legal rules. In the baseline model, there is no compensation through taxes and transfers, for example due to political economy failures (Fennell and McAdams, 2016; Liscow, 2018a). In the model, legal rules disproportionately reduce the wages of the poor, leading to multiplying inequality, as we will explain. Suppose that there is a neighborhood of poor (i.e., low-wage) individuals and a neighborhood of rich (i.e., high-wage) ones. A continuum of toxic-waste dumps that emit a fixed amount of pollution are deciding where to locate. Pollution reduces productivity and thus yearly wages through lost days of work; wages are thus reduced in proportion to the pollution to which the workers are exposed.[1] Harm to wages lasts for two periods.

[1] This modeling assumption is consistent with evidence showing that pollution can lead to lost days of work (Moretti and Neidell, 2011), causing proportional wage declines.

Firms decide where to locate based on two factors. First, the polluters pay damages resulting from a strict liability rule.[2] Damages equal the foregone wages resulting from the pollution. Second, the firms have a continuum of preferences (e.g., based on closeness to customers or quality of the land) between the two locations. As a result, a dump indifferent between the two locations for reasons other than the damages it will pay will locate in the poor neighborhood, knowing that the damages will be lower under tort law's economic damages rule, since pollution causes less monetary damage to lower wages than higher ones.[3] So, all else equal, dumps will tend to locate in the poor neighborhood.

In the next period, after the existing toxic-waste dumps are full, polluters again decide where to locate. The pivotal question for further increases in the share of firms in the poor neighborhood is how past pollution affects the harm from current pollution. Here we introduce a new critical parameter: the "feedback" effect. It could be the case that pollution in previous period has no effect on the harm from pollution in the current period. In this scenario, we would see no further changes in the share of firms located in the poor neighborhood, and there is no feedback. However, we show that if pollution from last period reduces the harm to wages from pollution this period—because wages are lower due to that earlier pollution—then yet more polluting firms will locate in the poor neighborhood. And the same snowballing occurs for the third, fourth, etc., periods: more pollution leads to lower wages, leading to more pollution. We call this process in which the harm governed by the policy becomes increasingly adverse to the poor by operation of an efficient legal rule "policy snowballing."

Of course, it is unsurprising that a legal rule that benefits richer individuals and is uncompensated by taxes and transfers benefits the rich. That is tautological. What is

[2] Of course, in practice, there may be a negotiated settlement, but—assuming that settlements are negotiated in the shadow of the strict liability rule—the implications should be similar regardless of whether or not parties go to trial.

[3] One response might be that the parties can have insurance. However, first, it is unlikely that such insurance exists in the real world (at least to a sufficient extent) due to adverse selection and moral hazard. Second and more importantly, after risk adjustment, the poor would have to pay a larger amount in premia to gain actuarially fair insurance (due to the greater risk of having the dump placed nearby). So while insurance does remove the risk component, it does not solve the distributive issue.

novel is that, solely by repeated operation of the same legal rule, the poor could be further immiserated over time.

The intuition of the model is that, by focusing on only static damages, typical efficient legal rules ignore the dynamic harm that they can cause to lower-income individuals when there is a feedback effect. In particular, static damages can ignore a key dynamic impact: since wages decrease in later periods because of the pollution, yet more polluters will pile into the poor area. And, since that pollution is disproportionately—and increasingly—located on the poor because damages are lower there, wages will decline more rapidly in the poor areas. An efficient static rule does not consider that.

Importantly, policy snowballing is invariant to the compensation received by those harmed. The driver of policy snowballing is diverging wages because of the disproportionate pollution on the poor. Compensation—in the form of either damages from polluters or transfers from the government—does not increase the wages themselves, which drive the policy snowballing, and thus does not slow the snowballing.

In the model, there may be incomplete compensation because a share of damages goes to legal fees, as is the case with the "American rule" under which each party pays its own legal fees. (Note that, to be efficient, legal rules need not involve any compensation at all.) Thus, the application of the efficient legal rule may progressively increase income inequality as well, which we call "income snowballing."[4] The example shows the possibility that not only are efficient legal rules not neutral but also that they may also become increasingly harmful to the poor with time, thereby exacerbating income inequality.

The article's main contribution is identifying and modeling the policy and inequality snowballing mechanisms. The article also has four additional contributions.

[4] One response might be that the pollution will lower rents, benefitting the poor renters. That is probably true to some extent, but there is little reason to believe that there would be full offset of the income loss by lower rents. For example, with an infinitely elastic housing supply, there should be no price response at all. Furthermore, recent work by Pat Kline and Enrico Moretti has emphasized the importance of "inframarginal" individuals who are not on the margin between moving between one place and another (Kline and Moretti, 2014a; Kline and Moretti, 2014b). These individuals are harmed when the quality of their current residence declines in value, since they are staying there and paying the rent regardless.

First, we identify the key feedback parameter that is a necessary and sufficient condition for policy snowballing and a further necessary and sufficient condition—incomplete compensation—for income snowballing.

Second, we characterize the equilibrium level of pollution in the rich and poor communities and explore its comparative statics. For example, we show that, the higher the level of initial inequality, the more unequal the equilibrium pollution will be. We also identify the parameters that determine the speed at which the equilibrium is reached.

Third, we find a simple formula for the extent to which ignoring dynamics—that is, the additional harm from pollution that arises simply from snowballing—causes a failure to capture harm to the poor. It is proportional to the same key feedback parameter. We show in an example that these dynamic losses can be quite large. So, part of this article's policy stakes is that, when efficient legal rules are adopted, policymakers trying to compensate for losses may calculate the wrong distributional consequences if they only account for the standard static distributional effects rather than the dynamic distributional effects considering snowballing inequality through repeated application of efficient legal rules.

Finally, we develop a formula for the welfare-maximizing policy. Depending on the welfare weights and the level of existing inequality, it can be welfare-maximizing to adopt an *inefficient* policy because of the dynamics that immiserate the poor over time.

As the article explains, similar dynamics in which inequality affects the outcome of the legal rule, which in turn exacerbates inequality, may exist in many areas of law, including transportation spending, healthcare spending, eminent domain law, the provision of amenities like parks, and a host of others. Take the example of efficient cost-benefit analysis of transportation spending across rich and poor areas, which has similar features to the torts example (Liscow, 2022). In particular, it has the three key features for snowballing: First, all else equal, it is efficient to spend more in rich places than poor ones (because the rich are willing to pay more for faster transportation) and, second, application of the legal rule may further the disparity between the rich and the poor (because the rich receive more transportation funding and thus access to higher incomes). These two features set up policy snowballing. If there is additionally a third factor that there is insufficient compensation for the greater spending on the rich, there may be income snowballing. So cost-benefit analysis for transportation spending is promising for snowballing as well. As the article explains, many other areas are not promising, including those where efficient legal rules do not benefit the rich more than the poor and those where there is no feedback loop.

Inequality has been rising in developed countries over the past few decades (Piketty, 2014; Piketty and Saez, 2003).[5] There are many proposed explanations for the increase in inequality, like the institutional forces described by Piketty (2014), skill-biased technological change (Autor, Katz, and Kearney, 2008), globalization of trade (Harrison, McLaren, and McMillan, 2011), and others. In this article, we raise the intriguing possibility that efficient legal rules could lead to repeated multiplication of inequality over time, suggesting that the dynamics of efficient laws may be an additional possible explanation. We believe that this is the first economics article to lay out a model providing a proof of possibility for how such multiplication would happen—of course, without making any claim as to whether such a mechanism has in fact been at play in increasing income inequality.

As a proof of possibility, this model of course does not account for all relevant factors. One important factor is the absence of potential benefits from having a toxic waste dump located nearby, like employment. The extensions section shows that labor demand can easily be incorporated into the model, producing similar results so long as the benefits to wages from the positive labor demand shock do not outweigh the harm to wages from pollution. Other potentially relevant aspects of reality, some of which could mitigate the severity or eliminate the presence of snowballing, are discussed below.

The paper touches on several literatures. The distributional impacts of efficient legal rules have been much discussed—quite sensibly, since the Kaldor-Hicks efficient prescription is the standard one in law and economics (Posner, 2014; Shavell, 2004; Cooter and Ulen,

[5]Note that others argue that claims about increasing income inequality are overstated (Burkhauser et al, 2012).

2012). Perhaps most famously, Ronald Dworkin (1980) described how it could be efficient to take a book from a poor person and give it to a rich person who values it more. Essentially, because the rich are typically willing to pay more for things by virtue of their greater wealth, there is a tendency for efficient legal rules to allocate more to the rich, since efficient legal rules are based on willingness to pay (Liscow, 2018a). While valuable, this analysis is static, in a one-period model, and thus does not consider the dynamics of how repeated application of efficient legal rules could amplify or counteract these distributional consequences.

There has, of course, been considerable work on dynamics in economics (e.g., Stokey, Lucas, and Prescott, 1989). And there has long been speculation about the long-run effects of efficient rulemaking (e.g., Hicks, 1941).[6] But little has been done formally on the dynamics of efficient legal rules, which is the focus of this paper.

Similarly, the "environmental justice" literature has long discussed how governments may tend to pollute more on the poor than the rich, perhaps due to racism (e.g., Bullard, 1994). This article introduces a new legal mechanism explaining disproportionate environmental harm to the poor, formally models it, and explores its dynamics.

Finally, while it has been noted that tort law's assessment of economic damages based on income tends to lead to higher damages for the rich than the poor (e.g., Chamallas, 2005), the consequent dynamics over time developed here are new. Likewise, the "American rule's" requirement that parties pay their own fees has been criticized for under-compensating those who are harmed, but again the dynamics over time described here, including the interaction with economic damages, is new (Ehrenzweig, 1966).

This article proceeds as follows. Section 2 sets up the model. Section 3 defines the two forms of inequality snowballing—policy snowballing and income snowballing—and describes the broad circumstances under which the model exhibits either one. Section 4 discusses model

[6] The question of how efficient legal rules affect distribution in aggregate over time is one that has divided scholars. For example, Richard Zerbe and Tyler Scott (2014) argue that cost-benefit analysis yields results that are close to Pareto superior. See also Polinsky (1972). Indeed, Hicks (1941, pg. 111) called this notion the "classical creed," meaning that if society made "all alterations" that met the Kaldor-Hicks criterion, then "there would be a strong probability that almost all (individuals) . . . would be better off after the lapse of a sufficient length of time." See also Persky (2001) for further exploration of the "classical creed."

dynamics, comparative statics, the importance of dynamics for calculating needed taxes and transfers after a change in legal rules, and the implications for welfare. Section 5 offers an extension to the model, including localized labor demand impacts from firms' locational choices. Section 6 discusses factors that were not considered in the model and how they might impact the presence of snowballing. Section 7 considers other policy scenarios that could similarly lead to inequality snowballing.

2 Model Setup

Setting and Wages

There are two equally populated neighborhoods, one high-wage, denoted by H, and one low-wage, L. People are immobile and infinitely lived. In each neighborhood $N \in \{H, L\}$, the representative individual has base wages $\overline{w}_N$. By assumption, the individual in the high-wage neighborhood begins with a higher wage than the one in the poor neighborhood: $\overline{w}_H > \overline{w}_L$. Workers supply one unit of labor inelastically each time period, so, in a world without pollution, the earnings of a worker in neighborhood N are $\overline{w}_N$.

However, the workers are polluted on. Each time period, there is a unit-measure of polluting firms that decide where to locate. Think of the firms as toxic waste dumps that do not employ any labor and that fill up each period. All firms produce the same, fixed amount of pollution that reduces the wages of workers in the neighborhood in which they locate.[7] The total harm to the wages of individuals in the neighborhoods depends on the share of firms that locate (and have located) in the neighborhood. Specifically, let n_t be the share of polluting firms that locate in neighborhood N at time t. The realized wage of individuals in neighborhood N at time t is

$$w_{Nt} = [1 - \theta_1 n_t - \theta_2 n_{t-1} + \theta_3 n_t n_{t-1}]\, \overline{w}_N. \tag{1}$$

[7] A variety of evidence suggests that pollution reduces workers' productivity (Heyes, Neidell, and Saberian, 2016; Moretti and Neidell, 2011; Sanders, 2012).

Harm from pollution thus may last two periods, with $\theta_1 n_t$ the harm from pollution this period, $\theta_2 n_{t-1}$ the harm from pollution last period, and $\theta_3 n_t n_{t-1}$ the harm from the interaction of this period and the last one. For example, when base wages $\overline{w}_N$ are multiplied by $\theta_1 n_t$, this multiplies pollution, n_t, by the harm from that pollution, θ_1, reducing realized wages. We call θ_1 and θ_2 "main effects." Again, it is natural to think about this as pollution reducing the share of days that someone can work. Note that, since it is added rather than subtracted, a higher θ_3 *decreases* the marginal harm of pollution today for a given level of pollution yesterday. This is a way of saying that, once wages have already been reduced, a further proportional reduction has less of an impact. We call this the "feedback" effect. This effect might seem like a relief to the harmed parties, since it reduces harm; we will show that it can, in fact, have the opposite effect.

This modeling choice for wages is a flexible generalization of harm to wages. For example, consider a wage process in which pollution affects wages multiplicatively. Such a process could be expressed as

$$w_{Nt} = (1 - \theta n_t)\,(1 - \theta n_{t-1})\,\overline{w}_N.$$

This is the case of wage function (1) in which $\theta_1 = \theta_2 = \theta$ and $\theta_3 = \theta^2$. Here, pollution reduces wages this period a certain percent, and next period the same percent. But, because the pollution effects are applied multiplicatively (by reducing days worked), the past pollution effectively reduces the wage base on which the current pollution acts. Pollution in the previous period makes the base on which pollution acts on $(1 - \theta n_{t-1})\,\overline{w}_N$ instead of $\overline{w}_N$. A lower base means a smaller absolute reduction in wages from a given level of pollution. The θ^2 term in this special case captures the feedback effect.

Our setup allows for pollution harm to have different effects depending on time since pollution. Such a process could be expressed as

$$w_{Nt} = (1 - \theta_1 n_t)\,(1 - \theta_2 n_{t-1})\,\overline{w}_N.$$

Now the impact of past and present pollution is allowed to vary but it is still applied multiplicatively. For example, here $\theta_3 = \theta_1 \cdot \theta_2$.

Equation 1 also nests the possibility that there is no feedback effect at all. In such a case, the harm from pollution today, and the harm from pollution yesterday are always applied to the same base: $\overline{w}_N$. The process could be expressed as:

$$w_{Nt} = (1 - \theta_1 n_t - \theta_2 n_{t-1})\,\overline{w}_N.$$

This is the case of wage function (1) in which $\theta_3 = 0$.

We set the following conditions on the wage parameters:

$$0 \leq \theta_1 \leq 1, \qquad 0 \leq \theta_2 \leq 1, \qquad \theta_3 < \theta_1, \qquad \theta_3 < \theta_2, \qquad \theta_1 + \theta_2 - \theta_3 \leq 1 \qquad (2)$$

The only restrictions we place are that θ_3 must be less than both θ_1 and θ_2. This ensures that an increase in pollution, either of n_t or n_{t-1}, cannot increase the realized wage of the individual. All are nonnegative. The constraint $\theta_1 + \theta_2 - \theta_3 \leq 1$ ensures that, for any possible combination of past and present pollution, current wages can never be negative.

Liability and Firm Location

Firms face a strict liability rule for the harm their pollution causes to wages. This strict liability rule is efficient in this context because the polluters pay for their marginal harm, and those polluted upon are immobile, so their behavior cannot be distorted by the compensation (Shavell, 2004, p. 178-80). Let D_{Nt} be the damages a given firm has to pay if it locates in neighborhood N at time t. Then

$$D_{Nt} = (\theta_1 + \theta_2 - \theta_3 n_{t-1})\,\overline{w}_N \qquad (3)$$

Note that the firms have to pay for damage that will be realized next period but is caused by pollution this period (the θ_2 term in the damages equation). [8] There are no punitive damages.

There are a couple of points to note about the damages charged to a firm locating in neighborhood N. The first is that damages (and harm) are a percentage of the base wage $\overline{w}_N$, since with "economic damages" in tort law, higher wages mean higher damages for the same lost number of hours. This will tend to push firms toward the low-wage neighborhood. For example, suppose that $\theta_3 = 0$. Then, a firm will cause the same harm as a percentage of wages in either neighborhood. Because $\overline{w}_H > \overline{w}_L$, this means that absolute damages are higher in the high-wage neighborhood than in the low-wage neighborhood. Since the choice of neighborhood depends on the absolute harm and not relative harm (as we will describe below), this will tend to push firms toward the low-wage neighborhood.

The second aspect to note about the damages from current-period pollution shown in (3) is that a decrease in n_{t-1} (pollution in the neighborhood in the *previous* period) leads to an increase in damages as long as $\theta_3 > 0$. As polluting firms move out of the neighborhood, the marginal harm from a polluter locating in that neighborhood in the next period increases. In a strict liability legal regime, this, in turn, will tend to push more firms out of the neighborhood. This basic dynamic drives policy and inequality snowballing.

A firm's choice of neighborhood depends on the value of damages to be paid as well as firm-specific preferences for either the high-wage neighborhood or the low-wage neighborhood. These firm-specific preferences can be thought of as coming out of features of each neighborhood that affect profitability, such as the quality of the geography for building a dump; proximity to natural resources, customers or suppliers; cheap supplies of energy; transportation infrastructure; or low construction costs. We assume that pollution affects only

[8] As noted by a leading treatise on torts, "If future wage increases are to be expected, either because of a general increase in industrial productivity or because of the plaintiff's reasonably expected advancement, those increases have also been lost and are thus recoverable as damages."(Dobbs, Hayden, and Bublick, 2011, § 479). Thus, the tort rule reflects the lost wage growth. Results are similar without compensation for future wage gains.

the wages of the neighborhoods and does not change the characteristics of the neighborhoods that determine the firm's preferences for either neighborhood.

Specifically, let α_k be firms' idiosyncratic (non-damages) relative value from locating in the high-wage neighborhood compared to the low-wage neighborhood. Firm k has preference $\alpha_k \sim F$, where F is smooth cdf whose pdf has compact support, and keeps this preference for the rest of time. We impose an additional condition on F: that it has median 0. This is merely so that, before damages calculations, firm preferences do not tilt the balance of firms to one neighborhood or the other.

Under the strict liability legal regime, firm k will locate in the low-wage neighborhood if

$$\alpha_k < D_{Ht} - D_{Lt} \tag{4}$$

That is, the firm will locate in the low-wage neighborhood if the value of its relative preference for the high-wage neighborhood is smaller than the damage payments loss from locating in the high-wage neighborhood.

Let $G(l_{t-1})$ be the difference in damage payments between the high-wage neighborhood and low-wage neighborhood at time period t when the previous time period's pollution in the low-wage neighborhood was l_{t-1} so that $G\left(l_{t-1}\right)$ is

$$G\left(l_{t-1}\right) = D_{Ht} - D_{Lt} = \overline{w}_H\left(\theta_1 - \theta_3 h_{t-1} + \theta_2\right) - \overline{w}_L\left(\theta_1 - \theta_3 l_{t-1} + \theta_2\right)$$

where h_{t-1} is the share of firms in the high-wage neighborhood at time period $t-1$. Because $l_{t-1} = 1 - h_{t-1}$, G can be written as simply a function of l_{t-1}, and we will rarely use the variable h_t, instead relying on its expression in terms of l_t. Making this change gives

$$G\left(l_{t-1}\right) = \left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \theta_3\overline{w}_H + \theta_3 l_{t-1}\left(\overline{w}_H + \overline{w}_L\right) \tag{5}$$

$G\left(l_{t-1}\right)$ plays an important role in the model so it is useful to examine it. First, increases in θ_3 increase the influence of past location decisions on this period's damage differences. Moreover, a higher share of firms locating in the low-wage neighborhood in the previous period increases the damages difference.

By the location decision condition (4), the share of firms that will locate in the low-wage neighborhood at time t is

$$l_t = F\left(G\left(l_{t-1}\right)\right) \tag{6}$$

This equation is the law of motion for the share of firms in the low-wage neighborhood.

All that is left to determine the system is an initial condition, l_0, for the share of firms in the low-wage neighborhood in the initial period. We set $l_0 = \frac{1}{2}$. Thus, our model studies a world that starts with the firms equally split between the two neighborhoods and then sees a change in the legal regime to the efficient strict liability framework described above. We focus on this case because one of our goals is to study what a legislature would have to account for to offset the inequality that would arise due to the switch from a more equitable but inefficient legal regime to an efficient legal regime.

Note that, in the baseline model, there are not taxes and transfers that compensate for the losses that the poor suffer. Theoretically, there are many reasons for legislative inertia that would lead to incomplete distributional offsetting (Fennell and McAdams, 2016; Liscow, 2018a). It is an open empirical question how much compensation happens after a change in legal rule impacts distribution. And there is evidence both ways (Autor, Dorn, and Hanson, 2013; Boylan and Mocan, 2014; Liscow, 2018b). For our purposes, the point is that there is a plausible case that compensation is incomplete.

3 Policy and Income Snowballing

In this section, we describe the conditions in which the switch to the efficient legal regime leads not only to an increase in the share of firms in the low-wage neighborhood, but also an increasing share of firms in the low-wage neighborhood over time. We call a situation in which the share of firms in the low-wage neighborhood is increasing over time because of these dynamics *policy snowballing.* We also provide conditions in which the policy snowballing translates to increasing income inequality over time, which we call *inequality snowballing.* The importance of both of these phenomena is not just that the efficient regime leads to more pollution and income inequality for the low-wage neighborhood over time. The dynamic, compounding increase in pollution will also make it difficult for a legislature to redistribute to compensate for the increased inequality, a point we elaborate on in Section 4.

3.1 Policy Snowballing

Formally, we define *policy snowballing* as occurring when $l_t > l_{t-1}$ for all $t \geq 1$ or when there exists a $t^* \in \{\mathbf{2}, \mathbf{3}, \mathbf{4}, ...\}$ such that for $t \geq t^*, l_t = 1$ (all firms locate in the low-wage neighborhood) and for $1 \leq t < t^*, l_t > l_{t-1}$. The law of motion described in equation 6 determines the presence of policy snowballing. Our first result establishes the necessary and sufficient conditions for policy snowballing. The proof for this result, as well as later results, are in the Appendix.

Result 1: Policy Snowballing. *Suppose that $l_1 < 1$.*[9] *There is policy snowballing if and only if $\theta_3 > 0$.*

According to Result 1, the key ingredient for policy snowballing is θ_3. Policy snowballing occurs and, given our other assumptions in the set-up of the model, can only occur when pollution today decreases the marginal harm from pollution that occurs next period. Returning to the two simple harm processes discussed in the Introduction and Section 2, the result establishes that if the harm process is one in which pollution acts on the same base wage $\overline{w}_H$ irrespective of past pollution, then there can be no policy snowballing. Policy snowballing requires and is in essence caused by the feedback effect. All else equal, if a higher share of firms located in a neighborhood in the previous period, then the damages from locating in that neighborhood are lower in this period. Without the feedback effect, the adjustment to

[9]Having all firms immediately locate in the low-wage neighborhood is uninteresting as the number of firms can no longer increase, and there is no scope for θ_3 to play a role in dynamics.

the change in legal regime would occur in one period.

To see how this leads to perpetual policy snowballing, consider the how θ_3 affects the location decision across multiple time periods. At time period t, firms calculate the damage differences (5) between the neighborhoods and compare it to their own idiosyncratic preferences to choose in which neighborhood to locate. Suppose that the balance of these decisions means that more firms have moved into the low-wage neighborhood. Hence, $l_t > l_{t-1}$. This means that when firms are calculating the damages from neighborhood choice at period $t+1$, the damages to be paid from locating in the low-wage neighborhood now will have decreased precisely because $l_t > l_{t-1}$. Conversely, the damages to be paid from locating in the high-wage neighborhood will increase because $h_t < h_{t-1}$. So the damage differences (5) increases, and the share of firms in the low-wage neighborhood will increase in time period $t+1$.

3.2 Income Snowballing

Next, we examine when we would see a repeated increase in income inequality and its relation to policy snowballing. To do so, we first define income. Formally, we define income in neighborhood N at time t as

$$I_{Nt} = I_{Nt}(n_{t-1}, n_t) = w_{Nt} + \Omega\left(\left(\theta_1 - \theta_3 n_{t-1}\right) n_t + \theta_2 n_{t-1}\right) \overline{w}_N \tag{7}$$

where $0 \leq \Omega \leq 1$. This new parameter represents the wedge between damages paid by the firms (which are the efficient amount) and the payments received by the harmed individuals. These can be thought of as lawyers' fees that plaintiffs have to pay to take the firms to court. This assumption is consistent with the "American Rule," the standard U.S. practice by which each party pays for its own legal fees (Derfner and Wolf, 2018, ¶ 1.01).

The first term in the income equation is the wages of individuals in neighborhood N at time t. The second term in the expression is the amount of income received from damage payments. It includes payments for harms from two different periods: harm today $((\theta_1 - \theta_3 n_{t-1})\, n_t)$ and harm from pollution in the previous period $(\theta_2 n_{t-1})$. Damage payments are timed so that individuals receive payments today for harm realized today. At time t, a firm located in neighborhood N will be ordered to pay $(\theta_1 - \theta_3 n_{t-1})\, \overline{w}_N$ today and $\theta_2 \overline{w}_N$ tomorrow. Summing up over the number of firms that located in the neighborhoods gives the n_t and n_{t-1} in Equation 7. This definition of income allows for an easy comparison of today's income with a counterfactual income absent pollution or absent a change in pollution.

Define the level of income inequality Q_t at time t as the ratio of high-wage neighborhood wages to low-wage neighborhood wages: $Q_t = \frac{I_{Ht}}{I_{Lt}}$. Furthermore, we formally define *income snowballing* analogously to *policy snowballing.* Define *income snowballing* as occurring when $Q_t > Q_{t-1}$ for all $t \geq 1$ or when there exists a $t^* \in \{\mathbf{2}, \mathbf{3}, \mathbf{4}, ...\}$ such that for all $Q_t = c \in \mathbb{R}$ for all $t \geq t^*$ (after some time income inequality is constant) and for $1 \leq t < t^*, Q_t > Q_{t-1}$. Then we have the following result.

Result 2: Income snowballing. *If there is policy snowballing, then a necessary and sufficient condition for income snowballing is* $\Omega < 1$.

Although policy snowballing will occur as long as there is a feedback effect, income snowballing requires an extra ingredient: that damages received by harmed individuals are incomplete. If there are no frictions preventing harmed individuals from receiving the full damages that firms pay, the income of individuals in both neighborhoods will be the same as their counterfactual income absent the change in regime, keeping income inequality the same. But a change in the legal regime to the efficient rule and a wedge between harm and damage payments received combine to increase income inequality over time.

4 Model Dynamics

In this section, we describe the model's steady-state outcome, how its parameters affect the steady state, and how quickly the steady state is arrived at. We also analyze the quantitative importance of snowballing and describe optimal policy. To do so, we take the completely general law of motion, equation (6), and specify a uniform distribution for it.

In subsection 4.1, we establish that a steady state always exists (excepting one knife-edge condition), and we characterize when there is an interior solution. We also discuss how changes in the model parameters affect the location of the final steady state, providing additional intuition about the forces at play.

Moreover, defining "speed" of convergence as (roughly) the number of time periods needed to get close to the stead state, we show in subsection 4.2 that the speed of convergence is governed by a push-and-pull between firm preference dispersion and the feedback parameter.

In subsection 4.3, we illustrate the importance of the snowballing described here by asking how much would be missed if it was ignored by a legislature compensating those harmed by a change in legal regime. We characterize the amount of harm to the poor missed. And we provide an example showing the large amount that feedback dynamics can impact the harm of a change in legal rules on the poor.

Finally, in subsection 4.4, we turn to welfare and characterize the welfare weights required to prefer the inefficient regime over the efficient one if compensation is incomplete.

4.1 Steady-State Existence and Comparative Statics

Throughout this section we assume that firm preferences are uniform. Specifically, we assume that the cdf of firm preferences is given by

$$F_U \sim U\left[-M \cdot \left(\overline{w}_H + \overline{w}_L\right), M \cdot \left(\overline{w}_H + \overline{w}_L\right)\right] \tag{8}$$

where $M > 0$ is a constant that controls the dispersion of firm preferences. The uniform preferences allow for a more detailed analysis of the model dynamics.[10]

First, we establish, with a very simple technical assumption, that the process set in motion by the change in legal regime will eventually reach a steady state. Formally, the process defined by (6) will converge to a steady state. Additionally, we derive an equation for the steady state. Result 3 below summarizes the main conclusions, proven in the Appendix.

Result 3: *Let $\theta_3 \neq 2M$. Suppose that $\theta_3 > 0$ and hence there is policy snowballing when the efficient legal regime is implemented. Then the process converges to a steady-state share of firms in the low-wage neighborhood, $l^* > \frac{1}{2}$. Generally, the steady state is given by the equation*

$$l^* = \frac{M\left(\overline{w}_H + \overline{w}_L\right) + \left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \overline{w}_h\theta_3}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)}$$

However, if the above expression is greater than or equal to one, then $l^* = 1$. In that case, all of the firms will eventually locate in the low-wage neighborhood.

After the switch to the efficient legal regime, if $\theta_3 > 0$, then the share of polluting firms in the low-wage neighborhood will increase every period. With the uniform preferences for the firms, we can say that in many cases the increase in subsequent periods will become small enough so as to never lead all firms to locate in the low-wage neighborhood. Instead, the share of firms in the low-wage neighborhood will approach a steady state l^*. We can also know exactly what the steady state is using the model parameters.

Furthermore, note that, if $\theta_3 = 0$, so that there is no policy snowballing, the steady state is reached immediately, as there are no dynamics. Thus, with the exception of the knife-edge condition, there is always a steady state.

[10] In technical terms, the uniform preferences turn the first-order difference process (6) into a linear difference function, which allows us to draw from well-established first-order difference equation results.

Comparative Statics Results

We now examine how changes in the model parameters affect the final share of firms that will locate in the low-wage neighborhood. To do so, we will focus on the cases in which the steady state is less than 1, so that we can simply analyze the expression for l^* in Result 3. Note, too, that this assumption implies that $\theta_3 < 2M$ (see the Appendix proof for Result 3), an inequality useful for signing the comparative statics.

The first comparative statics results are

$$\frac{\partial l^*}{\partial \theta_1} > 0, \qquad\qquad \frac{\partial l^*}{\partial \theta_2} > 0.$$

Increasing the main effects from pollution, either the effects from pollution this period or the effects from pollution emitted in the previous period, increases the steady-state share of firms in the low-wage neighborhood. This is because increasing the harm from pollution, which works as a percentage of base wage level, leads to a bigger absolute increase in damages to pay in the high-wage neighborhood than the low-wage neighborhood since high-wage individuals have a higher base wage level. So, the steady state can sustain a higher share of firms in the low-wage neighborhood when either main effect increases.

Next, we have that

$$\frac{\partial l^*}{\partial M} < 0.$$

The intuition between this result is that a larger M means that firms have more dispersed preferences for location, which means that—with a higher M—a larger share of firms will have a strong preference to locate in the rich neighborhood. This reduces the share that locate in the poor neighborhood in equilibrium.

We deal with the how changes in base wages affect l^* together. The comparative static results are

$$\frac{\partial l^*}{\partial \overline{w}_H} > 0 \qquad\qquad \frac{\partial l^*}{\partial \overline{w}_L} < 0.$$

As the inequality in the initial wages grows, the low-wage neighborhood becomes increasingly attractive. Because firms care only about absolute damages, and harm is a percentage of base wages, then an increase in the gap between the base wages increases the absolute damages difference between locating in the high-wage neighborhood and locating in the low-wage neighborhood. In short, the higher the level of initial inequality, the more unequal equilibrium pollution will be.

The final comparative static to examine is how l^* changes with changes in θ_3.

$$\frac{\partial l^*}{\partial \theta_3} = \frac{(\overline{w}_H - \overline{w}_L)(\theta_1 + \theta_2 - M)}{(2M - \theta_3)^2 (\overline{w}_H + \overline{w}_L)}.$$

The effect of a change in θ_3 on the steady state is ambiguous. Its sign depends on the sign of $\theta_1 + \theta_2 - M$. If $\theta_1 + \theta_2 > M$, then $\frac{\partial l^*}{\partial \theta_3} > 0$: a higher feedback parameter increases the share of polluting firms in the poor neighborhood. If $\theta_1 + \theta_2 < M$, then the reverse is true: $\frac{\partial l^*}{\partial \theta_3} < 0$. Note that, intriguingly, a larger feedback parameter—which mitigates the harm from pollution—ultimately ends up causing *more* pollution in the poor areas in the former case: something that seems like it would make things better for the poor actually can make them worse.

This is because an increase in θ_3 has two effects. First, it dampens the harm from pollution from the main effects. Because this dampening effect acts as a percentage of base wage, this effect will tend to make the high-wage neighborhood more attractive, as it will tend to see a larger absolute fall in damages from an increase in θ_3. The importance of this effect depends on the main effects, θ_1 and θ_2. If these are large, then the effect of an increase in θ_3 will be minimal since the fall in damages difference will be small compared to the total damages difference between the neighborhoods.

Second, an increase in θ_3 will make firms more sensitive to the past level of pollution, which will tend to push them towards the low-wage neighborhood. This second effect is closely tied to the proof of Result 1, which showed that if there has been policy snowballing in the previous period, there will be policy snowballing this period. This is because an increase in the share of firms in the low-wage neighborhood unambiguously decreases the cost of locating in the low-wage neighborhood. A higher θ_3 amplifies the decrease in the cost of locating in the low-wage neighborhood from policy snowballing. The impact of this second effect depends on M. With a high M, firm preferences are more dispersed, and hence the share of firms in the low-wage neighborhood is less sensitive to changes in the damages difference between the neighborhoods.

In sum, there are two effects that come out of an increase in θ_3. The first effect will tend to push firms away from the low-wage neighborhood, and the intensity of the effect is inversely related to the size of $\theta_1 + \theta_2$. The second effect will tend to push firms towards the low-wage neighborhood, and the intensity of the effect is inversely related to the size of M. Hence, the key for signing $\frac{\partial l^*}{\partial \theta_3}$ is the sign of $(\theta_1 + \theta_2 - M)$, a simple comparison of $\theta_1 + \theta_2$ and M. If $\theta_1 + \theta_2$ is bigger than M, then the second effect dominates and the steady-state share of firms in the low-wage neighborhood increases with an increase in θ_3. If not, then the reverse is true.

4.2 Speed of Convergence

With uniform firm preferences, we can also evaluate how fast the process converges to steady state. We measure speed of convergence by the rate of change of distance from the steady state; formally, we express this as $\frac{l_t - l^*}{l_{t+1} - l^*}$ (see, e.g., Süli and Mayers 2003). Loosely, the speed of convergence relates to the number of periods it takes to get "close" to the steady state. Assuming the steady state is an interior point, we can apply the law of motion and the formula for the steady state to measure the speed of convergence in terms of the parameters.

Result 4: *With firm preferences (8) and $l^* < 1$, then the speed of convergence is*

$$\frac{l_t - l^*}{l_{t+1} - l^*} = \frac{2M}{\theta_3}$$

See Appendix for proof.

A small $\frac{2M}{\theta_3}$ means that $l_{t+1} - l^*$ is close to $l_t - l^*$ and hence the distance from steady state changed little between period t and $t+1$.

The primary intuition for why the ratio $\frac{2M}{\theta_3}$ determines the speed of the convergence process is that the more sensitive that firm location choice is to changes in neighborhood firm shares, the more steps the process needs to converge to steady state; hence, the slower the process is.

The logic for why a higher M increases the speed of convergence is similar to why a higher M leads to fewer firms in the poor neighborhood in equilibrium. M governs the dispersion of firm preferences. A higher M means that aggregate firm location decisions are less sensitive to the changes in relative damages between the neighborhoods. If firm preferences are more dispersed, an increase in the share of firms in the low-wage neighborhood leads to fewer firms choosing to move into the low-wage neighborhood compared to a scenario with less dispersed firm preferences. A higher M, therefore, means that the process of converging to steady state is faster since firms are not very sensitive to changes in damage differences between the neighborhoods. The process needs fewer steps before settling down.

In contrast to its ambiguous effect on the equilibrium share of firms in the poor neighborhood, however, θ_3 has an unambiguously negative effect on the speed of convergence. Indeed, this result provides useful intuition into the dynamics of policy snowballing. Recall that θ_3 governs how sensitive the relative damages between neighborhoods are to changes in the share of firms in the neighborhoods. A higher θ_3 means that a given increase in the share of firms in the low-wage neighborhood leads to a larger decrease in the relative cost of locating in the low-wage neighborhood. Thus, firm location decisions are more sensitive to changes in the share of firms in the low-wage neighborhood. Because of this increased sensitivity, the process takes more steps before it settles down, and thus it is slower.

4.3 Accounting for Dynamics: What Static Compensation Misses

Typical analysis of legal rules is static, considering costs and benefits in one period. In this subsection, we consider what static accounting misses given the snowballing dynamics. Specifically, we consider the change from an inefficient, but more egalitarian, regime where half of the firms are required to be in each neighborhood to an efficient rule in which the firms are made to pay damages according to strict liability. We suppose that the legislature provides static compensation to the poor from this change. That is, we illustrate how much the legislature can miss by failing to account for the dynamic process of firm movement into poor areas first by measuring the gap between the one-period statically calculated compensation and the transfers needed to make up for the growth of income inequality in steady state. Then, we present a numerical example showing how the gap between statically calculated transfers and needed transfers can grow over time due to policy snowballing. Throughout, we suppress the formal expressions defining the different lump-sum taxes and transfers as well as derivations of results, leaving these instead for the Appendix.

4.3.1 The Gap Between Static and Dynamic Transfers

We consider a scenario in which the half of the firms had been locating in each neighborhood for many periods, perhaps because the government mandated this arrangement. Suppose that the government changes this legal regime so that now firms have strict liability for the harms from pollution, setting into motion the model and dynamics described above. We assume throughout that there is policy and income snowballing. The government wishes to compensate for the inequality that comes about from moving to the efficient regime. To do so, it observes how many firms moved into the low-wage neighborhood after the first period of the new regime. Then, it calculates the appropriate lump-sum transfer to make to return income inequality to its original level.

Specifically, after the first period of the new regime, the government observes the new share of firms in the low-wage neighborhood, l_1, and calculates a lump-sum transfer $\tau_{1,s}$ to return inequality to its original level. Having observed the share of firms in the low-wage neighborhood, for the rest of time the legislature assumes that the share of polluting firms in the low-wage neighborhood will remain at l_1. So for the second period and beyond, it picks a lump-sum transfer, τ_s, to keep inequality at its original level, assuming that the share of firms in the low-wage neighborhood will remain at l_1. We call τ_s the *static transfer*. This taxes-and-transfers scheme will fail to fully compensate for the change in income inequality, as it will not take into account the continuing policy and income snowballing that will occur period after period. Let $\tau_{d,t}$ be the transfer at time t that reproduces the original level of inequality. Call this the *dynamic transfer*.

We are interested in the *transfer gap*, which we define as the difference between the static and the dynamic transfer as a share of the dynamic transfer: $\frac{\tau_{d,t}-\tau_s}{\tau_{d,t}}$. Because from Result 2 we know that income inequality is increasing over time, it must also be that $\tau_{d,t}$ increases over time. As a result, the transfer gap will also increase over time. With the uniform firm preferences assumption, we can be more precise and compare what the transfer should be at steady state to preserve the original income inequality to the static transfer calculated above. Let τ_d be the steady-state lump-sum tax that produces income inequality in steady state equal to the original income inequality. Assuming that $\theta_3 > 0$, $\Omega < 1$, and that $l^* < 1$,[11] we calculate that the transfer gap as a fraction of the steady-state transfer is

$$\frac{\tau_d - \tau_s}{\tau_d} = \frac{2\left(l^* - l_1\right)}{2l^* - 1}.$$

A simple takeaway from this expression is that the further way l_1 is from l^*, the larger that gap is between the static transfers and correct dynamic transfers. Since both are determined by

[11] The first two assumptions ensure that there is policy and income snowballing, guaranteeing that $\tau_d \neq 0$. The last assumption allows us to substitute in the expression for l^* from Result 3.

model primitives, however, it is better to understand how the model parameters themselves determine this gap. In the Appendix, we show that this expression in terms of primitives is

Result 5:

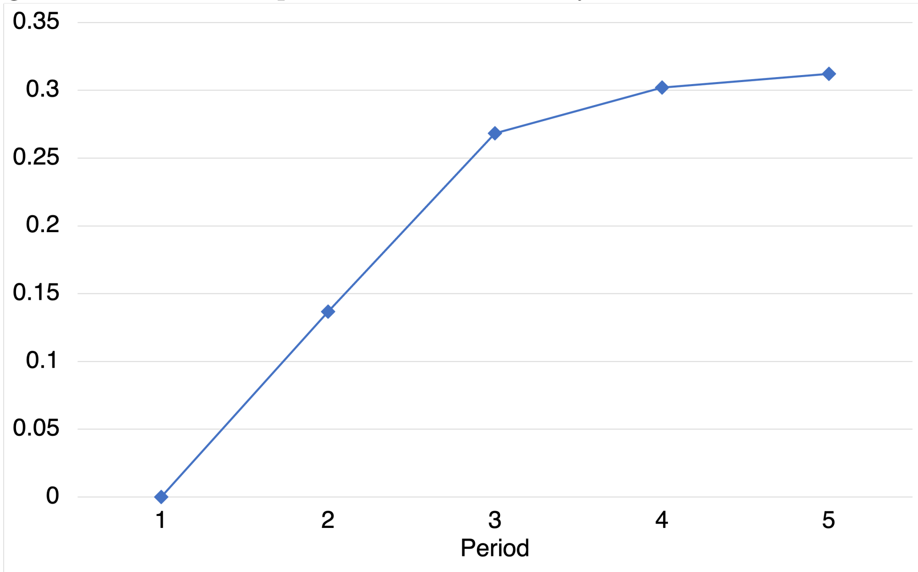
$$\frac{\tau_d - \tau_s}{\tau_d} = \frac{\theta_3}{2M}.$$

This expression demonstrates that the transfer gap as a share of the steady-state transfer is equal to the inverse of the speed of snowballing. A faster speed of convergence (i.e. a lower $\frac{\theta_3}{2M}$) will tend to produce smaller transfer gaps. Intuitively, this is because when convergence is faster there are fewer steps needed to get close to the steady state and, hence, l_1 will not be as far from l^*. As a result, the myopic legislature will miss the correct transfer by less when the convergence process is faster. Note that again, like with the equilibrium comparative statics, the feedback parameter also has a perverse effect here: though θ_3 mitigates the harm from pollution in the short run, it ends up causing a larger compensation gap because it also increases the uncompensated feedback drawing more firms into the poor neighborhood over time.

4.3.2 An Example

We present an example economy, showing how the gap between the static transfer and the transfer that correctly returns inequality to its original level evolve over a few periods. The example shows the large compensation gap that can arise due to ignoring snowballing. We use the following parameters: $\overline{w}_H = 100,000, \overline{w}_L = 25,000, \theta_1 = 0.2, \theta_2 = 0.2, \theta_3 = 0.19$, and $M = 0.3$. The ratio of rich income to poor income, our measure of income inequality, in this economy at time $t = 0$, when the share of firms in each neighborhood has been set to $\frac{1}{2}$, is 4. After the government institutes strict liability for damages, the steady state produces a level of income inequality of 4.83. When the government introduces the statically-calculated transfer from above, τ_s, the level of inequality is only 4.28. Figure 1 plots the evolution of $\frac{\tau_{d,t}-\tau_s}{\tau_{d,t}}$. over five periods. In the second period, the statically calculated transfer misses 15% of the correct transfer, and this number keeps increasing over time until it reaches 31.7% by

Figure 1: Transfer Gap As a Share of the Dynamic Transfer Over Time



the fifth period, at which point the share of firms in the low-wage neighborhood is close to its steady-state value and so the gap will not grow much after that. So, in the end, a large share—about a third—of the compensatory is missed by ignoring snowballing.

4.4 Welfare

In this subsection, we further quantify the importance of the snowballing and the failure to compensate by characterizing the welfare weights that would lead a social planner to prefer one regime to another. We mainly consider two regimes: the inefficient regime in which half of the firms locate in each neighborhood and the efficient regime without compensating transfers. We again assume here that $\theta_3 > 0$, meaning that there is policy snowballing, and that $\Omega < 1$, meaning that there is income snowballing and damages fail to fully compensate harmed individuals. Importantly, we find that it can be welfare-maximizing to adopt the *inefficient* policy, and we specify when that is the case.

Suppose all workers share an increasing, twice-differentiable utility function $u(y_i)$, where y_i is the income of person i. To think about welfare, we will consider the welfare weights on the low-wage individuals, g_L, and the welfare weight placed on the high-wage individuals,

g_H, that would justify choosing one regime over the other. Because all that matters is the size of the weights relative to each other, we set $g_H = 1$. We can then interpret g_L as how much more the social planner weights the utility of the low-wage individuals relative to the high-wage individuals. The social planner chooses the regime to maximize

$$g_L u\left(I_L\right) + u\left(I_H\right).$$

To simplify matters for characterizing the set of welfare weights that would lead the social planner to choose one regime over another, further assume that the individuals' utility function is linear in income. This focuses relative weighting of dollars in the hands of the poor versus the rich on only the welfare weights, rather than also mixing in the curvature of a utility function. Then we obtain the following result:

Result 6: *With a utility function that is linear in income, the social planner is indifferent between the efficient and inefficient regimes when the welfare weight $\hat{g}_L$ satisfies*

$$\hat{g}_L = \frac{\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]\overline{w}_H}{\left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]\overline{w}_L}.$$

Furthermore, the social planner would choose the efficient regime without transfers over the inefficient regime if

$$g_L < \frac{\overline{w}_H}{\overline{w}_L},$$

and she would choose the inefficient regime over the efficient regime without transfers if

$$g_L > \left(\frac{\theta_1 + \theta_2 - \frac{1}{2}\theta_3}{\theta_1 + \theta_2 - \frac{3}{2}\theta_3}\right)\left(\frac{\overline{w}_H}{\overline{w}_L}\right).$$

With a linear utility function, we can obtain a simple expression for the welfare weight that makes the social planner indifferent between the inefficient regime and the efficient regime without transfers. The expression for $\hat{g}_L$ shows that as the steady-state share of firms in the low-wage neighborhood increases, the indifference weight also increases. This follows simply from more pollution leading to lower wages. A higher steady-state share of firms in the low-wage neighborhood means a lower income in the low-wage neighborhood and a higher income in the high-wage neighborhood, increasing steady-state inequality. Because a higher level of steady-state inequality reflects a higher level of inefficiency in the initial regime, it takes a higher concern for the low-wage neighborhood to justify sustaining the inefficient regime.

Result 6 also shows a couple of cases in which one does not need to know the steady-state share of firms l^* to know which regime the social planner would prefer. If the preference for the low-wage neighborhood is smaller than the ratio of the high-wage base to the low-wage base, then in no circumstance would the social planner prefer the inefficient regime. In that case, the total utility gains from switching to the efficient regime always overcome any preference for the low-wage individuals. The other case shows that if the weight on the low-wage earner is high enough, greater than $\left(\frac{\theta_1+\theta_2-\frac{1}{2}\theta_3}{\theta_1+\theta_2-\frac{3}{2}\theta_3}\right)\left(\frac{\overline{w}_H}{\overline{w}_L}\right)$, then there is no circumstance in which the social planner would choose the efficient regime. No total utility gains are high enough to overcome the planner's objection to the rising inequality.

Finally, we can characterize the relationship among the indifference weights making each of three regimes optimal versus the efficient regime: the efficient regime without transfer, the efficient regime with myopic transfer (transfer τ_s), and the efficient regime with fully compensatory steady-state transfer (transfer τ_d).

Result 7: *Let $\hat{g}_{no-transfer}$ be the weight that makes the social planner indifferent between the inefficient regime and the efficient regime without transfers. Let $\hat{g}_{myopic}$ be the weight that makes the social planner indifferent between the inefficient regime and the efficient regime with the myopic transfer τ_s. Suppose that these exist.*[12] *Then we have the following relationship*

[12]Note that $\hat{g}_{myopic}$ may not exist if the static transfer to the low-wage individuals makes the total income of the low-wage in steady state higher than the income of the low-wage in the inefficient regime. In that case, the efficient regime with myopic transfer pareto dominates the inefficient regime.

$$\hat{g}_{no-transfer} < \hat{g}_{myopic}$$

Moreover, there is no welfare weight that would lead the social planner to prefer the inefficient regime over the efficient regime with the correct transfer.

Intuitively, because the transfer lowers steady-state inequality, the social planner would have to put a much bigger weight on the utility of the low-wage individuals to still prefer the inefficient regime compared to the the weight she would need to prefer the inefficient regime to the efficient regime without a transfer. And, because the dynamic transfer that keeps inequality at the same level as before means that both low-wage and high-wage individuals see a higher income than in the inefficient regime, the efficient regime with the dynamic transfer Pareto-dominates the inefficient regime. No social planner would prefer the latter to the former.

5 Extension: Labor Demand Impacts

The model so far has assumed that, when firms move into an area, workers are harmed because of pollution, but do not benefit because of increased labor demand. In this section, we relax this assumption. For simplicity (to avoid the complication of polluting firms paying higher wages because of increased labor demand), we still assume that the polluting firms do not employ anybody from the neighborhoods. Rather we imagine that the presence of the firms can lead to investment, local government revenue, or the transit of people from outside the neighborhoods that could positively impact labor demand. Of course, if workers benefitted because of increased labor demand, then that would partially offset the extent of inequality snowballing. We explain how our results change as a result of labor demand impacts.[13]

Suppose that wages increase by parameter ϕ such that the next period's wages are

[13] The proofs for the result changes are available upon request. Since they just replace one parameter and follow the same math that is already in the Appendix, we do not include them to economize on space.

$$w_{Nt} = \left[1 - (\theta_1 - \phi)\, n_t - \theta_2 n_{t-1} + \theta_3 n_t n_{t-1}\right] \overline{w}_N.$$

Furthermore, damages do not account for the firm's effect on labor demand. The damages to be paid from locating in neighborhood N are still:

$$D_{Nt} = (\theta_1 + \theta_2 - \theta_3 n_{t-1})\, \overline{w}_N.$$

Results 1, 3 and 4 are unchanged. These results depend only on firm decisions; since damages are the same as without the labor demand effects, these results are the same. Result 5 and 7 also remain the same. Even though both results involve income, which is affected by labor demand impacts, the labor demand impacts get cancelled out of the expressions.

Results 2 and 6 do change because they involve the income and do not have labor demand effects that are cancelled out. Below, we begin with the change to Result 2:

Result 2LD: Income snowballing with labor demand impacts. *If there is policy snowballing and $\phi < (1 - \Omega)(\theta_1 - \theta_3)$, then a necessary and sufficient condition for income snowballing is $\Omega < 1$.*

The second result now needs the additional condition that $\phi < (1 - \Omega)(\theta_1 - \theta_3)$. This guarantees that income always decreases with an increase in pollution. Note that the conditions that previously guaranteed that income decreased with increased pollution were $\theta_1 > \theta_3$ and $\theta_2 > \theta_3$. However, because an increase in current period pollution, n_t, now can also increase income by ϕ, we need to ensure that this increase is less than the decrease in income from current pollution: $(1 - \Omega)(\theta_1 - \theta_3)$. Hence, high enough labor demand effects may break the connection between policy snowballing and income snowballing.

Result 6 is modified in a similar way:

Result 6LD: *With a utility function that is linear in income, the social planner is* *indifferent between the regimes when the welfare weight* $\hat{g}_L$ *satisfies*

$$\hat{g}_L = \frac{\left(\phi - (1-\Omega)\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]\right)\overline{w}_H}{\left(\phi - (1-\Omega)\left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]\right)\overline{w}_L}.$$

Furthermore, we can give simpler characterizations of when the social planner would choose one regime over the other. She would choose the efficient regime without transfers over the inefficient if

$$g_L < \frac{\overline{w}_H}{\overline{w}_L},$$

and she would choose the inefficient regime over the efficient regime without transfers if

$$g_L > \frac{\phi - (1-\Omega)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{\phi - (1-\Omega)\left(\theta_1 + \theta_2 - \frac{3}{2}\theta_3\right)}\left(\frac{\overline{w}_H}{\overline{w}_L}\right).$$

There are two differences in the expression for the welfare weight. First, the expression now includes ϕ in the denominator and numerator. The within-neighborhood income differences across the two regimes still depend on the uncompensated damages, but the effect is now attenuated by labor demand impacts. For the low-income, the increase in pollution and attendant damages are dampened by the increase in labor demand from more firms moving into the neighborhood. Similarly, the boon to the high-income neighborhood from polluting firms moving out is dampened by the reduced labor demand. A higher labor demand effect increases the indifference welfare weight, since labor demand reduces the inequality effect of the efficient regime. The second difference is that the term for incomplete compensation, $(1-\Omega)$, is now in the welfare weight expression. Because the impact of labor demand does not depend on incomplete compensation, $(1-\Omega)$ can no longer be factored out of the numerator and denominator.

6 Factors Outside the Model

In the interest of parsimony, we did not include many features of reality in the model. These other factors could change the results. For example, we did not include the firms' profits in the model. Including a model for the firms' profits would allow us to give people in the neighborhoods ownership in the companies and see how the distribution of profits affects inequality snowballing. For example, if profits are generally positive and the people in the rich neighborhood own the majority of the companies, then that might exacerbate inequality snowballing as the rich would have another source of income advantage over the poor. Alternatively, if the poor owned most of the firms, the profits could compensate for the losses in wages or fewer firms would locate in the poor neighborhood as the firms would internalize the losses to wages from locating in the poor neighborhood.

Except for the policy change hypothetical, the model does not consider the availability of taxes and transfers, which could undo the income snowballing inequality, but not the policy snowballing. Another factor that is not modeled is mobility, which could mitigate the results by allowing people to escape the harm. The model also ignores the possibility of a correlation between firm preferences and local wages. And, the functional form of proportional harm to wages is important; if pollution caused the same dollar harm to rich and poor, the results would be different. This section is not intended to provide a comprehensive list of all the assumptions in the model (e.g., the model also assumes the same number of people live in rich and poor places), but rather to suggest that—while the model is quite general—more work remains to be done to determine the scope of that generality.

7 Other Possible Settings with Inequality Snowballing

Commenting on the degree to which the dynamics discussed in this article contribute to increasing inequality is beyond the article's scope. Nevertheless, it is worth discussing other policies that have features similar to the model here and could thus lead to inequality snowballing. Recall that there are three key policy features that can lead to inequality snowballing. In the specific model here, (1) wages affect application of a legal rule, (2) the legal rule in turn affects wages in the future, creating a feedback loop ($\theta_3 < 1$), and (3) compensation is incomplete ($\Omega < 1$). More generally, what is needed is that (1) parties' willingness to pay affects application of a legal rule, (2) the legal rule in turn affects willingness to pay in the future, creating a feedback loop, and (3) compensation is incomplete. And, of course, rich and poor people need to be differentiated somehow: by geography (rich vs. poor neighborhoods), by means of consumption (flying on airplanes vs. riding public busses), or otherwise. For example, a policy wherein (1) more resources are given to the rich because they are willing to pay more for them, (2) those resources in turn increase willingness to pay in the future, and (3) compensation is incomplete would satisfy all three conditions.

One example is cost-benefit analysis of transportation infrastructure. When allocating funding for competitive grant programs, the federal Department of Transportation requires cost-benefit analysis that places a higher value on time saved in high-speed rail and airports than in bus lines because the former are used largely by richer people and the latter largely by poorer people.[14] The analysis follows federal requirements for producing efficient regulations by measuring individuals' willingness to pay using their wages because of the time value of money. Consider how such policymaking follows the analysis in this paper. If an individual has low wages, then her willingness to pay for transportation is lower. As a result, the government neglects transportation that benefits the poor and invests, all else equal, in more transportation that benefits the rich. This results in longer commutes for the poor, which may increase fatigue or limit their ability to pursue training or educational opportunities. In turn, this reduces productivity and lowers wages (and, in any case, arguably reduces willingness to pay because of the relatively reduced wealth of the poor). The analysis then repeats in the next period, leading to snowballing inequality in the absence of compensation to the poor for the smaller amount of transportation spending that they receive. This

[14]See explanation in Liscow (2018a), p. 1688-91.

possibility is consistent with recent research suggesting the importance of transportation for income mobility (Chetty and Hendren, 2018).

Another set of policies that is promising for exhibiting snowballing are those in which wealth alone drives the vicious cycle, since greater wealth tends to increase willingness to pay. Consider, for example, pollution that harms housing values, leading to a tort (Chay and Greenston, 2005; Currie et al., 2015). To the extent that housing values reflect willingness to pay to avoid pollution, the rich will be able to recover a larger amount than the poor. This would deter other polluters from locating nearby, thereby forcing more polluters onto poor neighborhoods, disproportionately reducing the property values—and therefore—the wealth of the poor. In subsequent periods, the poor would be even further immiserated relative to the rich. Similar analysis applies outside of torts, to the panoply of regulatory areas (e.g., zoning and administrative approvals) in which governments decide on the siting of polluting facilities. And, indeed, many argue that such facilities are disproportionately sited in poor areas (Been, 1993).

Similar analysis could also be applied to eminent domain and redevelopment efforts. Poor people may be more likely to be subjected to eminent domain because their homes are worth less, and eminent domain may in turn result in uncompensated income losses as people's lives are disrupted through displacement.

This analysis can be flipped as well: When cities are analyzing where to build parks, they may look at the economic benefits as reflected by increases in housing values. As a result, richer areas are more likely to have parks built in them, generating increases in property values and thus wealth that would in turn drive greater demand for amenities in the future—and thus yet more spending on amenities in the richer areas.

One could imagine a whole host of other mechanisms: Poorer people are—all else equal—willing to pay less for road safety, effective policing, good hospitals, and communications infrastructure, which could affect willingness to pay in subsequent periods through wages or wealth. But the point here is not to lay out the range of areas in which such a mechanism could be at play, but rather to suggest that the range could be significant.

At the same time, there are many circumstances in which there would not be policy snowballing because of the lack of a feedback effect θ_3. We note here some cases where we would not expect policy snowballing. Doing so helps emphasize that the presence of snowballing is not obvious and that the setting developed here can help shed light on where it occurs. Most basically, there are efficient legal rules that do not disproportionally benefit the rich—what Liscow (2018a) calls "neutral" efficient legal rules. For example, consider a tort in which a polluter causes damages to a laundromat, requiring that the laundromat purchase an air purifier for \$10,000 and thereby causing economic losses of \$10,000. It does not matter whether the laundromat is owned by a poor person or a rich person; the damages are \$10,000. Since there is no bias, there is no policy snowballing.

Likewise, for there to be policy snowballing, there must be a particular kind of feedback loop, in which the legal rule leads to harms to the thing (like wages) that determines the application of the legal rule in the next period. Consider, for example, a modified case of the main example in the article; here, regulators are deciding where to locate power plants producing pollution that reduces life expectancy but does not reduce one's working life or productivity. In this case, there would be no positive feedback loop because, even as the poor had more reduction in their lifespan because of the pollution, there would not be a disproportionate reduction in the poor's willingness to pay for additional life since their financial resources are constant. If anything, there may be a negative feedback as the years of life for the poor become scarcer and therefore more valuable.

8 Conclusion

This article is a proof of possibility for and exploration of how efficient policies can lead to a vicious cycle by harming the wages of the poor over time solely through the operation of the efficient legal rule. The model has three essential features: First, more of a disamenity like pollution (or, equivalently, less of an amenity) is allocated to the poor because they are willing to pay less than the rich to avoid it. Second, that disproportionate allocation in turn disproportionately reduces the willingness to pay of the poor to avoid the disamenity, setting up a feedback loop. These two together lead to what we call "policy snowballing," or spiraling disproportionate reductions in wages for the poor, leading to more pollution on them, and so on. And third, if there is incomplete compensation to offset these distributional consequences, then income snowballing can result. At least in principle, many legal rules may satisfy these conditions. In any case, it is important to know the size of the taxes and transfers needed to compensate for the distributional impacts of efficient legal rules. And static compensation may miss considerable dynamic harm.

Whether such dynamics are at play in the real world is a question beyond the scope of this article. More research should be done on this question. Theoretical work on other policy settings would be valuable for pinpointing the most credible settings for snowballing, determining what empirical parameters are most important to measure, and developing testable empirical implications. Empirically, qualitative legal research on when in fact efficient legal rules are used in relevant settings would be very valuable. Quantitative empirical work would also be useful. For example, there may be natural experiments available for testing for the presence of snowballing. One possible setting is how changes in state law (or federal Circuit splits) relate to subsequent changes in either the allocation of the amenity or disamenity (for policy snowballing) or income inequality (for income snowballing).

In the meantime, these results raise the stakes of such work, by showing that a common policy goal can have such perverse distributional consequences. It is helpful for policymakers to know the distributional impacts from the adoption of efficient legal rules so that they can appropriately compensate the various parties. And compensating for only the static harm may not nearly compensate for the actual, dynamic harm over time to the poor that results from the policy snowballing demonstrated here.

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Appendix

Proof of Result 1: Conditions for Policy Snowballing

To prove Result 1, first we establish that there is always an increase of firms in the low-wage neighborhood in the first period. Because the share of firms in either neighborhood is evenly split at $t = 0$, i.e. $l_0 = \frac{1}{2}$, there is an increase in the share of firms in the low-wage neighborhood in the first period when $l_1 > \frac{1}{2}$. Using the law of motion, this means there will be policy snowballing in the first period if $F\left((\overline{w}_H - \overline{w}_L)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)\right) > \frac{1}{2}$. Since the firm preference distribution has median zero by assumption, the condition for policy snowballing in the first period amounts to

$$(\overline{w}_H - \overline{w}_L)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right) > 0.$$

So long as there is a difference in permanent wage level between the two neighborhoods ($\overline{w}_H > \overline{w}_L$), the term $\theta_1 + \theta_2 - \frac{1}{2}\theta_3$ determines whether or not there is policy snowballing in the first period. Note that the assumptions in (2) imply that $\theta_1 + \theta_2 - \theta_3 > 0$. This inequality and the assumption $\theta_3 \geq 0$ further imply that $\theta_1 + \theta_2 - \frac{1}{2}\theta_3 > 0$. So after the first period, there is always policy snowballing.

Next, we show that if there is an increase in the share of firms in the low-wage neighborhood at period $t \geq 2$ i.e. $l_t > l_{t-1}$, then it must be that $\theta_3 > 0$. Applying the law of motion (Equation 6) and the expression for the damage difference (Equation 5) to $l_t > l_{t-1}$ implies that:

$$
\begin{aligned}
& F\left(G\left(l_{t-1}\right)\right) > F\left(G\left(l_{t-2}\right)\right) \\
\implies & G\left(l_{t-1}\right) > G\left(l_{t-2}\right) \\
\iff & \left(\overline{w}_H + \overline{w}_L\right)\theta_3\left(l_{t-1} - l_{t-2}\right) > 0 \\
\implies & \theta_3 > 0
\end{aligned}
$$

Finally we show that if $\theta_3 > 0$, then there is policy snowballing. We proceed first by induction. We have already demonstrated that $l_1 > l_0$. Now, suppose that $l_{t-1} > l_{t-2}$. We will show that this implies that either $l_t > l_{t-1}$ or, if $l_{t-1} = 1$, then $l_t = 1$. In either case, we have that $\theta_3 > 0 \implies \left(\overline{w}_H + \overline{w}_L\right)\theta_3\left(l_{t-1} - l_{t-2}\right) > 0$. From above, we know that $\left(\overline{w}_H + \overline{w}_L\right)\theta_3\left(l_{t-1} - l_{t-2}\right) > 0 \iff G\left(l_{t-1}\right) > G\left(l_{t-2}\right)$. Consider now the first case, $l_{t-1} < 1$. This means that, because the pdf of F has compact support, $F\left(G\left(l_{t-1}\right)\right) > F\left(G\left(l_{t-2}\right)\right)$. Hence, by the law of motion, $l_t > l_{t-1}$. Now suppose that $l_{t-1} = 1$. Then $G\left(l_{t-1}\right) > G\left(l_{t-2}\right) \implies F\left(G\left(l_{t-1}\right)\right) = F\left(G\left(l_{t-2}\right)\right)$ by the properties of cdfs. So, $l_t = l_{t-1} = 1$. Putting it all together, by the principle of induction we have that for all t where $l_{t-1} > l_{t-2}$, either $l_t > l_{t-1}$ or $l_t = l_{t-1} = 1$. To fully conclude the proof for policy snowballing, note that if $l_{t-1} = l_{t-2} = 1$, then $G\left(l_{t-1}\right) = G\left(l_{t-2}\right) \implies l_t = l_{t-1} = 1$. But since we know that $l_1 > l_0$ the first t such that $l_t = 1$ must have that $t \geq 2$.

Proof of Result 2: Conditions for Income Snowballing:

Using the definition of income (7) and the formula for wages at time t from (1) gives that the level of inequality at time $t = 0$ is

$$
\begin{aligned}
Q_0 = \frac{I_{H0}}{I_{L0}} &= \frac{\left[1 - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2 + \frac{1}{4}\theta_3\right]\overline{w}_H + \Omega\left(\left(\theta_1 - \frac{1}{2}\theta_3\right)\frac{1}{2} + \frac{1}{2}\theta_2\right)\overline{w}_H}{\left[1 - \frac{1}{2}\theta_1 - \frac{1}{2}\theta_2 + \frac{1}{4}\theta_3\right]\overline{w}_L + \Omega\left(\left(\theta_1 - \frac{1}{2}\theta_3\right)\frac{1}{2} + \frac{1}{2}\theta_2\right)\overline{w}_L} \\
&= \frac{\overline{w}_H}{\overline{w}_L}.
\end{aligned}
$$

The initial inequality is just the ratio of the permanent wages. This means that the polluting firms have no effect on income inequality (relative to the no pollution baseline) when half of the firms are located in each neighborhood for at least two consecutive periods.

In subsequent periods, inequality is

$$Q_t = \frac{1-(1-\Omega)\left(h_t\theta_1 + h_{t-1}\theta_2 - h_t h_{t-1}\theta_3\right)}{1-(1-\Omega)\left(l_t\theta_1 + l_{t-1}\theta_2 - l_t l_{t-1}\theta_3\right)} \cdot \frac{\overline{w}_H}{\overline{w}_L}.$$

Note that $\Omega = 1 \implies Q_t = \frac{\overline{w}_H}{\overline{w}_L}$. Hence, $Q_t > Q_{t-1} \implies \Omega < 1$; $\Omega < 1$ is necessary for income snowballing.

Suppose that $\Omega < 1$, $l_1 < 1$, and there is policy snowballing. We will first show that when $l_{t-1} < l_t$, then $Q_{t-1} < Q_t$. We begin by establishing that $l_{t-2} < l_{t-1} < l_t$ implies both that $I_{L,t-1} > I_{Lt}$ (the income of the low-wage individuals decreases from $t-1$ to t) and $I_{H,t-1} < I_{Ht}$ (the income of the high-wage individuals decreases from $t-1$ to t). The expression $I_{L,t-1} - I_{Lt}$ can be expressed as:

$$\begin{aligned} I_{L,t-1} - I_{Lt} &= 1-(1-\Omega)\left(l_{t-1}\theta_1 + l_{t-2}\theta_2 - l_{t-1}l_{t-2}\theta_3\right) - 1 + (1-\Omega)\left(l_t\theta_1 + l_{t-1}\theta_2 - l_t l_{t-1}\theta_3\right) \\ &= (1-\Omega)\left(\left(l_t - l_{t-1}\right)\theta_1 + \left(l_{t-1} - l_{t-2}\right)\theta_2 - \left(l_t - l_{t-2}\right) l_{t-1}\theta_3\right) \end{aligned}$$

The $1-\Omega$ is positive since Ω <1. We focus on the other term. Re-arrange it as

$$\left(\frac{(l_t - l_{t-1})}{(l_t - l_{t-2})}\theta_1 + \frac{(l_{t-1} - l_{t-2})}{(l_t - l_{t-2})}\theta_2 - l_{t-1}\theta_3\right)\left(l_t - l_{t-2}\right).$$

The term $\frac{(l_t - l_{t-1})}{(l_t - l_{t-2})}\theta_1 + \frac{(l_{t-1} - l_{t-2})}{(l_t - l_{t-2})}\theta_2$ is a convex combination of θ_1 and θ_2. Since both are larger than θ_3 and $l_{t-1}\theta_3 < \theta_3$, we can conclude that $I_{L,t-1} - I_{Lt} > 0$ and hence that the income of the low-wage individuals decreased from $t-1$ to t.

One can show $I_{H,t-1} < I_{Ht}$ through similar logic. $l_{t-2} < l_{t-1} < l_t$ implies that $h_{t-2} >$

$h_{t-1} > h_t$. We can use this to sign $I_{Ht} - I_{Ht-1}$:

$$
\begin{aligned}
I_{Ht} - I_{Ht-1} &= 1 - (1-\Omega)\left(h_t\theta_1 + h_{t-1}\theta_2 - h_t h_{t-1}\theta_3\right) - 1 + (1-\Omega)\left(h_{t-1}\theta_1 + h_{t-2}\theta_2 - h_{t-1}h_{t-2}\theta_3\right) \\
&= (1-\Omega)\left(\left(h_{t-1} - h_t\right)\theta_1 + \left(h_{t-2} - h_{t-1}\right)\theta_2 - \left(h_{t-2} - h_t\right)h_{t-1}\theta_3\right) \\
&= (1-\Omega)\left(h_{t-2} - h_t\right)\left(\frac{\left(h_{t-1} - h_t\right)}{\left(h_{t-2} - h_t\right)}\theta_1 + \frac{\left(h_{t-2} - h_{t-1}\right)}{\left(h_{t-2} - h_t\right)}\theta_2 - h_{t-1}\theta_3\right) \\
&> 0.
\end{aligned}
$$

Knowing that $l_{t-2} < l_{t-1} < l_t \implies I_{L,t-1} > I_{Lt}$ and $I_{H,t-1} < I_{Ht}$ allows us to conclude that $Q_t > Q_{t-1}$, income inequality increased from $t-1$ to t, since $Q_t = \frac{I_{Ht}}{I_{Lt}}$ and income is always positive (since wages are always at least zero).

Next, we show that $Q_0 < Q_1$. The critical term for signing $I_{L0} - I_{L1}$ is

$$
\left(\frac{\left(l_1 - \frac{1}{2}\right)}{\left(l_1 - \frac{1}{2}\right)}\theta_1 + \frac{\left(\frac{1}{2} - \frac{1}{2}\right)}{\left(l_1 - \frac{1}{2}\right)}\theta_2 - \frac{1}{2}\theta_3\right)\left(l_1 - \frac{1}{2}\right) = \left(\theta_1 - \frac{1}{2}\theta_3\right)\left(l_1 - \frac{1}{2}\right).
$$

Since $\theta_1 > \theta_3$ and we know that $l_1 > l_0 = \frac{1}{2}$, we can conclude that $I_{L0} > I_{L1}$. Similar reasoning allows us to conclude that $I_{H0} < I_{H1}$ and therefore that $Q_0 < Q_{1.}$. Thus, we have shown that whenever $l_{t-1} < l_t$, then $Q_{t-1} < Q_t$.

To finish the proof, we deal with the possibility that the policy snowballing is of the kind in which for some $\hat{t} \geq 2$, $l_t = 1$ for all $t \geq \hat{t}$. We thus have that $l_{\hat{t}-1} < l_{\hat{t}} = l_{\hat{t}+1} = 1$. To sign $Q_{\hat{t}+1} - Q_{\hat{t}}$, we need to sign $I_{L\hat{t}} - I_{L,\hat{t}+1}$, which depends on the sign of:

$$
\left(\frac{(1-1)}{\left(1 - l_{\hat{t}-1}\right)}\theta_1 + \frac{\left(1 - l_{\hat{t}-1}\right)}{\left(1 - l_{\hat{t}-1}\right)}\theta_2 - \theta_3\right)\left(1 - l_{\hat{t}-1}\right) = \left(1 - l_{\hat{t}-1}\right)\left(\theta_2 - \theta_3\right).
$$

Since $1 > l_{\hat{t}-1}$ and $\theta_2 > \theta_3$, we can conclude that the income of the low-wage neighborhood decreased from $\hat{t}$ to $\hat{t}+1$, while the income of those in the high-wage neighborhood increased. Hence, $Q_{\hat{t}+1} > Q_{\hat{t}}$: inequality also increased. For all $t > \hat{t}+1$, $l_{t-2} = l_{t-1} = l_t = 1$; for those t, income inequality will not be changing.

Proof of Result 3: Expression for the Law of Motion and Proof of Convergence to Steady State.

Applying the CDF of the uniform distribution gives the following form for the law of motion:

$$l_t = \begin{cases} 0 & \text{if } G\left(l_{t-1}\right) < -M \cdot \left(\overline{w}_H + \overline{w}_L\right) \\ T\left(l_{t-1}\right) & \text{if } -M \cdot \left(\overline{w}_H + \overline{w}_L\right) \leq G\left(l_{t-1}\right) \leq M \cdot \left(\overline{w}_H + \overline{w}_L\right) \\ 1 & \text{if } G\left(l_{t-1}\right) > M \cdot \left(\overline{w}_H + \overline{w}_L\right) \end{cases} \tag{9}$$

where

$$T\left(l_{t-1}\right) = \frac{1}{2} + \frac{\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \overline{w}_H \theta_3}{2M\left(\overline{w}_H + \overline{w}_L\right)} + \frac{\theta_3}{2M} l_{t-1}. \tag{10}$$

Let l^* denote the steady state. Let $\hat{l}$ be the value such that $T\left(\hat{l}\right) = \hat{l}$. This proof will proceed in multiple steps. First, we will argue that $\hat{l}$ such that $T\left(\hat{l}\right) = \hat{l}$ exists and is unique when $\theta \neq 2M$. Second, we will show that $\hat{l} > \frac{1}{2}$ implies that $\theta_3 < 2M$. Third, we will use last fact to show that if $\hat{l} \in \left(\frac{1}{2}, 1\right)$, then $\hat{l} = l^*$. Fourth, we will show that if $\hat{l} > 1$,then $l^* = 1$. Fifth, we end by showing that because $\hat{l} < \frac{1}{2}$ implies that $\theta_3 > 2M$, then $l^* = 1$.

1) Existence and Uniqueness of $\hat{l}$: Because $\theta \neq 2M$, $T(x) = \frac{1}{2} + \frac{(\overline{w}_H - \overline{w}_L)(\theta_1 + \theta_2) - \overline{w}_H \theta_3}{2M(\overline{w}_H + \overline{w}_L)} + \frac{\theta_3}{2M} x$ is a line with slope not equal to one. Therefore, it will intersect with $f(x) = x$ at one and only one point, meaning that an $\hat{l}$ such that $T\left(\hat{l}\right) = \hat{l}$ exists and is unique. Specifically, the expression for $\hat{l}$ is

$$\hat{l} = \frac{M\left(\overline{w}_H + \overline{w}_L\right) + \left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \overline{w}_h \theta_3}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)}.$$

2) $\hat{l} > \frac{1}{2} \implies \theta_3 < 2M$: Using the expression from above, if $\hat{l} > \frac{1}{2}$ then

$$
\begin{aligned}
& \frac{M\left(\overline{w}_H + \overline{w}_L\right) + \left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \overline{w}_h \theta_3}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)} > \frac{1}{2} \\
\implies \quad & \frac{M\left(\overline{w}_H + \overline{w}_L\right) + \left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \overline{w}_h \theta_3}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)} - \frac{1}{2} > 0 \\
\implies \quad & \frac{2\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \theta_3\left(\overline{w}_H - \overline{w}_L\right)}{2\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)} > 0 \\
\implies \quad & \frac{2\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{2\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)} > 0 \\
\implies \quad & 2M - \theta_3 > 0.
\end{aligned}
$$

All but the last of lines above follow from simple algebra. The last line follows from the constraint $\theta_1 + \theta_2 - \theta_3 > 0$. That constraint implies that $\theta_1 + \theta_2 - \frac{1}{2}\theta_3 > 0$. So, in the second-to-last line, for the left-hand expression to be positive, it must be that $2M - \theta_3 > 0$. Thus $\hat{l} > \frac{1}{2} \implies \theta_3 < 2M$.

3) $\hat{l} \in \left(\frac{1}{2}, 1\right) \implies \hat{l} = l^*$: To show this result, we first argue that the linear first-order difference equation $l_t = T(l_{t-1}) = \frac{1}{2} + \frac{(\overline{w}_H - \overline{w}_L)(\theta_1+\theta_2) - \overline{w}_H\theta_3}{2M(\overline{w}_H + \overline{w}_L)} + \frac{\theta_3}{2M} l_{t-1}$ has a unique, globally stable steady state. Formally, a steady state $\widehat{l}$ of a first-order difference equation is globally stable if for all possible initial conditions, $l_0 \in (-\infty, \infty)$, the associated sequence l_t induced by the first-order difference equation system converges to $\hat{l}$. The steady state is globally unique because the linear process has slope not equal to one. By the second result in this proof, we know that $\theta_3 < 2M$. This means that the slope of $l_t = T\left(l_{t-1}\right)$ is less than one. By theorems from the mathematics of linear difference equations, this means that the steady state $\hat{l}$ is globally stable.[15] In particular, we know that from a starting point $l_0 = \frac{1}{2}$, the first-order difference system $l_t = T(l_{t-1})$ will produce a sequence that converges to $\hat{l}$.

All that is left to show is that because $l_t = T(l_{t-1})$ converges to $\hat{l}$, the first-order difference equation described in (9) must also converge to $\hat{l}$. On the interval $(0, 1)$, the two systems are

[15] For example, see Theorem 2.2 in Acemoglu (2009).

identical. All we have to do is show that an edge case of $l_t = 0$ or $l_t = 1$ never occurs. This follows from Result 1 because it implies that l_t must be strictly increasing. Because $l_0 > \frac{1}{2}$ and $\hat{l} < 1$, this means that $l_t \in (\frac{1}{2}, 1)$ and so an edge case is never reached. Therefore, the steady state of the difference equation (9) is equal that of $l_t = T(l_{t-1})$ i.e $\hat{l} = l^*$.

4) $\hat{l} \geq 1 \implies l^* = 1$: Using the logic from the proof of part 3, if $\hat{l} \geq 1$, then the sequence produced by $l_0 = \frac{1}{2}$ and the linear first-order difference equation (9) is strictly increasing until $l_t = 1$. At that point, the share of firms can no longer change and the sequence remains there for all future time periods. Hence, $l^* = 1$.

5) $\hat{l} < \frac{1}{2} \implies l^* = 1$: Using the same argument as in part 2 of this proof, if $\hat{l} < \frac{1}{2}$ then it must be that $2M < \theta_3$. Thus, the slope of $l_t = T\left(l_{t-1}\right)$ will be greater than one in absolute value. By the theorems of first-order difference equations, this means that the steady state $\hat{l}$ will be unstable; all sequences with starting points $l_0 \neq \hat{l}$ will move away from the steady state. Since $\hat{l} < \frac{1}{2}$ and $l_0 = \frac{1}{2}$, this means that once again the sequence l_t produced by first-order difference equation (9) will be strictly increasing until it hits the edge case $l_t = 1$, where it will subsequently remain. Thus, $\hat{l} < \frac{1}{2} \implies l^* = 1$.

Expressions for Comparative Statics

The expression for the steady-state share of firms in the low-wage neighborhood is

$$l^* = \frac{M\left(\overline{w}_H + \overline{w}_L\right) + \left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \overline{w}_h \theta_3}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)}.$$

The partial derivatives of l^* are

$$
\begin{aligned}
\frac{\partial l^*}{\partial \theta_1} &= \frac{(\overline{w}_H - \overline{w}_L)}{(2M - \theta_3)(\overline{w}_H + \overline{w}_L)} \\
\frac{\partial l^*}{\partial \theta_2} &= \frac{(\overline{w}_H - \overline{w}_L)}{(2M - \theta_3)(\overline{w}_H + \overline{w}_L)} \\
\frac{\partial l^*}{\partial M} &= -\frac{2(\overline{w}_H - \overline{w}_L)\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{(2M - \theta_3)^2(\overline{w}_H + \overline{w}_L)} \\
\frac{\partial l^*}{\partial \overline{w}_H} &= \frac{2\overline{w}_L\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{(2M - \theta_3)(\overline{w}_H + \overline{w}_L)^2} \\
\frac{\partial l^*}{\partial \overline{w}_L} &= -\frac{2\overline{w}_H\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)}{(2M - \theta_3)(\overline{w}_H + \overline{w}_L)^2} \\
\frac{\partial l^*}{\partial \theta_3} &= \frac{(\overline{w}_H - \overline{w}_L)(\theta_1 + \theta_2 - M)}{(2M - \theta_3)^2(\overline{w}_H + \overline{w}_L)}
\end{aligned}
$$

Proof of Result 4: Speed of Convergence

We solve for the speed of snowballing in terms of model primitives using the equation for l^* when it is an interior solution and the law of motion.

$$
\begin{aligned}
\frac{l_t - l^*}{l_{t+1} - l^*} &= \frac{l_t - \frac{M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3}{(2M-\theta_3)(\overline{w}_H+\overline{w}_L)}}{\frac{M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3+\theta_3(\overline{w}_H+\overline{w}_L)l_t}{2M(\overline{w}_H+\overline{w}_L)} - \frac{M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3}{(2M-\theta_3)(\overline{w}_H+\overline{w}_L)}} \\
&= \frac{(\overline{w}_H + \overline{w}_L)\, l_t - \frac{M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3}{(2M-\theta_3)}}{\frac{M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3+\theta_3(\overline{w}_H+\overline{w}_L)l_t}{2M} - \frac{M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3}{(2M-\theta_3)}} \\
&= \frac{2M(2M-\theta_3)(\overline{w}_H+\overline{w}_L)\, l_t - 2M\left[M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3\right]}{(2M-\theta_3)\,\theta_3(\overline{w}_H+\overline{w}_L)\, l_t - \theta_3\left[M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3\right]} \\
&= \frac{2M\left[(2M-\theta_3)(\overline{w}_H+\overline{w}_L)\, l_t - \left[M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3\right]\right]}{\theta_3\left[(2M-\theta_3)(\overline{w}_H+\overline{w}_L)\, l_t - \left[M(\overline{w}_H+\overline{w}_L)+(\overline{w}_H-\overline{w}_L)(\theta_1+\theta_2)-\overline{w}_h\theta_3\right]\right]} \\
&= \frac{2M}{\theta_3}
\end{aligned}
$$

Proof of Result 5: Expressions for Lump-Sum Transfers and the Transfer Gap

The legislature selects the transfer in the first period, $\tau_{1,s}$ to satisfy the equation

$$\frac{I_{H1}\left(\frac{1}{2},1-l_1\right)-\tau_{1,s}}{I_{L1}\left(\frac{1}{2},l_1\right)+\tau_{1,s}}=\frac{\overline{w}_H}{\overline{w}_L}.$$

Here, the I_{N1} functions come from Equation 7 and reflect the fact that in the previous period the share of firms in each neighborhood was $\frac{1}{2}$. $\tau_{1,s}$ returns the level of income inequality to its original level, $\frac{\overline{w}_H}{\overline{w}_L}$.

The myopic legislature picks τ_s in the second period and beyond to satisfy

$$\frac{I_{Ht}\left(1-l_1,1-l_1\right)-\tau_s}{I_{Lt}\left(l_1,l_1\right)+\tau_s}=\frac{\overline{w}_H}{\overline{w}_L}.$$

Here, the legislature assumes that the share of firms in the low-wage neighborhood will remain at l_1 for the rest of time. Operating under that assumption, it chooses the lump-sum tax-and-transfer scheme to return the world to its original level of inequality.

The steady-state level transfer required to return the world to its original level of inequality is τ_d and solves:

$$\frac{I_{Ht}\left(1-l^*,1-l^*\right)-\tau_d}{I_{Lt}\left(l^*,l^*\right)+\tau_d}=\frac{\overline{w}_H}{\overline{w}_L}.$$

We begin calculating the transfer gap by first calculating the ratio of myopic transfer to the steady-state transfer:

$$\begin{aligned}
\frac{\tau_s}{\tau_d} &= \frac{\left[I_{Ht}\left(1-l_1,1-l_1\right)-\frac{\overline{w}_H}{\overline{w}_L}I_{Lt}\left(l_1,l_1\right)\right]\left(1+\frac{\overline{w}_H}{\overline{w}_L}\right)^{-1}}{\left[I_{Ht}\left(1-l^*,1-l^*\right)-\frac{\overline{w}_H}{\overline{w}_L}I_{Lt}\left(l^*,l^*\right)\right]\left(1+\frac{\overline{w}_H}{\overline{w}_L}\right)^{-1}} \\
&= \frac{\left(1-2l_1\right)\left(\theta_1+\theta_2-\theta_3\right)\left(1-\Omega\right)}{\left(1-2l^*\right)\left(\theta_1+\theta_2-\theta_3\right)\left(1-\Omega\right)} \\
&= \frac{2l_1-1}{2l^*-1}.
\end{aligned}$$

From the expression for l^* from Result 3, which we can freely apply since we assume $l^*<1$, we obtain an expression for the denominator:

$$
\begin{aligned}
2l^* - 1 &= \frac{2M\left(\overline{w}_H + \overline{w}_L\right) + 2\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{w}_h\theta_3}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)} - \frac{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)} \\
&= \frac{2\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{w}_h\theta_3 + \theta_3\left(\overline{w}_H + \overline{w}_L\right)}{\left(2M - \theta_3\right)\left(\overline{w}_H + \overline{w}_L\right)}.
\end{aligned}
$$

From the law of motion, we obtain the expression for the numerator:

$$
\begin{aligned}
2l_1 - 1 &= 2\left(\frac{1}{2} + \frac{\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - \overline{w}_H\theta_3}{2M\left(\overline{w}_H + \overline{w}_L\right)} + \frac{\theta_3}{2M}l_0\right) - 1 \\
&= \frac{2\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{w}_H\theta_3}{2M\left(\overline{w}_H + \overline{w}_L\right)} + \frac{\theta_3}{2M} \\
&= \frac{2\left(\overline{w}_H - \overline{w}_L\right)\left(\theta_1 + \theta_2\right) - 2\overline{w}_H\theta_3 + \theta_3\left(\overline{w}_H + \overline{w}_L\right)}{2M\left(\overline{w}_H + \overline{w}_L\right)}.
\end{aligned}
$$

Combining the two expression gives

$$
\begin{aligned}
\frac{\tau_s}{\tau_d} &= 1 - \frac{\theta_3}{2M} \\
\implies \frac{\tau_d - \tau_s}{\tau_d} &= \frac{\theta_3}{2M}
\end{aligned}
$$

Proof of Result 6: Social Planner's Preference for the Efficient Regime Over the Inefficient Regime

The income in neighborhood N under the inefficient regime is

$$
I_{N0} = \left[1 - (1 - \Omega)\left(\frac{1}{2}\theta_1 + \frac{1}{2}\theta_2 - \frac{1}{4}\theta_3\right)\right]\overline{w}_N.
$$

Under the strict liability regime without transfers, the steady-state income level in neighborhood N is

$$
I_N^* = \left[1 - (1 - \Omega)\left(n^*\theta_1 + n^*\theta_2 - \left(n^*\right)^2\theta_3\right)\right]\overline{w}_N.
$$

The welfare weight $\hat{g}$ that makes the social planner indifferent between the inefficient regime and the efficient regime without transfers satisfies

$$\hat{g_L} u\left(I_{L0}\right) + u\left(I_{H0}\right) = \hat{g_L} u\left(I_L^*\right) + u\left(I_H^*\right).$$

Re-arranging gives:

$$\hat{g}_L = \frac{u\left(I_H^*\right) - u\left(I_{H0}\right)}{u\left(I_{L0}\right) - u\left(I_L^*\right)}.$$

Assuming that utility is linear in income. Let u' be the slope of utility with respect to income. Then the condition becomes:

$$\hat{g}_L = \frac{u' \cdot \left(I_H^* - I_{H0}\right)}{u' \cdot \left(I_{L0} - I_L^*\right)} = \frac{I_H^* - I_{H0}}{I_{L0} - I_L^*}.$$

Plugging in the corresponding values for the incomes gives

$$\hat{g}_L = \frac{(1-\Omega)\left(\frac{1}{2} - h^*\right)\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]\overline{w}_H}{(1-\Omega)\left(l^* - \frac{1}{2}\right)\left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]\overline{w}_L}.$$

Because $l^* + h^* = 1$, then $\frac{1}{2} - h^* = l^* - \frac{1}{2}$. So, the condition boils down to

$$\hat{g}_L = \frac{\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]\overline{w}_H}{\left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]\overline{w}_L}.$$

This proves the first claim of the result.

Next, note that following the logic from above, the social planner prefers the efficient regime when she has welfare weight g_L such that

$$g_L < \frac{\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]\overline{w}_H}{\left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]\overline{w}_L}.$$

Since $l^* > \frac{1}{2}$, then it must be that

$$1 < \frac{\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]}{\left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]}.$$

So, if

$$g_L < \frac{\overline{w}_H}{\overline{w}_L},$$

then it must follow that $g_L < \frac{\left[\theta_1+\theta_2-(\frac{3}{2}-l^*)\theta_3\right]}{\left[\theta_1+\theta_2-(l^*+\frac{1}{2})\theta_3\right]}\frac{\overline{w}_H}{\overline{w}_L}$; and the social planner would prefer the efficient regime regardless of the value of l^*.

For the last result, observe that the indifference expression $\frac{\left[\theta_1+\theta_2-(\frac{3}{2}-l^*)\theta_3\right]}{\left[\theta_1+\theta_2-(l^*+\frac{1}{2})\theta_3\right]}\frac{\overline{w}_H}{\overline{w}_L}$ is increasing in l^*. Because $l^* \leq 1$, if the welfare weight satistfies

$$g_L > \frac{\left[\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right]}{\left[\theta_1 + \theta_2 - \frac{3}{2}\theta_3\right]}\frac{\overline{w}_H}{\overline{w}_L}$$

then regardless of the steady-state share of firms in the low-wage neighborhood the social planner will always prefer the inefficient regime without transfers to the ineffcient regime.

Result 7: Comparing the Different Regimes

Recall from above that the weight that makes the social planner indifferent between the inefficient regime and the efficient regime without transfer is

$$\hat{g}_{no-transfer} = \frac{I_H^* - I_{H0}}{I_{L0} - I_L^*}.$$

The income expression with the transfer simply adds the transfer to the income of the low-wage individuals and subtracts it from the income of the high-wage people. So, the corresponding expression for the welfare weight that makes the social planner indifferent between the inefficient regime and the efficient regime with the myopic transfer is

$$\hat{g}_{myopic} = \frac{I_H^* - \tau_s - I_{H0}}{I_{L0} - I_L^* - \tau_s}.$$

The assumption that $\hat{g}_{myopic}$ exists means the low-wage do not make a higher income in the steady state with the myopic transfer than in the inefficient regime. Hence, $I_{L0} - I_L^* - \tau_s > 0$. Now, evaluate the difference between the two:

$$
\begin{aligned}
\hat{g}_{myopic} - \hat{g}_{no-transfer} &= \frac{(I_H^* - \tau_s - I_{H0})\,(I_{L0} - I_L^*)}{(I_{L0} - I_L^* - \tau_s)\,(I_{L0} - I_L^*)} - \frac{(I_H^* - I_{H0})\,(I_{L0} - I_L^* - \tau_s)}{(I_{L0} - I_L^*)\,(I_{L0} - I_L^* - \tau_s)} \\
&= \frac{\tau_s\,(I_H^* - I_{H0}) - \tau_s\,(I_{L0} - I_L^*)}{(I_{L0} - I_L^* - \tau_s)\,(I_{L0} - I_L^*)} \\
&= \frac{\tau_s\,(1-\Omega)\left(l^* - \frac{1}{2}\right)\left(\left[\theta_1 + \theta_2 - (\frac{3}{2} - l^*)\theta_3\right]\overline{w}_H - \left[\theta_1 + \theta_2 - (l^* + \frac{1}{2})\theta_3\right]\overline{w}_L\right)}{(I_{L0} - I_L^* - \tau_s)\,(I_{L0} - I_L^*)} \\
&= \frac{\tau_s\,(1-\Omega)\left(l^* - \frac{1}{2}\right)\left(\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)(\overline{w}_H - \overline{w}_L) - \overline{w}_H\theta_3 + l^*\,(\overline{w}_H + \overline{w}_L)\,\theta_3\right)}{(I_{L0} - I_L^* - \tau_s)\,(I_{L0} - I_L^*)}.
\end{aligned}
$$

Note that the expression in the last line is increasing in l^*. Because $l^* > \frac{1}{2}$, this means that if the expression $\left(\left(\theta_1 + \theta_2 - \frac{1}{2}\theta_3\right)(\overline{w}_H - \overline{w}_L) - \overline{w}_H\theta_3 + l^*\,(\overline{w}_H + \overline{w}_L)\,\theta_3\right)$ is greater than 0 when evaluated at $l^* = \frac{1}{2}$, then it is always greater than 0. Evaluated at $\frac{1}{2}$, this expression is $(\theta_1 + \theta_2 - \theta_3)\,(\overline{w}_H - \overline{w}_L)$, which is greater than 0 since $\theta_1 > \theta_3$. Thus, $\hat{g}_{myopic} - \hat{g}_{no-transfer} > 0$.