NEW YORK UNIVERSITY SCHOOL OF LAW

COLLOQUIUM ON TAX POLICY
AND PUBLIC FINANCE
SPRING 2013

"Taxation, Risk, and Portfolio Choice: The Treatment of Returns to Risk Under a Normative Income Tax"

Jake Brooks
Professor of Law
Georgetown Law

February 5, 2013 (Tuesday)
NYU School of Law
Vanderbilt Hall-208
Time: 4:00-5:50pm
Number 3

SCHEDULE FOR 2013 NYU TAX POLICY COLLOQUIUM
(All sessions meet on Tuesdays from 4-5:50 pm in Vanderbilt 208, NYU Law School)

1. January 22 – David Kamin, NYU Law School, "Are We There Yet?: On a Path to Closing America's Long-Run Deficit."

2. January 29 – Edward McCaffery, USC Law School, "Bifurcation Blues: The Problems of Leaving Redistribution Aside."

3. **February 5 – Jake Brooks, Georgetown Law, "Taxation, Risk, and Portfolio Choice: The Treatment of Returns to Risk Under a Normative Income Tax."**

4. February 12 – Lilian Faulhaber, Boston University School of Law, "Charitable Giving, Tax Expenditures, and the Fiscal Future of the European Union."

5. February 26 – Peter Diamond, MIT Economics Department, "The Case for a Progressive Tax: From Basic Research to Policy Recommendations."

6. March 5 – Darien Shanske, University of California at Hastings College of Law, "A Proposal for a New Property Tax Infrastructure."

7. March 12 – Dhammika Dharmapala, U. of Illinois Law School, "Competitive Neutrality among Debt-Financed Multinational Firms."

8. March 26 – Sarah Lawsky, University of California at Irvine Law School, "Unknown Probabilities and the Tax Law."

9. April 2 – Alan Viard, "American Enterprise Institute, Progressive Consumption Taxation: The Choice of Tax Design."

10. April 9 – Brian Galle, Boston College Law School, "A Nudge is a Price."

11. April 16 – Leslie Robinson, Tuck Business School, Dartmouth College, "Internal Ownership Structures of Multinational Firms."

12. April 23 – Larry Bartels, Department of Political Science, Vanderbilt University, "Inequality as a Political Issue in the 2012 Election."

13. April 30 – Itai Grinberg, Georgetown Law School, "A Governance Structure to Mediate the Battle Over Taxing Offshore Accounts."

14. May 7 – Raj Chetty, Harvard Economics Department, "Active vs. Passive Decisions and Crowd-Out in Retirement Savings Accounts: Evidence from Denmark."

Taxation, Risk, and Portfolio Choice: The Treatment of Returns to Risk Under a Normative Income Tax

66 TAX L. REV. ___ (forthcoming 2013)

John R. Brooks II[†]

Draft of January 14, 2013
Please do not quote or cite without permission

Abstract

Many articles in the legal and economic literature claim that a pure Haig-Simons income tax cannot effectively tax investment income. This is because an investor can use leverage to gross up her investments in risky assets such that the increased gain (or loss) exactly offsets any income tax (or deduction) on the returns to risk-taking. This article argues, however, that while it is possible *for an investor to make such portfolio shifts, she almost certainly will not because of the increased risk of doing so.*

Central to any discussion of the effects of taxation on investment risk-taking is the meaning of risk *itself. The central claim of this article is that a better conception of investment risk is the* risk of loss *and not merely the* variance *of returns. Applying this notion of risk—one that is well supported in the finance literature but new to the taxation-and-risk literature—to an investor's portfolio choice question shows that an investor will not increase her investment in risky assets by enough to offset the tax. As a result, there is an effective tax on investment risk-taking under a normative income tax.*

† Associate Professor of Law, Georgetown University Law Center. I am grateful to Jennifer Bird-Pollan, Stephen Cohen, Lillian Faulhaber, Mihir Desai, Michael Doran, Michael Graetz, Itai Grinberg, Daniel Halperin, Louis Kaplow, Wojciech Kopczuk, Alex Raskolnikov, David Schizer, Stephen Shay, Theodore Sims, Joshua Teitelbaum, David Walker, Alvin Warren, Ethan Yale, Kathryn Zeiler, and participants in workshops at Columbia Law School, Georgetown University Law Center, and Harvard Law School for helpful conversations, comments, and suggestions. I am also especially grateful for the valuable research assistance of Yingchen Luo and Eric Remijan.

I. Introduction

It is commonly accepted in the tax law literature that a normatively "pure" income tax—also referred to as a Haig-Simons income tax—does not tax returns to risk (the "Domar-Musgrave result").[1] Under an income tax, it is argued, investors will build portfolios that generate the same after-tax return as if the tax fell only on the risk-free rate of return and exempted the risk

[1] Domar and Musgrave being the progenitors of the taxation and risk literature. Evsey D. Domar & Richard A. Musgrave, *Proportional Income Taxation and Risk-Taking*, 58 Q.J. ECON. 388 (1944).

premium.[2] Indeed, it has been shown that, under certain strong assumptions, an income tax is equivalent to a tax only on wages plus the risk-free return to capital.[3] From this result, some scholars conclude that a normative income tax does not tax investment risk-taking at all,[4] and thus that attempts to tax returns from investment risk taking—"risky returns"—are misguided.[5]

This paper will argue, by contrast, that even if a normative income tax and a tax on the risk-free return are equivalent, it does not follow that there is no tax[6] on risky returns. Under plausible assumptions about investor risk preferences a normative income tax will indeed tax risky returns.

In the Domar-Musgrave result, in order to completely erase the tax on risky returns, investors must fully "gross up"—that is, investors must reallocate their portfolios toward risky assets by enough for the increased expected return to pay the expected tax. However, an investor will fully gross

---

[2] *See, e.g.,* Alvin C. Warren Jr., *Would a Consumption Tax be Fairer Than an Income Tax*, 89 YALE L.J. 1081 (1979) [hereinafter Warren, *Consumption Tax*]; Alvin C. Warren Jr., *How Much Capital Income Taxed under an Income Tax Is Exempt under a Cash Flow Tax?*, 52 TAX L. REV. 1 (1996) [hereinafter Warren, *Capital Income*]. For the components of capital income *see infra* note 22.

[3] Louis Kaplow, *Taxation and Risk Taking: A General Equilibrium Perspective*, 47 NAT'L TAX J. 789, 792-93 (1994).

[4] *See, e.g.,* Joseph Bankman & Barbara H. Fried*, Winners and Losers in the Shift to a Consumption Tax*, 86 GEO. L.J. 539, 541-42 (1998) ("[M]arginal returns to risk [are] arguably subject to a zero rate of tax [under an income tax]."); Joseph Bankman & Thomas Griffith, *Is the Debate Between and Income Tax and a Consumption Tax a Debate About Risk? Does It Matter?*, 47 TAX L. REV. 377, 378 (1992) ("[U]nder certain assumptions, investors in risky assets are able to offset the effects of government taxation of the risk premium by changing their investment portfolios."); Noël B. Cunningham, *The Taxation of Capital Income and the Choice of Tax Base*, 52 TAX. L. REV. 17, 21 (1996) ("[T]he income tax will not reach the premium a sophisticated investor receives for investing in risky investments."); David Elkins & Christopher H. Hanna, *Taxation of Supernormal Returns*, 62 TAX LAW. 93, 93 (2008) ("As is generally accepted, under certain assumptions an accrual income tax system taxes the risk-free rate of return on capital but does not tax the risk premium."); Adam H. Rosenzweig, *Imperfect Financial Markets and the Hidden Costs of a Modern Income Tax*, 62 SMU L. REV. 239, 241-42 (2009) ("[U]nder certain assumptions, relatively simple changes by both taxpayers and the government can result in risky returns … avoiding the impact of an income tax altogether."); Daniel Shaviro, *Replacing the Income Tax With a Progressive Consumption Tax*, 2004 TAX NOTES 91, 101-02 ("[D]ue to portfolio adjustments, an income tax fails to affect either ex ante risk premiums or ex post risky outcomes."); David A. Weisbach, *The (Non)Taxation of Risk*, 58 TAX. L. REV. 1, 2 (2004). ("[I]ndividuals, even in a Haig-Simons system, can, and will, eliminate the tax on [the return to bearing risk].").

[5] *See, e.g.,* Cunningham, *supra* note 4, at 20; Deborah Schenk, *Saving the Income Tax With a Wealth Tax*, 53 TAX L. REV. 423, 435 (2000); Weisbach, *supra* note 4, at 2-3.

[6] I use the term "tax" here to describe not only the nominal tax itself, but also the effects on expected returns due to portfolio shifts. *See infra* Part IV. To be clear, I am not referring to excess burden or deadweight loss. Although the full cost of the tax is partly because of portfolio shifts, those shifts also cause a direct one-to-one increase in government revenues. Thus one could think of the full "tax" as simply the government revenues from the policy. *See, e.g.,* William M. Gentry & R. Glenn Hubbard, *Implications of Introducing a Broad-Based Consumption Tax*, 11 TAX POL'Y & ECON. 1, 7 (1997).

up only if the tax does not change either a) the risk-aversion of the investor or b) the overall risk of her portfolio. Neither is the case. Even a tax on the risk-free return will make an investor poorer and thus likely to be more risk-averse than in the absence of the tax.[7] In addition—and central to this article's argument—the tax will expose the investors to *greater risk of loss* than they would assume in a world with no tax. As a result of the changes to risk aversion and portfolio risk, investors will not shift their portfolio investments sufficiently toward risky assets to offset the full effects of the income tax—they will not fully gross up their investment in risky assets in order to achieve the same after-tax returns as if there were no tax. Thus, a taxpayer will end up paying an effective tax on risky returns, even under a pure normative income tax.

The first effect mentioned above—increased risk aversion due to lower expected wealth—is often known as the "wealth effect," and is a well-understood prediction of expected utility theory.[8] The wealth effect has been known to the economic literature on taxation and risk for some time,[9] though it makes only brief appearances in the legal literature.[10] Because the tax will necessarily reduce wealth as compared to the no-tax world, we would not expect an investor to try to recreate the same portfolio risk as before the tax.

---

[7] This effect is described in the economic literature as decreasing relative risk aversion or decreasing absolute risk aversion, depending on the specific behaviors. The general idea is that a person with less wealth will also have less appetite for risk—losing $100 is much worse for a person who only has $500 in wealth vs. $500,000. *See, e.g.,* Louis Kaplow, *Accuracy, Complexity, and the Income Tax*, 14 J. L. ECON. & ORG. 61, 74 (1998); Theodore S. Sims, *Capital Income, Risky Investments, and Income and Cash-Flow Taxation*, at 19-21 (2008) (unpublished manuscript, on file with author).

[8] Expected utility theory is the standard economic account of decision-making under uncertainty. *See, e.g.,* John A. List & Michael S. Haigh, *A Simple Test of Expected Utility Theory Using Professional Traders*, 102 PROC. NAT'L ACAD. SCI. U.S. 945, 945 (2005) ("Expected utility (EU) theory remains the dominant approach for modeling risky decision-making and has been considered the major paradigm in decision making since World War II."). Much of the economic literature approaches the taxation-and-risk question through an expected utility framework, with the key exception of Domar and Musgrave's paper. To be clear, in what follows below I use some tools of expected utility theory to examine the taxation-and-risk question, but my analysis is not limited to that theory. Indeed, I also rely on portfolio choice models that, while strongly supported, do not comply with all the assumptions of expected utility theory. *See infra* Part III.

[9] *See, e.g.,* ANTHONY B. ATKINSON & JOSEPH E. STIGLITZ, *Taxation and Risk-Taking*, in LECTURES ON PUBLIC ECONOMICS 97, 106-07 (1980); AGNAR SANDMO, *The Effects of Taxation on Savings and Risk Taking*, in 1 HANDBOOK ON PUBLIC ECONOMICS 265, 295-97 (Alan J. Auerbach & Martin Feldstein eds., 1985); Agnar Sandmo, *Portfolio Theory, Asset Demand and Taxation: Comparative Statics with Many Assets*, 44 REV. ECON. STUD. 369, 377 (1977).

[10] *See, e.g.,* Weisbach, *supra* note 4, at 18 (noting the existence of a wealth effect under both an income tax and a tax on the risk-free return); Sims, *supra* note 7; Ethan Yale, *The Cary Brown Theorem and the Income Taxation of Risk: A Reappraisal* (2005) (unpublished manuscript, on file with author); Lawrence Zelenak, *The Sometimes-Taxation of the Returns to Risk-Bearing Under a Progressive Income Tax*, 59 SMU L. REV. 879, 895 (2006).

However, there is still the question of portfolio risk itself. Much of the tax law literature approaches the taxation-and-risk question as essentially a portfolio choice question. In doing so, the literature implicitly claims that an investor will measure the risk of an investment portfolio only by its *variance*, that is, the volatility of potential returns around an expected return, or mean,[11] and will attempt to hold variance constant, or with a small adjustment for wealth effects. While variance[12] is a common measure of portfolio risk, however, it has well-known flaws and does not reflect actual investor risk preferences,[13] nor does it capture more rigorous conceptions of risk.[14] Indeed, even the most orthodox models of portfolio choice do not suggest that an investor should hold variance constant in the face of a tax that lowers expected returns.

The major problem with variance is that it measures only volatility, and thus implies, *inter alia*, that a risk-averse investor dislikes above-normal returns just as much as below-normal returns. It also measures only dispersion around the mean, not the size of potential losses. Other risk measures, such as those focusing on *risk of loss* better capture these more realistic concerns of investment risk-taking.

As this article will show, replacing variance with a downside risk measure in the Domar-Musgrave result leads to the conclusion that there is an effective tax on risky returns, and a larger one than predicted by considering the wealth effect alone.[15] Thus, this article uses more nuanced ideas of portfolio theory and risk management to correct the existing conclusion of most of the taxation-and-risk legal literature: A normative income tax will effectively tax returns to investment risk-taking.

In the course of making this argument, this article will also provide the first extensive discussion in the legal literature of some of the competing conceptions and measures of investment risk that have been developed in the financial economics and mathematical risk literature, along with these

---

[11] Variance is defined as the expected value of squared deviations from the expected return. Thus if $p(s)$ is the probability of each scenario and $r(s)$ is the actual return in each scenario, variance is: $\sigma^2 = \sum_s p(s)[r(s) - E(r)]^2$

See ZVI BODIE, ALEX KANE, AND ALAN J. MARCUS, INVESTMENTS 129 (9th ed. 2011).

[12] Or standard deviation, which is the square root of the variance.

[13] *See* STEPHEN F. LEROY & JAN WERNER, PRINCIPLES OF FINANCIAL ECONOMICS 183 (2001) ("[V]ariance does not in general provide an accurate measure of risk."); HARRY M. MARKOWITZ, PORTFOLIO SELECTION: EFFICIENT DIVERSIFICATION OF INVESTMENTS 194 (1970) (suggesting that analyses based on semi-variance, a measure of downside risk, "tend to produce better portfolios than those based on [variance]," but that "[v]ariance is superior with respect to cost, convenience, and familiarity"); *infra* Part III.A.

[14] *See infra* Part III.

[15] *See, e.g.,* Yale, *supra* note 10, at TK (deriving a relatively small effective tax rate under an expected utility approach to the Domar-Musgrave result).

measures' particular strengths and weaknesses.[16] In addition to serving this article's arguments, this discussion is also relevant to scholars of investment management, trust, and fiduciary law, and to legal scholarship generally. The practice of law is, after all, largely about managing risk, and legal scholarship has not generally engaged with the implications of some of the more sophisticated ways of quantifying and measuring risk.

If a normative income tax does in fact tax risky returns, what are the implications? The taxation-and-risk question is relevant to, among other things, the comparison between an income tax and a consumption tax, and in particular to the cash-flow tax version of a consumption tax.[17] Some scholars have argued that a normative income tax reaches so little capital income as to be vanishingly close to a cash-flow consumption tax.[18] Thus, David Weisbach argues, supporters of a more pure Haig-Simons income tax ought to in fact prefer a cash-flow consumption tax to our imperfect income tax system.[19]

However, if an income tax does reach capital income, the theoretical relationship between an income tax and a cash-flow consumption tax changes in important ways. If an income tax taxes capital income, then it will raise more revenue than a cash-flow tax at the same rate, because the tax base is larger—it includes labor and capital, not just labor. For the two tax systems to raise the same revenue, the cash-flow tax rate must be higher than the income tax rate. However, the additional tax will fall on wages, rather than capital income.[20]

Thus, a cash-flow consumption tax places a higher burden on labor income while largely exempting capital income, while an income tax can place a lower burden on labor income because it also captures some tax revenue from capital income. While this result is consistent with the conventional view

---

[16] *See infra* Part III.

[17] A consumption tax means a tax levied on a tax base of consumption (as opposed to a tax levied on a tax base of income, estate size, wealth, or other tax base). Typical consumption taxes include retail sales taxes and value-added taxes (VATs), but can also include wage taxes and cash-flow taxes. To see that a wage tax is equivalent to a consumption tax, consider the Haig-Simons definition of income as consumption plus changes in wealth: $Y = C + \Delta W$. HENRY C. SIMONS, PERSONAL INCOME TAXATION: THE DEFINITION OF INCOME AS A PROBLEM OF FISCAL POLICY 50 (1938). Thus the difference between a consumption tax base and an income tax base is the inclusion of changes in wealth, or savings. But because total income is essentially a combination of labor income and capital income, the exemption of savings is also the difference between a comprehensive income tax and a wage tax. Thus the two are equivalent. *See* E. Cary Brown, *Business-Income Taxation and Investment Incentives*, in INCOME, EMPLOYMENT AND PUBLIC POLICY: ESSAYS IN HONOR OF ALVIN H. HANSEN 300 (1948); William D. Andrews, *A Consumption-Type Or Cash Flow Personal Income Tax*, 87 HARV. L. REV. 1113 (1974) (showing that a cash-flow tax is a consumption tax); Kaplow, *supra* note 3, at 793 (showing equivalence of consumption and wage taxes).

[18] *See supra* note 4.

[19] *See* Weisbach, *supra* note 4, at 2-3.

[20] *See infra* Part V.A

that a consumption tax is likely to be less progressive than an income tax in practice, to my knowledge it has not before been argued under the strong assumptions of the taxation-and-risk literature.

In addition, if an investor only partially grosses up in the face of an income tax, the tax system will end up treating winners and losers differently ex post. One defense of an income tax over a consumption tax is that it focuses on ex post results, rather than ex ante expectations.[21] The existing taxation-and-risk literature challenges that view by arguing that an income tax will not be successful in reflecting ex post differences as a result of ex ante risky investments. But where there is a real, material tax on risky returns, as I argue here, we would see different treatment of winners and losers.

This article proceeds as follows. Part II explains the Domar-Musgrave result and discusses why some descriptions of the result implicitly adopt variance as a measure of investment risk. Part III reviews different conceptions of investment risk, emphasizing that economists and mathematicians have long understood that variance is a simplified measure of investment risk not suited to all applications. It will also discuss other risk measures that focus instead on risk of loss, especially the increasingly dominant Value at Risk risk measure. Part IV will return to the numerical examples from Part II and show that the Domar-Musgrave result changes when using a downside risk measure. Part IV will also address the question of the appropriate risk-free rate of return. This article's argument depends in part on the risk-free rate being materially greater than zero, and there are good reasons to believe it is. Part V extends the result to consider the comparison between an income tax and a cash-flow consumption tax. Part VI concludes.

II. The Domar-Musgrave Result

The taxation-and-risk literature has taken several different approaches to showing the potential effects of taxation on investment risk-taking.[22] One

[21] *See infra* Part V.B.

[22] Investment income can be thought of as containing three elements: a risk-free return, a risky return, and an inframarginal return. *See* Cunningham, *supra* note 4, at 23; Gentry & Hubbard, *supra* note 6, at 2 (though Gentry & Hubbard consider the ex post return, and thus also consider the actual "lucky" return, not just the expected risk premium). The risk-free return is essentially a time-value-of-money return, and is equal to the return from investing in risk-free or virtually risk-free assets, such as U.S. Treasury bonds. The risky return is the potentially greater, but more variable, return from investing in a risky asset, such as stocks. However, the total return from a risky asset includes a risk-free, or time-value-of-money, return as well. Thus the expected risky return is essentially the risk premium above a risk-free return that investors demand from risky assets. The inframarginal return is an above-normal return, even above the risk premium, that is sometimes available because of limited, unique opportunities. *See, e.g.,* Cunningham, *supra* note 4, at 23; Weisbach, *supra* note 4, at 19. However, in perfect capital markets (which is the setting for this model), we would not expect to see inframarginal returns.

strand of the literature essentially takes a portfolio choice approach, showing how to potentially build an optimal portfolio given an income tax (the "portfolio approach"). This approach is typical in the legal literature[23] and was also used by Domar and Musgrave in their original work on the subject.[24] Another strand applies expected utility theory, the standard economic model that addresses choice under uncertainty (the "expected utility approach"). This approach is more typical in the economics literature,[25] but has also appeared in the legal literature.[26] The idea of a wealth effect originates in the expected utility approach. Finally, a third strand shows the algebraic equivalence[27] of an income tax and a tax on the risk-free rate of return, but without reliance on assumptions about portfolio behavior or investor utility (the "equivalence approach"). This approach is associated particularly with Louis Kaplow and Alvin Warren.[28]

Each approach leads to slightly different expressions of the Domar-Musgrave result. The portfolio approach usually concludes that *an income tax does not tax risky returns*.[29] The expected utility approach reaches the same conclusion, provided that the risk-free rate is zero. When the risk-free rate is positive, the expected utility approach predicts a wealth effect and thus concludes that *an income tax partially taxes risky returns*,[30] with the degree of

---

Such returns are essentially economic rents due to market power, particular skills, particular access to investment opportunities, or unique ideas. *See* Cunningham, *supra* note 4, at 23; Elkins & Hanna, *supra* note 4. As such, they are either returns to labor or returns to capital due to imperfect markets or information asymmetries. While the existence of inframarginal returns should play an important role in policy discussions, I do not consider them here, and instead focus only on risk-free and risky returns. Furthermore, inframarginal returns are thought to be taxed under a consumption tax, *see* Cunningham, *supra* note 4, at 24; Warren, *Capital Income*, *supra* note 2, at 4-6; Weisbach, *supra* note 4, at 23, and therefore need not be part of the comparison between a normative income tax and a normative consumption tax.

[23] *See, e.g.,* Cunningham, *supra* note 4; Schenk, *supra* note 5; Shaviro, *supra* note 4; Weisbach, *supra* note 4.

[24] *See* Domar & Musgrave, *supra* note 1.

[25] *See supra* note 9.

[26] *See* Sims, *supra* note 7, Yale, *supra* note 10.

[27] Kaplow has defined equivalence of two tax regimes for these purposes as follows:

(1) in every state of nature, individuals have the same after-tax wealth in period 1 under both regimes;

(2) in every state of nature, the government has the same revenue in period 1 under both regimes; and

(3) total investment in each asset in period 0 is the same under both regimes.

See Louis Kaplow, *Taxation and Risk-Taking*, NBER Working Paper No. 3079, at 6 (1991). By "total investment" Kaplow means the total market-wide investment in the asset. *Id.* n.7. Thus the fact that investors shift their portfolios does not upset the equivalence, because, as discussed below, the government makes offsetting shifts in its portfolio such that total investment is unchanged.

[28] *See* Kaplow, *supra* note 3; Warren, *Capital Income*, *supra* note 2.

[29] *See, e.g.,* Weisbach, *supra* note 4, at 2.

[30] *See supra* note 9.

taxation dependent on the investor's risk aversion. Finally, the equivalence approach concludes that *an income tax is equivalent to a wage tax plus a tax on the risk-free return*.[31]

Not all of these conclusions can be true, of course (provided that the risk-free rate is positive). At first glance, it may seem that the first and third strands, the portfolio approach and the equivalence approach, reach the same conclusion: if an income tax is equivalent to a tax only on the risk-free return, then would it not follow that risky returns are untaxed? But in fact the two conclusions are not the same. Even if we assume that an income tax is equivalent to a wage tax plus a tax on the risk-free return, it does not follow that risky returns are untaxed, as this article will show.[32]

The appeal of the portfolio approach is its explanatory power without resort to mathematical abstractions. As this Part will show, this approach typically involves an investor making shifts in her portfolio in response to the income tax. But therein is also the essential problem with the portfolio approach; by skipping critical math, it often skims over implicit assumptions that ignore the role of risk. Thus, writers rarely emphasize that they are assuming (unrealistically) constant relative risk aversion,[33] or that variance is their implied choice of risk measure.[34]

For the sake of clarity, this article will also generally follow the portfolio approach, but while engaging the important question of risk. I begin with examples of the basic portfolio approach as typically presented in the literature, starting first with the taxpayer's perspective, then turning to the government's. However, when we return to these examples in Part IV, I will show how measuring risk differently changes the result. I will also show, however, that my conclusion does not upset the robust conclusion of the equivalence approach, that an income tax is equivalent to a wage tax plus a tax on the risk-free rate of return.

A. The Model

In order to isolate the effects of taxation on risk and risk-taking, the standard model used in the literature assumes a simplified and idealized version of an income tax. Thus, we assume the tax base to be comprehensive Haig-Simons income,[35] taxed on an accrual basis.[36] Furthermore, the tax must

---

31 *See, e.g.,* Kaplow, *supra* note 3, at 792; Warren, *Capital Income*, *supra* note 2, at 8-15.
32 *See infra* Part IV.A.
33 *But see* Weisbach, *supra* note 4, at 18 (discussing the wealth effect where there is decreasing relative risk aversion). Constant relative risk aversion means that person will not change the portion of their wealth invested in risky assets as wealth changes. *See supra* note 7.
34 *See infra* notes 70 & 71.
35 "Personal income may be defined as the algebraic sum of (1) the market value of rights exercised in consumption and (2) the change in value of the store of property rights between

be proportionate, that is, the tax allows full offsetting of losses (as opposed to the limitation of losses under the current income tax system[37]); the tax has only a single rate; and the government participates in the market by actively managing a portfolio of risk-free and risky investments.[38] In addition, we assume only two assets, a risk-free and a risky asset (e.g., a Treasury bond and a stock) and no constraints on borrowing or lending.[39] In short, the only factors that will affect the tax are the tax rate, the risk-free rate, the return distribution on the risky asset, and the particular risk preferences of the investor. Later, I discuss the importance of some of these assumptions.[40]

B. Taxpayer Perspective

In brief, the intuition behind the Domar-Musgrave result is that the imposition of a proportionate income tax narrows the potential gains and losses from a risky investment, because the government takes a portion of the gains and covers a portion of the losses. The government becomes in effect a partner in the risky venture, taking on part of the risk. This, in turn, can allow investors to take on more of that risky investment, possibly enough so that the increased potential return offsets, at least somewhat, the imposition of the tax (and likewise, the increased potential loss is offset by the increased deduction for the loss). We start first with the case where the risk-free rate is 0%[41]:

> *Example 1*: Consider a risky asset A with a 50% chance of returning 30% and a 50% chance of losing 10%, and a risk-free asset B that returns 0%. There is no tax. Asset A therefore has an after-tax

---

the beginning and end of the period in question." SIMONS, *supra* note 17, at 50. I.e., income is defined as consumption plus changes in wealth, or $Y = C + \Delta W$. *See supra* note 17.

36 In contrast to our actual realization-based system. *See* I.R.C. § 1001(a) (calculating gain as amount "realized" over adjusted basis).

37 I.R.C. §§ 1211(b), 1212.

38 *See* Cunningham, *supra* note 4, at 32 n.56; Schenk, *supra* note 5, at 432 n.44; Kaplow, *supra* note 3, at 794.

39 *See* Kaplow, *supra* note 3, at 790-91.

40 *See infra* Part II.D.

41 For consistency with later examples I treat the investor as owning a risk-free asset that yields 0%. For example, the investor could simply hold cash as the risk-free asset. However, a more intuitive example might be where an investor held only risky assets, but could borrow at 0% to gross up.

To an investor with a portfolio of risk-free assets (e.g., Treasury bonds) and risky assets, selling the risk-free assets is equivalent to borrowing, assuming that his borrowing rate is the same as the bond interest rate. In each case, the net cost to the investor is $r(1 - t)$. In the case of borrowing at r, the interest is deductible, which lowers the net after-tax cost of borrowing to $r(1 - t)$. In the case of selling bonds, the investor forgoes the after tax return of $r(1 - t)$ on the bond. From the government side, the government has to pay r on any outstanding bonds. Buying back bonds thus lowers its net costs by r times the value of the bonds repurchased. This is equivalent to not repurchasing the bonds and instead lending at r.

> expected return of 10%. Thus if Investor has \$100 in A and \$100 in B, he has a 50% chance of earning \$30 and a 50% chance of losing \$10. Investor's expected return is thus \$10, or 10%.
>
> Now the government imposes a 40% income tax with full loss offsets. With no changes in the portfolio, Investor's gains and losses have been cut by 40%. Investor now has a 50% chance of earning \$18 (after tax) or of losing \$6 (after deduction). Investor's expected portfolio return, after tax, is reduced to \$6, or 6%.
>
> However, Investor can simply increase his investment in A by enough to offset the new tax (or "gross up" his investment). He can sell \$66.67 worth of the risk-free asset B and buy \$66.67 more of A. This returns his portfolio to having a 50% chance of earning \$30 or of losing \$10, and an expected return of \$10. (30% return on \$166.67 is \$50, or \$30 after tax, etc.)

Because in both situations the investor faces the same expected after-tax return and same return distribution, it is said that an income tax does not actually tax returns from investment risk-taking (when the risk-free rate is zero, or equivalently, when borrowing is costless[42]). Additionally—and importantly for the discussion that follows—the investor's overall portfolio volatility has not changed. He still faces a 50% chance of earning \$30 and a 50% chance of losing \$10.

Next, consider the case where there is a positive risk-free rate:

> *Example 2*: Now assume that the risk-free asset B returns the risk-free rate of 5% in all cases. Before imposition of the tax, Investor has \$100 invested in A and \$100 invested in B. Thus Investor has a 50% chance of his portfolio returning \$35 (\$30 from A and \$5 from B), a 50% chance of losing \$5 (-\$10 from A and +\$5 from B), and an expected return of \$15.
>
> Now the government imposes a 40% proportionate income tax. As in the example above, Investor can sell \$66.67 of B and buy \$66.67 of A. If he did so, he would forgo the 5% risk-free return on that \$66.67, lowering his pre-tax returns from B by \$3.33 (the same pre-tax cost as if he had borrowed \$66.67 at the risk-free rate in the market).
>
> Investor would then have \$166.67 invested in A, which means that the after-tax returns on A would be the same as \$100 invested in A in the

[42] *Id.*

> no-tax world—a 50% chance of earning $30 (.6 * $50) and a 50% chance of losing $10 (.6 * $16.67). However, the $33.33 remaining in B would earn only $1.67 before tax, which would be reduced to $1 by the tax. Thus, the overall portfolio would have a 50% chance of earning $31, a 50% chance of losing $9, and an expected return of $11.

In this example, the investor's only cost is the $4 reduction in the return on the risk-free asset (the forgone $3.33 return, plus the $0.67 tax on the remaining return). But this is equivalent to a tax on the risk-free return on the entire $200 portfolio. The net risk-free return on a $200 portfolio is $10 and a 40% tax on that $10 is $4. Thus, the explanation goes, an income tax accomplishes the same thing as a tax on only the risk-free return—either tax would have the same effect on the potential portfolio returns and would raise the same in tax revenue—therefore, it is said, the two tax systems are equivalent.

Note the steps in this reasoning, however. The examples follow the portfolio approach and conclude that $4 is raised from both an income tax and a tax only on the risk-free return, and thus that the two taxes are equivalent. Because that conclusion is the same as the conclusion of the equivalence approach of Kaplow, it is implicitly accepted that a rational taxpayer would make such portfolio shifts.

But this conclusion does not necessarily follow; the equivalence shown by Kaplow means only that the after-tax results under one tax could be replicated under the other, whether the investor grosses up fully, partially, or not at all. Below, I question whether we should actually expect to see the full gross-up shown in Example 2, and thus whether $4 is the full cost of *either* tax.[43] Before doing so, however, we need to complete the introduction to the Domar-Musgrave result.

C. Government Perspective

The prior section discussed how an income tax could affect individual taxpayer behavior. There is another side of any taxation question, however—government revenue. Kaplow's major contribution to the taxation-and-risk literature was to show that the an income tax was equivalent to a tax on the risk-free return not just in a partial equilibrium setting—looking just at taxpayers—but also in a general equilibrium setting where government behavior was also considered, at least under certain stringent assumptions.[44] As will be seen below,[45] the government side of the equation is important to this article's argument. Therefore, I briefly review it here.

[43] *See infra* Parts II.E and IV.
[44] Kaplow, *supra* note 3.
[45] *See infra* Part IV.

The examples above assume that the investor is able to purchase as many risky assets as he would like. But this assumption raises questions: Where do these extra risky assets come from? Who does the extra lending to finance the purchases? Assuming that investors writ large would already hold all existing risky assets in the no-tax world, how can they increase their holdings further after the imposition of the income tax?[46] In Kaplow's model, the additional risky assets come from the government. The government sells risky assets in order to meet the increased demand—either by selling short or by selling assets held in the government's portfolio. Furthermore, the government finances the purchases by buying back Treasury bonds, the risk-free asset.

Government portfolio policy not only helps to meet the increased demand for risky assets under this model, but also has two other important effects for the general equilibrium result. First, it causes government revenue to remain equivalent under an income tax and under a tax only on the risk-free return. Second, it causes overall social risk to remain equivalent under either tax, despite the increased risk-taking by investors.

First, consider government revenue. A major difference between an income tax and a tax on only the risk-free return is the source of direct government revenue from the tax. Under an income tax, the government collects a share of both risky and risk-free returns; under a tax only on the risk-free return, the government forgoes any share of risky returns. So how is it that government revenue remains constant? In the simple case where there is no expected return from risky returns—the risky part of an investment is a "fair bet" with an equal chance of gains or losses—then the tax on risky returns produces no expected revenue ex ante (nor does the government's portfolio policy). The same would be true ex post in the case where winners balanced out losers and there was no net return to risk in the market as a whole.

In the case where there is a positive expected return, the investors' gains would be offset by government losses. Recall that, under the Domar-Musgrave result, investors gross up their investments by enough to fund the tax on investment returns. But those additional gains come essentially out of the government's pocket. If the government sold the additional risky assets short in the market, for example, then its losses would exactly match the investors' gains, a portion of which gains are going right back to the government in the

[46] The comparison between a no-tax world and a world with an income tax is obviously somewhat stylized—the world with no taxes also has no government. Readers may prefer to imagine simply increasing an already existing income tax. If portfolio holdings were in equilibrium under a tax, the Domar-Musgrave result implies that, if the tax were increased, portfolios would shift somewhat toward risky assets. Again, assuming investors already held all risky assets, it is not obvious where the additional risky assets would come from. There might be some private short-sellers in the market, but if the market generally assumed a positive expected return, as my examples do, there would not be enough private short-sellers to meet demand.

form of tax revenue. The government is made whole, and left in the same position as if there were no tax on investment returns.

> *Example 3*: As in Example 2, Investor sells \$66.67 of B in order to purchase \$66.67 more of risky asset A. Assume Investor purchases the additional assets from the government, which sells A short to Investor. Investor's total investment in A produces an expected pre-tax return of \$16.67, and the investment in B produces an expected pre-tax return of \$1.67, for a total pre-tax expected gain of 18.33, 40% of which—\$7.33—goes to the government as tax revenue. The government has an offsetting expected loss on the trade of \$6.67 (by short-selling \$66.67 worth of A, which had a positive expected return of 10%). However, it also receives \$66.67 in cash at the beginning of the period for selling A, which, in this model, it uses to buy back B from Investor (thus giving Investor the cash to finance the purchase of B).[47] That lowers the government's interest payments by \$3.33. Therefore the government earns \$7.33 + \$3.33 = \$10.67, but loses \$6.67. The government thus nets \$4, which, as in the above section, is equivalent to simply levying a 40% tax on the 5% risk-free return on Investor's \$200 portfolio.

Second, consider social risk. The early literature on the Domar-Musgrave result concluded that an income tax increases overall private risk-taking, since, as shown above, investors increase their holdings of risky assets as a result of the tax.[48] However, under Kaplow's general equilibrium model, the increase in private holdings of risky assets is entirely offset by the fact that the government has divested itself of the same amount of risky assets. The overall amount of risky assets in the economy has not changed, merely the allocation of risky assets between private investors and the government. Thus the total social risk remains unchanged.[49]

D. Assumptions and Conditions

The purpose of this article is to show why even under the idealized taxation-and-risk model, an income tax does effectively tax returns to risk. This is not to say, however, that the idealized model realistically portrays our actual tax system. As noted in Part II.A, the Domar-Musgrave result,

---

[47] Another way of saying this is the government is assumed to earn the risk-free rate on cash it receives, whether by buying back bonds, lending the money out, or financing productive investment.

[48] *See* Domar & Musgrave, *supra* note 1, at 411-12; Kaplow, *supra* note 3, at 789 (discussing this implication of the earlier taxation-and-risk literature).

[49] Kaplow, *supra* note 3, at 794.

particularly the Kaplow general equilibrium result, depends on several strong assumptions that do not hold in practice. To what degree the relaxation of these assumptions affects the analysis has been addressed elsewhere[50] and is not a subject of this article. Nonetheless, for purposes of thoroughness, it is worth underscoring some of these assumptions and the effects that they have.

1. Proportionate Tax

For an investor to rationally make the sort of portfolio shifts described in Part II.B the government must participate in both the upside and downside of any risky investment—the government becomes essentially a partner in the risky venture. If, on the other hand, the government takes a share of the upside, but without sharing the cost of any losses (by disallowing a deduction), the riskiness of the investment changes (even when using the flawed variance measure of risk). In that case, the investor would compute gain at an after-tax rate, but losses at a pre-tax rate, thus skewing the expected after-tax return downward. This would minimize her interest in taking on more risk.[51]

Our current tax system allows only limited deductions for investment losses. First, all but $3000 of losses can be deducted only against capital gain, not ordinary income.[52] Second, even the netting of losses against gains is limited by the basketing of long- and short-term gain and loss,[53] and by other loss limitation rules.[54] As a result, an investor can only assume the deductibility of losses if she is confident of having sufficient capital gain to offset them,[55] which would likely only be the case for investors with substantial and diverse portfolios.

Not surprisingly, many scholars have concluded that having limited or no loss offsets affects the Domar-Musgrave analysis.[56] In particular, investors are not likely to shift as much toward risky assets without the assurance that the government will also share their losses. Some have noted, however, that the inability to deduct all losses will affect different investors in different ways.

---

[50] *See, e.g.,* Cunningham, *supra* note 4, at 37-39; Schenk, *supra* note 5, at 428-35; Zelenak, *supra* note 10.

[51] *See, e.g.,* ATKINSON & STIGLITZ, *supra* note 9, at 112-15; Thomas Brennan, *Certainty and Uncertainty in the Taxation of Risky Returns* (2009) (unpublished manuscript, on file with author); Domar & Musgrave, *supra* note 1, at 403-09; Schenk, *supra* note 5, at 429-30.

[52] I.R.C. §§ 1211(b), 1212.

[53] I.R.C. § 1222.

[54] *See, e.g.,* I.R.C. §§ 465 (at-risk limitations), 469 (passive loss limitations).

[55] An additional effect is that it forces investors to realize gains in order to offset losses. That creates transaction costs for investors, as well as forgone gains due to wash sale rules and other rules limiting the ability of the investor to enter back into the investment after wiping out gains. *See* I.R.C. § 1091 (wash sale rules).

[56] *See, e.g.,* ATIKINSON & STIGLITZ, *supra* note 9, at 112-15; Domar & Musgrave, *supra* note 1, at 403-09; David A. Weisbach, *Taxation and Risk-Taking with Multiple Rates*, 57 NAT'L TAX J. 229 (2004); Zelenak, *supra* note 10.

Sophisticated investors with broad and diverse portfolios and with significant other capital income are more likely to be able to fully deduct any losses, leading to the conclusion that the ability to take advantage of the Domar-Musgave portfolio shifts in order to offset the nominal tax on risky returns is not equitably distributed among all taxpayers.[57]

2. Single Tax Rate

Related to the assumption of unlimited deductibility of losses is the assumption that there is only a single rate of tax. Indeed, the lack of proportionality is in some ways a special case of the more general fact that losses and gains can be subject to different tax rates. For example, long- and short-term gains are subject to different tax rates;[58] certain basketing rules and other limitations can raise the effective tax rate on certain investments;[59] some investment assets are not considered "capital assets," and thus are subject to the progressive marginal tax rates for ordinary income;[60] and even certain capital assets are subject to different tax rates.[61] The denial of full loss offsets is essentially just applying a 0% rate to certain types of capital losses. All these factors can affect the degree to which investors are willing to increase their investment in risky assets. In general, if gains will be subject to a higher rate than losses,[62] then we would not expect an investor to fully gross up.

3. Government Portfolio Policy

As discussed in Part II.C, it is essential to the Domar-Musgrave result that the government supply the market with the additional risky assets needed by investors to gross up. Otherwise, demand for risky assets would outstrip supply, driving up prices and driving down returns. As a result, additional risky assets would not be able to generate returns sufficient to cover the tax.

In the examples above, Investor purchases an additional $66.67 of the risky asset in order to fund the tax on the returns to his original portfolio. It was assumed above that the marginal investment in the risky asset would also have an expected return of 10%. But if the expected return on the marginal investment shrank to only 5% because of rising share prices, there would be

---

[57] *See, e.g.,* Schenk, *supra* note 5.

[58] I.R.C. §§ 1(h), 1222.

[59] *See supra* note 54.

[60] *See, e.g.,* I.R.C. §§ 1221 (definition of capital asset), 475 (inventory and mark-to-market accounting for dealers in securities).

[61] *See, e.g.,* I.R.C. §§ 1(h)(1)(B) (0% rate for low-income taxpayers), (h)(4) (28% rate for collectibles and "section 1202 gain"), 11 (graduated rate on corporate income), 1231 (mixed ordinary/capital treatment for property used in a trade or business).

[62] Most of the rules mentioned above are intended to limit the government's share of tax losses, so we would expect that the overall affect of the rules are to create a higher rate on gains than on losses.

essentially no net gain, since the foregone risk-free return (or, equivalently, the borrowing cost) would fully offset the incremental gain on the risky asset. This would leave the original pre-tax expected return of $10, a tax of $4, and thus an after-tax return of $6, rather than $8 as in the example. (Expected marginal returns between 5 and 10% would therefore generate after-tax returns between $6 and $8.)

In reality, the government does not participate in the market in the way the model assumes, nor does anyone truly advocate that it should (though Kaplow suggests that a combination of other government actions could in theory approximate a government portfolio of risky assets).[63] Nonetheless, the notion is not unreasonable. Note that as a result of imposing the income tax, the government is newly exposed to volatile tax revenue from risky returns. We could imagine, therefore, that the government may wish to hedge that risk somewhat by selling risk into the market.[64] Nonetheless, because the government does not appear to manage its portfolio in this way, the equivalence between an income tax and a tax on the risk-free return cannot hold in practice.[65] This in turn means that an income tax does tax risky returns, even before taking into account the complications discussed below.

This additional tax would, however, be the same under a normative cash-flow consumption tax, assuming the other simplifying conditions of the model hold. The exemption of capital income under a cash-flow consumption tax requires the same grossing-up behavior as in the Domar-Musgrave analysis.[66] In that case, however, the grossing up is funded not by borrowing at the risk-free rate or selling risk-free assets, but by the tax deductions from expensing the amount invested.[67] But if the ability to gross up is similarly limited by the overall available pool of risky assets, and government portfolio policy does not alleviate the shortage, then we would see the same effect under a cash-flow consumption tax as under an income tax—some additional tax on risky

[63] For example, by shifting tax rates on capital and government expenditures in order to approximate short- and long-position returns depending on market states. *See* Kaplow, *supra* note 27, at 19.

[64] *See* Gentry & Hubbard, *supra* note 6, at 8 ("Kaplow concludes that neither an income tax nor a consumption tax taxes risk because the government offsets the effects of both taxes on the uncertainly of government revenues by decreasing its position in risky assets and increasing its position in safe assets."); Kaplow, *supra* note 3, at 794-95.

[65] *See* Kaplow, *supra* note 3, at 794.

[66] *See, e.g.,* Andrews, *supra* note 17.

[67] Briefly, a cash-flow consumption tax starts with a comprehensive income tax base (thus including income from labor and capital), but then provides for full expensing of capital investments. As Cary Brown and others have shown, full expensing of an amount invested is equivalent to full exemption of the income derived from that investment. *See* Brown, *supra* note 17. This is because the government provides a tax deduction for the amount invested. The reduction in tax from that deduction essentially funds an additional investment, which causes an additional deduction, and so on. The end result is a grossed-up investment, the additional returns from which pay for the nominal tax on any gain. *See infra* Part V.

returns.[68] Thus, rather than saying that neither an income tax nor a cash-flow consumption tax taxes risky returns, some writers instead say that both taxes treat risky returns the same, however that might be.[69]

Therefore, as unrealistic as the notion of an active government portfolio policy might be, the assumption does not change the essential question of the relationship between an income tax and a cash-flow consumption tax with respect to the treatment of capital income. I return to this issue in Part V.

E. The Problem

Return to Example 2 in Part II.B. In the no-tax world, the investor had a portfolio with a 50% chance of earning $35, a 50% chance of losing $5, and an expected gain of $15. Next, after imposition of the tax and fully grossing up the investor has a portfolio with a 50% chance of earning $31, a 50% chance of losing $9, and an expected gain of $11.

Even though the investor faces greater downside risk—growing from a potential loss of $5 to a potential loss of $9—the *variance* of the returns is the same in both examples. In either case, variance is constant. The potential gain and potential loss are +/- $20 away from the expected return, or *mean*, of each portfolio ($15 in the no-tax portfolio, $11 in the fully grossed-up after tax portfolio). If variance is used as the measure of investment risk, as it is in the portfolio approach, each portfolio is treated as equally risky,[70] *despite the greater risk of loss in the second portfolio.*

In stating that an investor would gross up despite the risk of greater loss, most of the tax law literature is essentially adopting an investor behavior model of maximizing expected returns while holding variance—volatility—constant. But this is bizarre approach to constructing a portfolio, and no portfolio choice model suggests that it is reasonable or appropriate. Defining risk as variance and ignoring the size of the potential loss contradicts not only common sense, but also the conclusions of much of the financial economics and mathematical risk literature. Part III discusses both more appropriate definitions of risk and the consequence of those definitions, namely that a full gross-up is unlikely to occur and therefore that even a pure Haig-Simons income tax will tax the return to risky investments.

---

68 *See* Weisbach, *supra* note 4, at 7.

69 *See, e.g.,* Gentry & Hubbard, *supra* note 6, at 8.

70 *See, e.g.,* Cunningham, *supra* note 4, at 33 ("[An investor] can increase her investment in the risky asset … without exposing herself to more risk."); Schenk, *supra* note 5, at 426 ("[T]he investor can make riskier investments … while maintaining the same risk exposure he found desirable in a tax-free world."); Reed Shuldiner, *Taxation of Risky Investments*, at 11 (2005) (unpublished manuscript, on file with author) ("An important point to note is that the risk of the taxpayer's portfolio, measured by its standard deviation or variance is unchanged.") *But see* Shuldiner, *id.*, (pointing out that the grossed-up after tax portfolio is less optimal than the pre-tax portfolio due to the wealth effect).

III. Investment Risk and Portfolio Theory

The prior section suggests that the conventional "portfolio approach" description of the Domar-Musgrave result overstates the degree to which even a purely rational investor would shift her portfolio toward risky assets in the face of income tax. The portfolio approach treats the grossed-up after-tax portfolio and the original portfolio in the no-tax world as equally risky, because each has the same *variance* in portfolio returns, and variance is often treated as a measure of investment risk.

In this section, I argue, first, that using variance in this way contradicts even the most orthodox theories of portfolio choice, such as the mean-variance model and expected utility theory. I argue further, however, that the mean-variance model in particular applies only under narrow and unrealistic assumptions about investor behavior and asset distributions. Finally, I discuss alternative theories of portfolio choice that focus on "safety first" principles, i.e., minimizing the risk of large losses. The Value at Risk model in particular has become a leading risk management tool for large financial institutions and their regulators.

A. The Problems with Variance

Numerous articles in the legal literature assume without question that variance is a sufficient measure of risk.[71] This view of variance is likely derived from the central role that variance plays as a risk measure in much of finance and portfolio theory. But it is nonetheless a misunderstanding of this literature to assume that variance alone suffices to measure risk.

That variance and volatility are so often treated as synonymous with risk is understandable, since the finance and risk literature uses the word "risk" in multiple ways and contexts. At one level, the literature does frequently define "risk" as "volatility."[72] But risk-as-volatility can be distinguished from particular *types* of risk: e.g., systematic risk, specific risk, credit risk, business

[71] *See, e.g.,* Cunningham, *supra* note 4, at 30-31 n.54; Lee Anne Fennell, *Unbundling Risk*, 60 DUKE L.J. 1285 (2011); Shuldiner, *supra* note 70, at 2; Stewart E. Stark, *Rethinking Trust Law Reform: How Prudent is the Modern Prudent Investor Doctrine?*, 95 CORNELL L. REV. 851 (2010); Robert J. Rhee, *Tort Arbitrage*, 60 FLA. L. REV. 125, 128 n.9 (2008) ("In this Article, 'risk,' 'uncertainty,' and 'variance' are used interchangeably."). *But see* Douglas O. Edwards, *An Unfortunate "Tail": Reconsidering Risk Management Incentives After the Financial Crisis of 2007-2009*, 81 U. COLO. LAW REV. 247 (2010) (discussing the Value at Risk measure and some of the limits of the mean-variance and capital asset pricing models).

[72] *See, e.g.,* BODIE, *supra* note 11, at 129 ("The standard deviation [the square root of variance] of the rate of return is a measure of risk."); PHILIPPE JORION, VALUE AT RISK: THE NEW BENCHMARK FOR MANAGING FINANCIAL RISK 2 (2d ed. 2001) ("Risk can be defined as the volatility of unexpected outcomes.").

risk, counterparty risk, liquidity risk, legal risk, reputational risk, etc.[73] In particular, in the investment portfolio context, risk scholars frequently focus on *market risk*, meaning risk of portfolio *losses* due to fluctuations in market prices.[74] As will be discussed below, the models for and measures of market risk tend to incorporate volatility (though not without problems, as we will see), but only as an input in calculating a measure of market risk. Thus, volatility is better understood as the *source* of market risk, rather than a stand-alone *measure* of market risk.

Nonetheless, because volatility is a source of market risk, minimizing volatility will naturally minimize market risk.[75] This is the central contribution of modern portfolio theory ("MPT") and the mean-variance portfolio selection model. The theory, first developed by Harry Markowitz in 1952,[76] suggests that an optimal portfolio can be determined based only on the *mean* and *variance* of the portfolio. An optimal portfolio, according to Markowitz, is one where the expected return—the mean—cannot be increased without also increasing the risk of the portfolio.[77] To measure risk, Markowitz settled on variance, but with clear reservations. He understood that variance was a simplification of the idea of risk. But since using variance made the optimization calculations far simpler than using other measures, variance nonetheless became the key risk measure for MPT.[78]

As will be discussed below, the model has significant flaws under more realistic assumptions about investor risk preferences and asset price distributions. But even if we applied it in its orthodox form to the taxation-and-risk question, it would be unlikely to suggest that an investor would fully gross up in response to an income tax, as the portfolio approach implies. The reason is that the fully grossed-up after-tax portfolio is clearly inferior to the before-tax portfolio; in MPT terms it is less efficient, and thus the two

---

[73] *See* MICHEL CROUHY, DAN GALAI & ROBERT MARK, RISK MANAGEMENT 22 (2001) ("The word 'risk' has many meanings and connotations.").

[74] *See* JORION, *supra* note 72, at 15; CROUHY ET AL., *supra* note 73, at 34. Often the assumption is that market risk is what remains after portfolio diversification removes the diversifiable, or nonsystematic, risk (market risk is also sometimes called systematic risk). *See, e.g.,* BODIE, *supra* note 11, at 197. In the stylized examples of this article the portfolio clearly is not diversified, but we could instead imagine the single risky asset as the market portfolio.

[75] If there were no volatility in expected returns, then we would know with certainty what an asset or a portfolio would be worth at some time in the future. Assuming we held only assets with positive expected returns, there would be no risk of loss. Thus volatility is clearly essential to any measure or definition of risk. However, that still leaves the question how to incorporate volatility into a more comprehensive measure of market risk.

[76] *See* Harry Markowitz, *Portfolio Selection*, 7 J. FIN. 77 (1952).

[77] MARKOWITZ, *supra* note 13, at 129.

[78] MARKOWITZ, *supra* note 13, at 194 (suggesting that analyses based on semi-variance, a measure of downside risk, "tend to produce better portfolios than those based on [variance]," but that "[v]ariance is superior with respect to cost, convenience, and familiarity").

portfolios would not be on the same "efficient frontier."[79] Because the investor would be facing a different efficient frontier in the after-tax case, we cannot assume that she would choose the same variance as in the before-tax case. Indeed, it would be surprising if she did.[80]

Put more intuitively, our investor was willing to take on a certain amount of volatility—and thus market risk—to earn a certain expected return. If that expected return were lowered, we should not assume that the investor would continue to take on the same amount of volatility. If returns are likely to be lower, an investor, following a mean-variance optimization model, is likely also to desire less volatility. There is a trade-off. This is another way of describing the wealth effect[81]: with lower expected wealth in the future, an investor is likely to want less risk of losing that wealth, where that risk of loss derives in part from portfolio volatility. The mean-variance model captures this effect to some extent; variance alone does not.

B. The Limits of the Mean-Variance Model

Orthodox modern portfolio theory predicts at least a partial wealth effect as the result of an imposition of the tax and the corresponding lower expected return. However, the mean-variance model itself has important flaws, and correcting for those flaws further increases the tax on risky assets.

It is well known in the finance literature that optimizing portfolios using only the portfolio mean and variance maximizes investor expected utility only in two narrow cases: where the portfolio returns are normally distributed or where the investor has a quadratic utility function.[82] Neither is likely to be the case.

First, if an asset's return distribution is normally distributed about the mean—meaning it follows the normal Gaussian bell curve—then mean and

---

[79] The "efficient frontier" is the set of portfolios that achieve the highest possible expected return for a given variance. *See* BODIE, *supra* note 11, at 211; Shuldiner, *supra* note 70, at 11

[80] *See* Shuldiner, *supra* note 70, at 17; *see also* Alan Auerbach & Mervyn King, *Taxation, Portfolio Choice, and Debt-Equity Ratios: A General Equilibrium Model*, 98 Q.J. ECON. 587, 596 (1983) (using a capital asset pricing model—a relative of MPT—to show that the optimal investor portfolio when taxed is a weighted average of the market portfolio and a tax-optimal portfolio, where the weight on each depends on the investor's risk preferences); JAMES M. POTERBA, *Taxation, Risk-Taking, and Household Portfolio Behavior* in 3 HANDBOOK OF PUBLIC ECONOMICS 1110, 1125 (Alan J. Auerbach & Martin Feldstein eds., 2002) (discussing Auerbach & King, *supra*).

[81] *See supra* note 8 and accompanying text.

[82] *See, e.g.,* JAKŠA CVITANIĆ & FERNANDO ZAPATERO, INTRODUCTION TO THE ECONOMICS AND MATHEMATICS OF FINANCIAL MARKETS 409-10 (2004); DAVID LUENBERGER, INVESTMENT SCIENCE 237 (1998); Dimitris Bertsimas, Geoffrey J. Lauprete, & Alexander Samarov, *Shortfall as a Risk Measure: Properties, Optimization and Applications*, 28 J. ECON. DYNAMICS & CONTROL 1353, 1353-53 (2004) (reviewing literature); Martin S. Feldstein, *Mean-Variance Analysis in the Theory of Liquidity Preference and Portfolio Selection*, 36 REV. ECON. STUD. 5 (1969).

variance are all that is needed to capture the potential distribution of returns from an investment.[83]

However, there are good reasons to believe that the price distribution for stocks do not fit the normal distribution.[84] Instead, it is likely to exhibit *skewness* (meaning that the distribution curve is weighted to one side or the other of the mean) or excess *kurtosis* (meaning that "extreme" events—highs or lows—are more frequent than under the normal distribution, the so-called "fat tail" problem).[85]

As a result of these and other variations, we have seen a far greater number of extreme events in financial markets than a normal distribution would predict. The details of these are well known by now:

> [O]n August 4, [1998,] the Dow Jones Industrial Average fell 3.5 percent. Three weeks later, as news from Moscow worsened, stocks fell again, by 4.4 percent. And then again, on August 31, by 6.8 percent.... The standard theories, as taught in business schools around the world, would estimate the odds of that final, August 31, collapse at one in 20 million—an event that, if you traded daily for nearly 100,000

---

[83] BODIE, *supra* note 11, at 129 ("As long as the probability distribution is more or less symmetric about the mean, variance is an adequate measure of risk. In the special case where we can assume that the probability distribution is normal—represented by the well-known bell-shape curve—mean and variance are perfectly adequate to characterize the distribution.").

[84] Some portfolio theorists have suggested that the class of Student *t*-distributions are better models of stock price distributions, since they allow for parameters beyond just mean and variance, and thus can be used to approximate distributions with "fatter tails." *See, e.g.,* JORION, *supra* note 72, at 93-94; Yalcin Akcay & Atakan Yalcin, *Optimal Portfolio Selection with a Shortfall Probability Constraint: Evidence From Alternative Distribution Functions*, 33 J. FIN. RES. 77, 80 (2010); Turan G. Bali, K. Ozgur Demirtas & Haim Levy, *Is There an Intertemporal Relation Between Downside Risk and Expected Returns?* 44 J. FIN. & QUANT. ANAL. 883, 888 (2009).

[85] *See* BODIE, *supra* note 11, at 145 ("[T]here is some evidence of greater exposure to extreme negative outcomes than would be the case under the normal distribution."); BENOIT B. MANDELBROT & RICHARD L. HUDSON, THE (MIS)BEHAVIOR OF MARKETS 96 (2004); Eugene F. Fama, *The Behavior of Stock-Market Prices*, 38 J. BUS. 34, 42 (1965) ("The presence, in general, of leptokurtosis in the empirical distributions [of stock price changes] seems indisputable."); Campbell R. Harvey & Akhtar Siddique, *Conditional Skewness in Asset Pricing Tests*, 55 J. FIN. 1263, 1264-65 (2000) (discussing evidence of skewness).

Some literature implies that skewness of returns is not important, provided that the returns are relatively "compact"—that is, that they are continuous and do not exhibit large jumps in price. *See* BODIE, *supra* note 11, at 169; Paul Samuelson, *The Fundamental Approximation Theorem of Portfolio Analysis in Terms of Means, Variances and Higher Moments*, 37 REV. ECON. STUD. 537, 537 (1970). Under that assumption and the assumption that investors will revise their portfolios over long periods of time, skewness becomes irrelevant. But the history of asset prices challenges the continuity assumption. *See infra* note 86 and accompanying text. Rather than being relatively smooth, risky assets have tended to exhibit sudden jumps in price. Some have argued that price discontinuity not only makes skewness relevant, but undermines most of the math central to portfolio theory and financial economics. *See, e.g.,* MANDELBROT & HUDSON, *supra*, at 237.

> years, you would not expect to see even once. The odds of getting three such declines in the same month were even more minute: about one in 500 billion.[86]

The paragraph above was written in 2004, before the financial crisis of 2007–08. And between 1998 and 2007 we also had the stock market crash in 2001 following the burst of the dot-com bubble. There can be little doubt that extreme events are not that rare, yet a simple mean-variance model behaves as if they were.

Second, the mean-variance model could maximize expected utility provided that the investor faces a quadratic utility function. This, however, is unlikely. Quadratic utility functions have several properties that make them unlikely to measure investor utility accurately, one of which is *increasing* absolute risk aversion—namely, investors' desire for risky assets decreasing as wealth increases.[87] Furthermore, quadratic utility functions are also indifferent to higher moments; that is, they do not reflect any changes in investor utility due to skewness and kurtosis, or to asymmetric return distributions generally. Because, as noted above, asset distributions tend to exhibit these properties, mean-variance theory is not sufficient to maximize investor utility.[88]

C. Stochastic Dominance

Although mean-variance is consistent with expected utility theory only under narrow assumptions, taking a more general approach is difficult, since it requires the specification of some utility function. Stochastic dominance provides an alternative approach to conceptualizing risk.[89] I discuss stochastic dominance briefly here in order to further criticize the notion that variance is synonymous with risk and because it clearly and robustly shows that the fully

---

[86] MANDELBROT & HUDSON, *supra* note 85, at 3-4; *see* NASSIM NICHOLAS TALEB, THE BLACK SWAN: THE IMPACT OF THE HIGHLY IMPROBABLE 276 (2d ed. 2010) ("If the world of finance were Gaussian, an episode such as the [1987 stock market] crash (more than twenty standard deviations) would take place every several billion lifetimes of the universe."); Darrell Duffie & Jun Pan, *An Overview of Value at Risk*, 4 J. DERIVATIVES 7, 11 (1997) (noting that the daily returns from the S&P 500 for 1986 to 1996 had a kurtosis of 111 while normally distributed returns would have had a kurtosis of 3).

[87] *See, e.g.,* ATKINSON & STIGLITZ, *supra* note 9, at 100; Bertsimas et al., *supra* note 82, at 1354; Samuelson, *supra* note 85, at 537; Sandmo, *supra* note 9, at 369.

[88] *See id;* Gary Chamberlain, *A Characterization of the Distributions That Imply Mean-Variance Utility Functions*, 29 J. ECON. THEORY 185 (1983).

[89] *See, e.g.,* HAIM LEVY, STOCHASTIC DOMINANCE: INVESTMENT DECISION MAKING UNDER UNCERTAINTY (2d ed. 2006); Josef Hadar & William R. Russell, *Rules for Ordering Uncertain Propsects*, 59 AMER. ECON. REV. 25 (1969); Haim Levy, *Stochastic Dominance and Expected Utility*, 38 MGMT. SCI. 555 (1992); Michael Rothschild & Joseph E. Stiglitz, *Increasing Risk: I, A Definition*, 2 J. ECON. THEORY 225 (1970).

grossed-up after-tax portfolio presented in Part II is in fact *riskier* than the no-tax portfolio.

Rather than reduce each possible return distribution to a single measure and then compare the two measures, a stochastic dominance approach instead compares each possible outcome along a return distribution, and then provides a preference ordering of different distributions. For example, consider the simplest case of *first-order stochastic dominance*. If there are two risky assets A and B, A is said to first-order stochastically dominate B, if for every possible future state,[90] A will always return more than B.[91] Suppose A had a 50% chance of returning $2 and a 50% chance of returning $4, while B had a 50% chance of returning $1 and a 50% chance of returning $3. In the worst-case scenario, A beats B ($2 vs. $1), and in the best-case scenario A also beats B ($4 vs. $3), and thus A first-order stochastically dominates B.

Stochastic dominance is a key feature of expected utility theory. Any expected utility maximizer will always prefer the asset that first-order stochastically dominates another.[92] Of course, with many assets in a portfolio, it is unlikely that one asset or portfolio dominates the other in every future state of the world—sometimes one portfolio is more likely to do better, sometimes the other. In those cases, we would look to *second-order stochastic dominance* in order to rank different options.[93]

For example, an asset A would second-order stochastically dominate an asset B if the total of the return premiums of A over B when A returns more than B is greater than the total premiums of B over A when B returns more than A.[94] For example, if A had a 50% chance of returning $1 and a 50%

---

[90] By "state of the world" I mean, more rigorously, "for every probability on the cumulative distribution function." The point is to compare worst-case scenarios, next-worse-case scenarios, etc. Not what each will do if it rains, or if the stock market collapses, or if the Cubs win the World Series.

[91] The more formal definition: For two cumulative distribution functions, F and G, of two risky assets (or, more generally, "lotteries") A and B, A first-order stochastically dominates B if $F(x) \leq G(x)$ for all outcomes x (with a strict inequality for at least one x). *See* LEVY, *supra* note 89, at 556. The direction of the inequality is because of the nature of cumulative distribution functions. Essentially, a cumulative distribution function measures the probability of being at or below an outcome x. Thus, $F(x) \leq G(x)$ means that A will return above x as or more often than B.

[92] *See* LEVY, *supra* note 89, at 556-57.

[93] Or higher-order stochastic dominance, if necessary.

[94] The more formal definition: For two cumulative distribution functions, F and G, of two risky assets, A and B, A second-order stochastically dominates B if:

$$\int_{-\infty}^{x} [G(t) - F(t)dt \geq 0$$

for all outcomes x (with strict inequality for at least one x). *See* LEVY, *supra* note 89, at 556. In other words, the area under F from $-\infty$ to x is less than or equal to the area under G from $-\infty$ to x. Under typical expected utility theory, risk-averse utility maximizers will prefer assets that second-order stochastically dominate others. *See id.* at 556-57.

change of returning $5, and B had a 50% chance of returning $2 and a 50% chance of returning $3, A would not first-order stochastically dominate B, because in the worst-case scenario, B returns more than A. But A would second-order stochastically dominate B, because B's worst case beats A's worst case by only $1, while A's best case beats B's best case by $2.[95]

Stochastic dominance thus provides a way to compare the riskiness of different risky options, or "lotteries," and does so in a robust way that is more likely to accurately describe how individuals perceive risky options than a single risk measure can.[96] However, it is limited to being comparative. It is not able to, say, identify a lower bound of acceptable losses, as Value at Risk attempts to do;[97] thus it can be more unwieldy for portfolio choice applications.[98] Nonetheless, it supplies both a stronger intuition and a greater rigor than a variance risk measure.

It would be complicated to use stochastic dominance to predict how the investor in the examples of Part II would behave in the face of the income tax. However, the fully grossed-up after-tax portfolio is clearly riskier in stochastic dominance terms than the original no-tax portfolio, since the fully grossed-up after-tax portfolio is first-order stochastically dominated by the original no-tax portfolio. This is because the effect of the tax, even after full gross up, is to shift the entire return distribution downward by the amount by which the tax lowers returns. The effect of the shift means that at each point in the return distribution, the fully grossed-up after-tax portfolio will return less than the original pre-tax portfolio; returns in any state of the world would be reduced by the amount of the tax. As result, the original portfolio first-order

---

95 Because a risk-averse utility maximizer prefers second-order stochastically dominating options, *see id.*, while even a risk-neutral person prefers a first-order stochastically dominating option, Hadar & Russell, *supra* note 89, at 27 (a first-order stochastically dominant prospect is preferred "regardless of the specifications of the utility function"), second-order stochastic dominance closely equates with the idea of risk. Indeed, for two distributions with the same mean, the distribution with the lower variance will second-order stochastically dominate the distribution with the higher variance. *See* R. Burr Porter, *Semivariance and Stochastic Dominance: A Comparison*, 64 AMER. ECON. REV. 200, 200 (1974). However, that result only holds generally where the means of the two distributions are the same. If they are not, lower variance no longer necessarily implies second-order stochastic dominance.

96 *See, e.g.,* R. Burr Porter & Jack E. Gaumnitz, *Stochastic Dominance vs. Mean-Variance Portfolio Analysis: An Empirical Evaluation*, 62 AMER. ECON. REV. 438, 445 (1972) (Where risk aversion is strong, "stochastic dominance rules are more consistent with the maximization of expected utility than is the mean-variance rule.").

97 *See infra* Part III.E.

98 *See* R. Burr Porter, *An Empirical Comparison of Stochastic Dominance and Mean-Variance Portfolio Choice Criteria*, 8 J. FIN. & QUANT. ANAL. 587, 589 (1973) ("Although the conceptual superiority of [stochastic dominance] over [mean-variance] is clear, its practical application requires a somewhat more sophisticated technology."). However, it has been shown that the Expected Shortfall risk measure discussed in note 133, *infra*, is consistent with second-order stochastic dominance. *See* Bertsimas et al., *supra* note 82, at 1357; Enrico De Giorgi, *Reward–Risk Portfolio Selection and Stochastic Dominance*, 29 J. BANK. & FIN. 895, 896 (2005).

stochastically dominates the grossed-up after-tax portfolio; the cumulative distribution function of the original portfolio is less than the cumulative distribution function of the fully grossed-up after-tax portfolio at every point in the distribution. Because first order stochastic dominance necessarily implies second-order stochastic dominance, the fully grossed-up after-tax portfolio is unambiguously riskier than the original pre-tax portfolio under expected utility theory.[99] This is true even if one disagrees with the downside risk approach of this paper. Because of the features of first-order stochastic dominance, this conclusion does not depend on an investor's risk preferences or the particular expected utility curve.[100]

D. Loss Aversion and Safety First

The criticisms of MPT and the mean-variance model are particularly relevant in a world of high volatility and extreme events—that is, a world very much like our own.[101] As long as asset distributions stay close to normally distributed, the mean-variance model can provide a reasonable approximation.[102] However, when "fat tails" and downward skewness appear, mean-variance loses traction. Because of the potential for frequent large losses, many risk and portfolio theorists argue for different approaches to optimizing portfolios and managing risk. Investors would do better, some argue, to focus on minimizing the risk of loss, not simply volatility.[103]

This approach—sometimes called a "safety first" approach—is especially relevant for investors who exhibit loss aversion. "Loss aversion refers to the phenomenon that decision-makers are distinctly more sensitive to losses than

---

[99] To see this more generally, recall the formal definition of first-order stochastic dominance in note 91, *supra*. Under this article's model, the fully grossed-up after tax portfolio returns a constant amount less in each scenario than the original pre-tax portfolio, i.e., the amount of the tax on the risk-free return (similarly, an actual tax on the risk-free return will be a constant amount in every situation). Thus assume F to be the cumulative distribution function of the original pre-tax portfolio and G to be the cumulative distribution function of the fully grossed-up after-tax portfolio. Then $F(x) = G(x - a)$, where a is a constant representing the amount of the tax owed. Since a is strictly positive (assuming a non-zero risk-free rate of return), $G(x - a) < G(x)$, or $F(x) < G(x)$, and thus the original portfolio stochastically dominates the fully grossed-up after-tax portfolio. Thus the fully grossed-up after-tax portfolio is unambiguously riskier.

[100] *See* Hader & Russell, *supra* note 89, at 27.

[101] *See supra* note 86 and accompanying text.

[102] *See supra* note 83.

[103] *See, e.g.,* Akcay & Yalcin, *supra* note 84; Bertsimas et al., *supra* note 82; Mei Choi Chiu, Hoi Ying Wong & Duan Li, *Roy's Safety-First Portfolio Principle in Financial Risk Management of Disastrous Events*, RISK ANALYSIS (forthcoming 2012); Peter Fishburn, *Mean-Risk Analysis with Risk Associated with Below-Target Returns*, 67 AMER. ECON. REV. 116 (1977); Guy Kaplanski & Yoram Kroll, *VaR Risk Measures Versus Traditional Risk Measures: An Analysis and Survey*, 4 J. RISK 1 (2002).

to gains."[104] Loss aversion is a feature of prospect theory,[105] which postulates, in part, that decision-makers derive utility from changes in wealth relative to a particular reference point, rather than absolute levels of wealth. Prospect theory thus conflicts with expected utility theory and provides an alternative model for individuals' decision-making under uncertainty. Indeed, prospect theory was developed, in part, to explain experimental results that were inconsistent with expected utility theory.[106]

The reference point for measuring losses and gains under prospect theory is typically treated as current wealth,[107] but it is consistent with loss aversion for the reference point to be some other threshold amount.[108] For example, some researchers have found that people may not be loss averse (in fact the reverse) for small losses,[109] which suggests that if loss aversion exists, it could apply only when losses become large enough.

There is substantial,[110] though not universal,[111] experimental evidence of loss aversion. In the investment context, for example, research shows that investors demand extra compensation for holding stocks with greater

---

[104] Arjan Berkelaar, Roy Kouwenberg & Thierry Post, *Optimal Portfolio Choice Under Loss Aversion*, 86 REV. ECON. & STAT. 973, 973 (2004); *see* Daniel Kahneman & Amos Tversky, *Prospect Theory: An Analysis of Decision Under Risk*, 47 ECONOMETRICA 263 (1979). Interestingly, Markowitz was an early observer of the behavior that came to be known as loss aversion. Harry M. Markowitz, *The Utility of Wealth*, 60 J. POL. ECON. 151, 153-54 (1952).

[105] On prospect theory, *see generally* Kahneman & Tversky*, supra* note 104; Amos Tversy & Daniel Kahneman, *Advances in Prospect Theory: Cumulative Representation of Uncertainty*, 5 J. RISK & UNCERTAINTY 297 (1992). Prospect theory and behavioral finance are playing an increasingly large role in finance generally. *See, e.g.,* BODIE ET AL., *supra* note 11, at 381; *see generally, e.g.,* Berkelaar et al., *supra* note 104; Carole Bernard & Mario Ghossoub, *Static Portfolio Choice Under Cumulative Prospect Theory*, 2 MATH. & FIN. ECON. 277 (2010); Enrico De Giorgi & Thorsten Hens, *Making Prospect Theory Fit For Finance*, 20 FIN. MKTS. & PORTFOLIO FIN. 339 (2006). Finance theorists continue to struggle somewhat with the best way to integrate prospect theory formally, however. *See id.*

[106] *See* Kahneman & Tversky, *supra* note 104, at 263.

[107] *See* Kahneman & Tversky, *supra* note 104, at 274.

[108] *See id.;* Bernard & Ghossoub, *supra* note 105 (using initial wealth plus a risk-free return as reference point).

[109] Fieke Harinck, Eric Van Dijk, Ilja Van Beest, and Paul Mersmann, *When Gains Loom Larger Than Losses: Reversed Loss Aversion for Small Amounts of Money*, 18 PSYCH. SCI. 1099, 1100-03 (2007).

[110] *See, e.g.,* Nicholas Barberis, Ming Huang & Tano Santos, *Prospect Theory and Asset Prices*, 116 Q.J. ECON. 1 (2001); David Genesove & Christopher Mayer, *Loss Aversion and Seller Behavior: Evidence from the Housing Market*, 116 Q.J. ECON. 1233 (2001); Sabrina M. Tom et al., *The Neural Basis of Loss Aversion in Decision-Making Under Risk*, 315 SCI. 515 (2007) (finding brain activity consistent with loss aversion during controlled experiments).

[111] *See, e.g.,* Eyal Ert & Ido Erev, *The Rejection of Attractive Gambles, Loss Aversion, and the Lemon Avoidance Heuristic*, 29 J. ECON. PSYCH. 715 (2008) (arguing that changes in experiment format can account for loss aversion); *see also* David Gal, *A Psychological Law of Inertia and the Illusion of Loss Aversion*, 1 JUDGMENT AND DECISION MAKING 23 (2006) (arguing that loss aversion can be explained by status quo bias).

downside risk than upside potential.[112] Loss aversion may also partly explain the observed "disposition effect," i.e., the tendency of investors to sell winners and hold onto losers.[113]

The safety-first approach to portfolio choice claims that investors would do better by focusing on the chance of a disaster-level loss in a portfolio, rather than the portfolio's volatility.[114] In fact, Domar and Musgrave proposed a similar risk measure in their original taxation-and-risk paper. They defined risk as expected loss,[115] emphasizing the intuition that an investor worries most about losing money. ("This is the essence of *risk*.")[116] The idea was further refined by A.D. Roy,[117] writing around the same time as Markowitz,[118] and yet further by William Baumol.[119] Baumol, in particular, noted the limits of using

---

112 *See* Andrew Ang, Joseph Chen & Yuhang Xing, *Downside Risk*, 19 REV. FIN. STUD. 1191, 1193-94 (2006) (finding that stocks that covaried highly with the market during market downturns had greater risk premiums); Bali et al., *supra* note 84, at 884 (finding a "strong positive relation between downside risk and excess market return across different left-tail risk measures," including Value at Risk, Expected Shortfall, and tail risk); *see also* Shlomo Benartzi & Richard H. Thaler, *Myopic Loss Aversion and the Equity Premium Puzzle*, 110 Q.J. ECON. 73, 85-86 (1995) (arguing that that the equity risk premium can be explained in part by loss aversion); Robert F. Dittmar, *Nonlinear Pricing Kernels, Kurtosis Preference, and Evidence from the Cross Section of Equity Returns*, 57 J. FIN. 369, 400 (2002) (finding that investors prefer stock with lower kurtosis); Harvey & Siddique, *supra* note 85, at 1277-78 (finding that investors demand a premium from stock exhibiting skewness).

113 While at first glance this might appear to be risk-seeking activity, what may be driving the behavior is investors shifting away from risky assets and toward risk-free assets as wealth increases. Such a "portfolio insurance" strategy is consistent with loss aversion. *See, e.g.,* Francisco Gomes, *Portfolio Choice and Trading with Loss-Averse Investors*, 78 J. BUS. 675, 676 (2005). *But see* Terrance Odean, *Are Investors Reluctant to Realize Their Losses?* 53 J. FIN. 1775 (1998) (finding that the disposition effect is not explained by portfolio rebalancing).

114 Interestingly, the safety-first model and the mean-variance model may converge to the same optimal portfolio when the disaster level is equal to the risk-free return. *See* Haim Levy & Marshall Sarnat, *Safety First: An Expected Utility Principle*, 7 J. FIN. & QUANT. ANAL. 1829, 1831-32 (1972). This suggests that a mean-variance model may still perform well enough for some investors. To be clear, this potential equivalence does not challenge my argument that a safety-first investor would face a higher tax than a mean-variance investor. While the potential loss in my examples is increased by the amount of the tax on the risk-free return, the disaster level itself is unrelated to the risk-free return.

115 Domar & Musgrave, *supra* note 1, at 395. More formally, they define risk as the total of the probability weighted returns below zero. Thus, risk is the sum of all potential returns below zero, each multiplied by the probability that such a return occurs.

116 *Id.* at 396; s*ee also* LEVY, *supra* note 89, at 11 ("[Domar & Musgrave's] measures of risk are very appealing. Indeed, they conform with our intuition.").

117 A.D. Roy, *Safety First and the Holding of Assets*, 20 ECONOMETRICA 431 (1952).

118 Markowitz later wrote that he was "often called the father of modern portfolio theory (MPT), but Roy (1952) can claim an equal share of this honor." Harry M. Markowitz, *The Early History of Portfolio Theory: 1600-1960*, 55 FIN. ANALYSTS J., no. 4, 1999 at 5.

119 William Baumol, *An Expected Gain in Confidence Limit Criterion for Portfolio Selection*, 10 MGMT. SCI. 174 (1963).

variance as a risk measure, since it is not sensitive to variable risks of loss.[120] Variance measures only dispersion around the mean, not the size of a particular loss. If the expected return is high enough, returns that fall one or even two standard deviations below the mean may still be positive.[121] Similarly, if the expected return is lower, the same distribution around that expected return starts to have a higher frequency of losses.[122]

Roy and Baumol each suggested that a better risk measure than variance was the likely lower bound of possible portfolio returns. Baumol in particular proposed a risk measure that used variance to measure the likely lower bound of an investment, and proposed that portfolios with the higher lower bound were less risky. The lower bound itself would depend on an individual's risk tolerance.[123] Baumol also explicitly incorporated this risk measure into the MPT portfolio optimization problem, but instead of using a mean-variance model, he used a mean–lower confidence limit model.[124]

E. Value at Risk

Baumol put forward his risk measure in 1963, but it was not until the 1990s that the safety-first approach was incorporated into what is currently the leading risk measure for financial firms, Value at Risk ("VaR").[125] VaR measures "the worst loss over a target horizon with a given level of confidence."[126] VaR, like Baumol's risk measure, starts by choosing a low point in the distribution that is deemed to be the maximum possible loss under normal conditions. The key is to decide what normal conditions are, and at

---

[120] *Id.* at 174 (1963) ("[A]n investment with relatively high standard deviation will be relatively safe if its expected value is sufficiently high."); *see also* LEROY & WERNER, *supra* note 13, at 104 (noting that it follows from a more general definition of risk that if one portfolio is riskier than another, it also has a higher variance, while also noting that the converse is not necessarily true—a portfolio with higher variance than another is not necessarily more risky).

[121] *See* Baumol, *supra* note 120, at 174.

[122] The failure of variance to account for variable amounts of losses means that it fails to be a "coherent" risk measure, as defined by Artzner et al., since it does not appear to exhibit the property of monotonicity (i.e., it does not account for the fact that a portfolio with the same variance could be superior because of a higher mean). *See* Philippe Artzner et al., *Coherent Measures of Risk*, 9 MATH. FIN. 203 (1999). This property is very similar to the idea of first-order stochastic dominance. *See infra* Part III.C and note 99. Variance also appears to fail the property of translation invariance.

[123] He defined the lower bound itself as k standard deviations below the mean, where the value of k depended on the subjective degree of risk an investor was willing to tolerate. Thus his risk index was $E - k\sigma$, where E is the expected return and σ is the standard deviation (or the square root of the variance). For a normal distribution, therefore, the probability of return below that threshold was $1/k^2$. For $k = 3$, for example, returns would be below the lower bound only 0.1% of the time, and thus could be ignored, according to Baumol. *See id.* at 177.

[124] *See id.*

[125] JORION, *supra* note 72, at 22.

[126] *Id.*

what confidence level. So, for example, the VaR at 1% would be the value below which returns will fall only 1% of the time. An investor might then ignore the possibility of falling below that and treat the VaR amount as the maximum possible loss (though this is perhaps unwise, as discussed below).

The main advance that VaR made over Baumol and others was to figure out, at a technical level, how to incorporate an institution's entire portfolio across all financial products, taking leverage and asset correlations into account.[127] It thus attempts to capture an institution's exposure to market risk—not merely volatility—in a single value.[128] VaR has been hugely influential. It is now the leading risk measure for financial institutions and has been incorporated into a number of banking and securities regulations.[129]

Value at Risk is not without its problems, however. Most obviously, like Roy's and Baumol's risk measures, it provides only a lower bound, but says nothing about what happens should returns fall below that bound.[130] The actual returns can be (and, as we have seen, often are) far below the VaR amount itself. For example, suppose an investor's portfolio has a VaR of -$100 at a 1% confidence level. Thus, the investor would expect to have returns below -$100 only 1% of the time. But when that time comes, the actual loss could be -$101, or it could be -$1001, or more. Furthermore, with over two hundred trading days a year, an institution should expect to fall below such a threshold at least twice a year, even assuming a normal distribution.

A second problem with VaR is that some applications of VaR derive the lower bound using variance and assuming a normal distribution.[131] Thus the VaR level itself is likely to be too low, in absolute terms. If a return distribution actually exhibits excess kurtosis, or a "fat tail," on the downside, then we would expect losses greater than the VaR amount more than 1% of the time.[132]

---

127 *Id.* at xxii.

128 *Id.* at 25-26.

129 *See, e.g.,* 12 C.F.R. § 932.5; 17 C.F.R. §§ 240.15c3-1e, -1f, & -1g; 229.305; BASEL COMMITTEE ON BANKING SUPERVISION, BASEL III: A GLOBAL REGULATORY FRAMEWORK FOR MORE RESILIENT BANKS AND BANKING SYSTEMS 31-33 (2011) [hereinafter BASEL] (incorporating VaR in calculating bank capital requirements), *available at* http://www.bis.org/publ/bcbs189.htm

130 *See* JORION, *supra* note 72, at 488; HANS FÖLLMER & ALEXANDER SHIED, STOCHASTIC FINANCE: AN INTRODUCTION IN DISCRETE TIME 180 (2002).

131 *See* MANDELBROT & HUDSON, *supra* note 85, at 272-73. Other methods of calculating VaR, such as the historical simulation approach and the monte carlo approach, rely less heavily on the assumption of a normal distribution. *See* CROUHY ET AL., *supra* note 73, at 216-18; JORION, *supra* note 73, at 215 *et seq.*

132 If there is excess kurtosis on the loss side of the curve, that means that the portfolio will exhibit extreme low returns more often than if there were no excess kurtosis. Thus returns would fall below the VaR amount more frequently than assumed, if the VaR amount were calculated assuming no excess kurtosis.

The combination of these two problems—underweighting the likelihood of losses greater than nominal Value at Risk and failing to measure the potential magnitude of such losses—means that VaR does not fully capture the risk of extreme events.[133] Indeed, some have pointed to an over-reliance on VaR as a partial cause of the financial crises of 2007–08.[134] But a full accounting of the strengths and weaknesses of VaR are beyond the scope of this article. The point is simply that many financial economists and sophisticated investors have worked to develop more precise risk measures by focusing on downside risk, and worst-case scenarios in particular. The intuition that VaR and its predecessors work to capture is that there is a point at which losses go from being acceptable to unacceptable. While such losses are undoubtedly related to the volatility of potential returns—and thus for some applications variance remains an acceptable short-hand—variance alone cannot tell us what those losses could be, and therefore does not fully measure an investor's market risk.

Finally, it should be noted that Value at Risk and safety-first approaches to portfolio choice are not simply intended to reflect likely investor risk preferences, but may produce higher performing portfolios than a mean-variance model. The research on optimal portfolio choice is quite diverse, with many finely tuned models intending to optimize this or that. But some researchers have found that portfolio choice models that incorporate a focus on risk of loss or other downside measures tend to produce returns as good as or better than the traditional mean-variance model.[135] Again, this is likely because extreme events are more common than the mean-variance model assumes, and managing a portfolio to minimize them is likely to preserve capital better.

---

133 *See* MANDELBROT & HUDSON, *supra* note 85, at 272-73; Suleyman Basak & Alexander Shapiro, *Value-at-Risk-Based Risk Management: Optimal Policies and Asset Prices*, 14 REV. FIN. STUD. 371, 372 (2001) (showing that VaR-based risk management can lead to large unprotected losses, because of a focus on the VaR level itself, rather than potential losses exceeding the VaR level). To address both concerns, risk managers are generally encouraged to use additional risk measures, such as Expected Shortfall (also referred to conditional VaR, conditional loss, tail loss, and several other names). *See* JORION, *supra* note 72, at 97. Expected Shortfall estimates the average loss should losses go below the VaR threshold amount. It relies on similar assumptions about distributions as VaR, however, and thus can still underweight the likelihood of extreme losses. Risk managers are thus also encouraged (and in some cases required) to "stress test" their portfolios in order to model worst-case scenarios. *See* BASEL, *supra* note 129, at 46; JORION, *supra* note 72, at 231-53.

134 *See, e.g.,* Joe Nocera, *Risk Mismanagement*, N.Y. TIMES, Jan. 2, 2009 (Magazine), at MM24.

135 *See, e.g.,* Artzner et al., *supra* note 122, at 204; Bali et al, *supra* note 84; Bing Liang & Hyuna Park, *Predicting Hedge Fund Failure: A Comparison of Risk Measures*, 45 J. FIN. & QUANTITATIVE ANALYSIS 199, 220 (2010) ("We find downside risk measures are superior to standard deviation in predicting both the attrition and the real failure of hedge funds"); Bing Liang & Hyuna Park, *Risk Measures for Hedge Funds: a Cross-Sectional Approach*, 13 EUR. FIN. MGMT. 333, 359 (2007).

F. Summary

The above discussion demonstrates that, at a minimum, there no support in portfolio theory or expected utility theory for the idea that an investor would fully gross up in the face of an income tax. The basic model of portfolio choice, the mean-variance model, suggests that an investor would be unlikely to accept the same portfolio variance if the expected return dropped. As I have shown, the income tax will lower expected returns, assuming a positive risk-free rate, even if the investor does fully gross up.[136] As a result, we should not start from the assumption that an investor would try to recreate the same portfolio variance after tax.

Applying expected utility theory, with which the mean-variance model is compatible under certain assumptions,[137] we also would not expect to see the investor fully gross up. Under normal assumptions about risk, an investor would desire less risk if she faced lower expected wealth. Again, the tax will lower expected wealth, so even using variance as the measure of "risk," we would not expect an investor to try to recreate the same portfolio variance. This is underscored by the fact that, under a stochastic dominance measure of risk, the fully grossed-up after-tax portfolio is actually *riskier* than the no-tax portfolio (even if the variances are equal).

But modern portfolio theory and expected utility theory are not the end of the story. There is also support for theories that deviate from the assumptions of expected utility theory, namely prospect theory and safety-first portfolio theory.[138] Loss aversion and other findings of prospect theory do not conform well to expected utility theory.[139] Similarly, a safety-first investor is also not likely to be an expected utility maximizer in the conventional sense.[140] But these approaches may nonetheless accurately describe human behavior, and even lead to more optimal portfolios. If that is the case, then it turns out that the tax on risky returns is actually substantial, as the next section demonstrates.

---

136 *See supra* Part II.B.

137 *See supra* Part III.B.

138 My criticism of MPT and expected utility theory here are relatively limited, but others have gone much further. *See, e.g.,* Eugene F. Fama & Kenneth R. French, *The Cross-Section of Expected Stock Returns*, 47 J. FIN. 427, 445 ("In a nutshell, market *ß* seems to have no role in explaining the average returns on NYSE, AMEX, and NASDAQ stocks for 1963-1990"); Daniel Friedman & Shyam Sunder, *Risky Curves: From Unobservable Utility to Observable Opportunity Sets* (unpublished manuscript, one file with author) (arguing that 60 years of empirical research provides scant support for the classic expected utility approach to decision-making under uncertainty); *see also supra* note 85.

139 *See* Haim Levy & Moshe Levy, *Prospect Theory and Mean-Variance Analysis*, 17 REV. FIN. STUD. 1015, 1015-16 (2003).

140 *See* Levy & Sarnat, *supra* note 114, at 1830; Roy, *supra* note 117, at 432-33.

IV. The Domar-Musgrave Result Under a Safety-First Risk Measure

The prior section showed that it is an error to focus only on portfolio variance in considering how an investor would respond to an income tax. The section further argues that, consistent with much of portfolio theory, an investor may be better off focusing on market risk, i.e., risk of loss from fluctuating market prices, when optimizing a portfolio. This section returns to the numerical examples from Part II, but describes how an investor would make different portfolio shifts if she takes a safety-first approach to her investment portfolio. In Part II, the portfolio shifts were enough to gross the investor up and out of any income tax on risky returns. As will be shown here, however, a loss-averse investor will not actually make sufficient portfolio shifts to fully offset the tax, thus resulting in at least partial taxation of risky returns.

The argument, in a nutshell, is that a tax on the risk-free return is, by definition, a tax that applies in all states of the world, even one in which the investor faces ex post losses. In that case, an investor is deemed to have made a positive risk-free return, but to have risky losses that more than outweigh that gain, with the net effect being an overall loss. The existence of the tax in effect shifts the entire return distribution for a portfolio down by the amount of that tax. If an investor who is measuring risk using a downside threshold, such as VaR, tried to maintain the same portfolio variance before and after the tax, he would find that the after-tax portfolio would be likely to exceed the VaR threshold more than 1% of the time (or whatever the confidence level is). By definition, that would be an unacceptable degree of risk, and the investor would reallocate his portfolio accordingly, by somewhat reducing his holdings of risky assets.

The examples below show a possible behavioral response to a tax for an investor who focuses on risk of loss. In particular, I consider the simple case where an investor has a downside threshold below which she is not willing to go. This simple model is thus consistent with Value at Risk, Baumol's risk measure, and the other "safety-first" approaches to investment risk; but this being a stylized example, it is not strictly adopting one or the other of those approaches. Furthermore, other risk measures may generate different results, both in kind and degree.

A. An Income Tax Taxes Risky Returns

1. Investor Perspective

To see how the use of a downside risk measure changes the result, consider the examples from Part II.B, but altered slightly. Recall Example 2 in Part II.B. In that example, after full gross-up the investor's expected return was reduced by $4 compared to the pre-tax world, from $15 to $11. That $4 is equivalent to a 40% tax on the risk-free return on the entire $200 portfolio.[141] Furthermore, the potential losses are also increased by that same $4, from -$5 to -$9. Thus the overall volatility of the portfolio remains the same before and after the tax: +/- $20 around the mean—it is only the mean, the expected return, that changes. If variance is the proper measure of Investor's investment risk, then this portfolio is no "riskier" than his portfolio before the imposition of the tax.[142] But what if instead Investor is not willing to lower his potential losses by $4, from -$5 to -$9? What if he conceives of investment risk more as a negative threshold—the maximum he is willing to lose (such as in VaR)? Suppose Investor's earlier portfolio already optimized for that approach to risk, such that his optimal portfolio is one that maximizes returns, given a maximum loss of $5? In that case, he would not shift nearly as much of his assets from B to A.

> *Example 5:* Investor has the same beginning portfolio as in the earlier example—$100 in A and $100 in B—prior to the imposition of an income tax. The government imposes a 40% income tax. Investor is not willing to have potential losses below -$5. In that case, Investor will only shift $22.22 from B to A. Investor's after-tax portfolio will consist of $122.22 in A and $77.78 in B. Her portfolio will thus have a 50% chance of earning $24.33 ($22 from A and $2.33 from B) after tax, a 50% chance of losing $5 (-$7.33 from A and +$2.33 from B), and an expected return of $9.67.

Using this downside risk measure lowers Investor's expected return by $1.33 compared to using a variance risk measure, and thus increases the total cost of the tax from $4 to $5.33. Because we already know that $4 is the equivalent of a tax on the risk-free return on the entire portfolio, that additional $1.33 functions essentially as a tax on the risky portion of the portfolio. This amounts to a tax of about 21.77% on the expected risky return.[143] While not as large as the 40% nominal tax, it is still substantial. But this example is obviously stylized and different results could be obtained in a more realistic portfolio or with a different cost of capital.

---

[141] The risk-free return on the whole $200 portfolio is $10. 40% of $10 is $4.

[142] *See supra* note 70.

[143] Calculation: Decompose A into risky and risk-free returns. Risk-free return is $6.11 (5% of $122.22). This means that risky returns are +$30.556 ($36.67 – $6.11) or -$18.33 (-$12.22 – $6.11). So expected pre-tax return from risk premium is also $6.11. $1.33/$6.11 = 21.77%

2. Government Perspective

In the previous example I describe the forgone risky return as effectively a tax. Due to her risk preferences, Investor was not willing to shift toward risky assets by enough to fully offset the tax on risky returns. She thus gave up a higher expected return—$1.33, in the example. But to be clear, this is not simply excess burden or deadweight loss—under this model that $1.33 also ends up directly in the government's hands as additional revenue. How?

As discussed in Part II.C, under Kaplow's general equilibrium model of the Domar-Musgrave result, the government acts as the supplier of the additional risky assets demanded by investors, by selling them short in the market. Without this assumption, investors would run into the problem of a limited supply of risky assets, making them unable to make the portfolio shifts at a price necessary for the equivalence to hold.[144]

In Example 4 above, with the full gross-up, Investor sells $66.67 of B and buys $66.67 of A. Her pre-tax expected return on a portfolio of $166.67 of A and $66.67 of B is $18.33, which generates $7.33 in direct tax revenue for the government. However, because the government would be on the other side of those trades, it bought back $66.67 of B (its own bonds) and sold short $66.67 of A. The expected net return on that pair of transactions would be -$3.33, thus bringing the government's overall revenue down to $4, or the equivalent of simply taxing the presumed risk-free return on Investor's entire portfolio.

But, as in Example 5 above, if instead of selling $66.67 of B and buying $66.67 of A, Investor sold only $22.22 of B and bought $22.22 of A, the result is different. Under the example's assumptions a portfolio of $122.22 of A and $77.78 of B has a pre-tax expected return of $16.11, which would generate direct tax revenue for the government of $6.44. But again, the government is on the other side of these portfolio transactions, which means that the government bought back only $22.22 of its bonds and shorted only $22.22 of A. That pair of transactions will net the government -$1.11, making the total net revenue for the government $5.33—or $1.33 more than the $4 that it would earn if Investor had fully grossed up her portfolio. This lower Investor's expected returns by $1.33 translates directly into $1.33 of additional expected revenue for the government—hence, it is a tax.

B. A Tax on the Risk-Free Return Taxes Risky Returns

If the overall tax in this situation is greater than the nominal tax on the risk-free return of the portfolio, how can it still be said that an income tax is equivalent to a tax on the risk-free return? Recall that the equivalence approach to the Domar-Musgrave result says only that an income tax is equivalent to a tax on only the risk-free return. It says nothing about how

[144] *See supra* Part II.D.3.

either tax treats returns to risk-taking. As shown above, a normative income tax is likely to tax risky returns under reasonable assumptions about risk preferences, even in this idealized model. This section will show that the same result obtains if we instead introduce a tax on only the risk-free return.

This is of course a counterintuitive result; it is odd to say that a tax on only the risk-free return still taxes risk-taking. But the reason for this is the same as in the prior section—the tax will act to increase the risk of loss in all scenarios, and an investor who cares about downside risk will respond by decreasing her exposure to risky assets. That portfolio shift amounts to a tax on risk-taking.

1. Investor Perspective

First, return to the example in the prior section:

> *Example 6:* In a no-tax world, Investor has $100 invested in risky asset A and $100 invested in risk-free asset B. As before, A has a 50% chance of gaining 30% and a 50% chance of losing 10%. B returns 5%. In the absence of taxes, Investor has a 50% chance of her portfolio returning $35 ($30 from A and $5 from B), a 50% chance of losing $5 (-$10 from A and +$5 from B), and an expected return of $15.
>
> Now the government imposes a tax of 40% on the risk-free return of an entire portfolio. Because a portfolio is deemed to earn the risk-free return regardless of actual ex post returns, the risk-free return in all cases is $10 (5% of $200), which results in a tax of $4 in all cases. Without any portfolio shifts, this would increase Investor's downside risk from $5 to $9 and decrease her expected return from $15 to $11.

If Investor measured risk using the variance of portfolio returns she would not adjust her portfolio at all following the imposition of a tax on the risk-free return; because she is presumed to earn the risk-free return on her entire portfolio no matter what, shifting her portfolio will not alter the tax. The volatility of the portfolio is unchanged from the pre-tax world—it is still +/- $20. Her return profile is the same as in the income tax example above before considering downside risk.

As before, now consider the effect using a downside risk measure focusing on risk of loss:

> *Example 7:* As in Example 5 above, assume that Investor's risk preference is such that she does not want her downside risk to be greater than $5. In order to reduce her downside risk, she must shift her assets away from the risky asset A and toward the risk-free asset B. She will sell $26.67 of A and buy $26.67 of B. Thus Investor's after-tax

> portfolio will consist of $73.33 in A and $126.67 in B. Her portfolio will have a 50% chance of earning $24.33 after tax ($22 from A and $6.33 from B, less $4 of tax) and a 50% chance of losing $5 (-$7.33 from A and +$6.33 from B, less $4 of tax), for an expected return of $9.67.

Note that the expected return and the distribution of possible returns in this example is identical to those in Example 5 in the prior section under an income tax using the downside risk measure. In the income tax case in that example, Investor shifted less from B to A than she did using the variance risk measure. Here, instead of leaving her asset mix unchanged, she shifts somewhat away from A toward B. As in the income tax case, her expected return is $1.33 lower in the downside risk measure case than in the variance case. That amount is again effectively a tax on risk-taking—here because she reduced her exposure to the expected return that the risky asset gave her. Thus, even though the tax is only on the risk-free return, the imposition of that tax still leads to an additional $1.33 tax on risk-taking.[145]

2. Government Perspective

The tax on the risk-free return takes 40% of the presumed risk-free return. In the example above, that amounts to $4—that is the total tax bill. As in the income tax example above, we still have the question of how the additional $1.33 gets into the government's pockets. Under an income tax, Investor sold some holdings of B in order to buy more A. In the example described in this section, under a tax on the risk-free return, the opposite would need to occur. Under such a tax, Investor sold some of A and bought more of risk-free asset B. In order for that to hold in equilibrium, the government must act as the buyer of A. In the example, Investor sells $26.67 of A—which will end up being bought by the government. A has a positive expected return of 10%, so this generates $2.66 in expected returns for the government. At the same time, the government sells $26.67 of B to Investor, thus giving up a return of $1.33. (Assuming that government bonds are the risk-free asset, this is the same as the government selling $26.67 in additional bonds carrying a 5% coupon—thus requiring a $1.33 annual payment from the government to Investor.) The net gain to the government is therefore $1.33 ($2.66 expected return from A, less $1.33 in additional interest payments). Thus a tax on only the risk-free

---

[145] I argued at *supra* note 147 and accompanying text that we could distinguish certain losses of wealth from uncertain portfolio losses when considering a loss averse investors' reference point for calculating losses. Potentially the same argument could apply to a loss-averse investor under a tax only on the risk-free return. Because portfolio shifts will not affect the tax, it could be that the amount of the tax would not be seen as part of the portfolio "loss" in that case. If that is true, then the equivalence between an income tax and a tax on the risk-free return breaks down. For simplicity, however, I treat them as continuing to be equivalent.

return will generate net government revenue above and beyond the nominal risk-free return on all assets in the market. That additional return thus functions as an effective tax on the risky return to those assets.

C. What Is Being Taxed?

Part IV.A.1, *supra*, showed that an investor focusing on downside risk will face a lower expected return relative to an investor who, unrealistically, focuses only on portfolio volatility. To be clear, this effect is largely because of the effect on investor risk-taking due to the potential lower wealth in the future. I describe this as effectively a tax on the risky return because, in the example, that is all that is left to tax. However, we could imagine a similar response by the investor because of a potential loss of wealth outside of her portfolio. For example, suppose the tax was instead on height,[146] or number of homes, or some other base. If such a tax lowered wealth by $4, we could possibly see a similar portfolio response as in Example 7, but it would perhaps be a stretch to say that a tax on height was in part a tax on risky returns.

However, it is important here that the effect being described is on the thing being taxed, and that the response to the tax may change the effects of the tax itself. A tax on height, for example, would not be affected by portfolio changes.[147]

Furthermore, for a loss-averse investor, the focus is not on lower expected wealth, but on potential portfolio *losses*. Arguably a tax on height or on wages would not be interpreted as loss in the same way that portfolio loss would. In particular, there is no risk involved—the tax would be certain, and thus could affect a loss averse investor's reference point for calculating losses. The portfolio loss, on the other hand, can be avoided or mitigated through portfolio choice. Thus, for a loss-averse investor, an income tax will affect risky returns differently than some other tax or wealth loss. For these reasons, I describe the tax as being on risky returns in particular.

A related criticism is that the response shown in Example 5 is a function not of the tax, but of the investor's risk-aversion. After all, the only difference between Example 2 and Example 5 is a change in the assumption of how an investor thinks about portfolio risk. And it is admitted that an investor *could* gross up out of the tax on risky returns is she desired.

---

[146] *See* N. Gregory Mankiw & Matthew Weinzierl, *The Optimal Taxation of Height: A Case Study of Utilitarian Income Redistribution*, 2 AMER. ECON. J.: ECON. POL. 155 (2010).

[147] Moreover, a main purpose of this paper is to rebut the claim that an income tax would have *no* effect on risky returns—the fact that some other tax might have similar effect on risky returns is beside the point.

For this reason, some have made the more subtle claim that there is no tax on risky returns in risk-adjusted presented value terms.[148] Assuming that the market risk premium compensates for any additional risk being taken on, then the additional government revenue generated in Example 7 is offset by the additional risk. In risk-adjusted present value terms, that additional revenue would be $0, no matter what the relative allocation between risky and risk-free assets.

But this is assuming that an investor and the government agree on the market price for risk in the form of the risk premium. If the market risk premium on risky assets were exactly sufficient to compensate any investor, and the government, for the risk, then it could follow that the government ought to be indifferent to the actual revenue raised. But this is likely not the case, for at least two important reasons.

First, a loss-averse investor is by definition judging risk in a personal way, relative to a wealth threshold. A market risk premium can only cover that to a degree—at some point the risk of an individual's loss cannot be compensated by a risk premium set by investors generally. Under this paper's model, a full gross-up is too expensive even in risk-adjusted present value terms.

In contrast, the government is likely even less risk-averse than the market, and certainly less risk-averse than a loss-averse investor. This is because the government can easily and cheaply borrow to offset revenue shocks. If so, the income tax would raise *positive* revenue in risk-adjusted present value terms, which means the government has an appetite for absorbing some of this risk from the market. This leaves us in a situation where the government's risk discounting factor is probably less than the risk premium, while the investor's is greater. Thus, the ex post allocation between risky and risk-free assets, for the government and the investor, may represent a meaningful equilibrium price for risk, and not simply a random point on an indifference curve.

Second, it is likely the case that the equity premium is not related solely to risk; the risk premium is actually quite a bit higher than we would expect if it were merely compensating for non-diversifiable market risk.[149] Thus, again, the government ought to be happy to absorb additional risk—if the government's discount rate for risk is actually less than the market risk premium, then additional revenue from taxing risky return would be positive in risk-adjusted present value terms.

---

[148] *See, e.g.,* Roger H. Gordon, *Taxation of Corporate Capital Income: Tax Revenues Versus Tax Distortions*, 100 Q.J. ECON. 1, 5 (1985); Ethan Yale, *Investment Risk and the Tax Benefit of Deferred Compensation*, 62 TAX L. REV. 377, 392-93 (2008).

[149] *See, e.g.,* Rajneesh Mehra & Edward C. Prescott, *The Equity Risk Premium: A Puzzle*, 15 J. MONETARY ECON. 145 (1985) (finding that the equity risk premium is six times higher than standard theory would predict); RAJNISH MEHRA & EDWARD C. PRESCOTT, *The Equity Premium in Retrospect*, 1 HANDBOOK OF THE ECONOMICS OF FINANCE 889, 923 (George M. Constantinides, Milton Harris, René M. Stulz, eds., 2003) (reviewing literature).

D. The Risk-Free Rate

The magnitude of the tax on risky returns shown above depends directly on the magnitude of the risk-free rate. It is the nominal tax on risk-free returns that drives the wealth effect and increases the market risk of a portfolio. Those effects are, in turn, what cause the tax on risky returns. In Example 1 in Part II.B where the risk-free rate was zero, there was no tax on either risk-free or risky returns. If an investor can gross up her risky investments without cost, then all the drivers of portfolio behavior remain the same—the expected return, variance, and any measure of downside risk are the same in both the no-tax and after-tax worlds. As Example 2 showed, it is only once we introduce a positive risk-free rate that an income tax and a tax on the risk-free return start to tax investment returns.

Therefore, the magnitude of the relevant risk-free rate is directly relevant to any conclusions about the tax on risky returns. While the risk-free rate is almost certainly not zero, it could be quite small, in which case the effective tax on risky returns would also remain negligible.

A number of scholars have asked the question—what is the relevant risk-free rate of return when considering the Domar-Musgrave result?—and have taken a range of positions. At the low end, Noël Cunningham argues for a real risk-free rate of around 0.6%, pointing to the average real return on short-term Treasury bonds.[150] Similarly, David Weisbach describes the real risk-free rate as "historically close to zero."[151] Cunningham points out, however, that even that rate is quite variable, a point that Deborah Schenk also underscores, noting that from 1985–1989 the real, risk-free rate of return was actually 3.14%.[152] Furthermore, as both Cunningham and Schenk note, what is relevant, at least in the income tax case, is not the risk-free rate but rather the investor's borrowing cost,[153] which in many cases is likely to be greater than the applicable risk-free rate.

150 Cunningham, *supra* note 4, at 21.
151 Weisbach, *supra* note 4, at 2.
152 Schenk, *supra* note 5, at 473.
153 Cunningham, *supra* note 4, at 37; Schenk, *supra* note 5, at 432-33. *But see* Weisbach, *supra* note 4, at 13 n.21. Weisbach argues that the investor's borrowing rate is not relevant, because the investor can instead simply shift from risk-free to risky assets within a portfolio. This is consistent with the examples in Part IV.A, *supra*. Thus, Weisbach implies, if an investor holds T-bills paying, say, 1% and chooses to sell those to buy more risky assets, it does not matter that the investor's borrowing rate might be 3%, 5%, or 10%—the cost is the forgone T-bill return, or 1%. However, that does not address the other concerns raised here. Namely, that a) T-bill rates are likely less than the risk-free market return; b) using a short-term T-bill rate exposes the investor to interest-rate risk during the longer-term holding of the risky asset (i.e., the relevant rate is not just the 1%, but all the weighted average T-bill rates during the entire holding period of the risky asset, and such rates could be substantially higher); and c) inflation is not considered. From a normative perspective, the fact that an investor could in fact finance grossing up his risky asset holdings by selling underpriced T-bills does not affect the

Reed Shuldiner questions whether short-term T-Bills are the appropriate risk-free asset from which to derive the risk-free rate. He notes, first, that T-Bill rates might be lower than the true risk-free yield, because a number of T-Bill holders are effectively forced to hold them for non-investment reasons, such as capital requirements or fiduciary obligations.[154] They are effectively paying for a service by taking a lower return.[155] Second, the history of short-term T-Bills contain some anomalous periods of negative returns, which then drive the overall average down, depending on the choice of period.[156] Third, economic theory predicts that the proper risk-free-rate should be about equal to the growth in real per capita income, which (the last few years notwithstanding) is closer to 2%. The fact that risk-free rates tend to be below that is a puzzle to financial economists.[157] Indeed, different economic approaches, such as real business cycle theory, predict a risk-free rate closer to 4%.[158]

Shuldiner also questions the use of a short-term rate generally, arguing that the relevant rate should be for a period equal to the holding period for the risky asset. If the borrowing is to fund the gross-up, then it follows that the borrowing period should be the same as the holding period for the grossed-up asset. If an investor were to simply roll over short-term debt, that would introduce interest-rate risk as the rates change; as noted above, even the short-term T-Bill rate can be volatile. If instead, a longer-term Treasury bond were used as the benchmark, the stated risk-free rate is much higher. From 1972 to 1999, the real return on 20-year Treasury bonds averaged 3.3%, for example.

Furthermore, as Lawrence Zelenak argues, there are good reasons to question our past assumptions about the relationship between the risk-free rate and the risk premium.[159] Recent work has shown that the equity risk premium

---

arguments here. If the actual cost of grossing up is below the true risk-free rate, that is a bug, not a feature, of our current system.

154 Shuldiner, *supra* note 71, at 27.

155 *Cf.* Yair Listokin, *Taxation and Liquidity*, 120 YALE L.J. 100 (2011) (arguing that certain assets provide benefits in the form of liquidity, and that because such benefits are compensated for by lower returns, they go untaxed).

156 During the 1945-1972 period, the real return on one-month T-Bills averaged -0.5%, due to high inflation. As a result, the average for the 1945-1999 period is 0.5%. However, the average for 1972–1999 is 1.5%, while the average for 1802–1997 is 2.9%. *See* Shuldiner, *supra* note 71, at 19.

157 *See* Rajneesh Mehra & Edward C. Prescott, *The Equity Risk Premium: A Puzzle*, 15 J. MONETARY ECON. 145, 158 (1985) ("The equity premium puzzle may not be why was the average equity return so high but rather why was the average risk-free rate so low."); Shuldiner, *supra* note 71, at 29.

158 *See, e.g.,* Ellen R. McGrattan & Edward C. Prescott, *Is the Stock Market Overvalued?* NBER Working Paper 8077, at 21 (2001); MEHRA & PRESCOTT, *supra* note 149, at 923.

159 The risk premium is the difference between the nominal rate of return and the risk-free return, i.e., the additional return that an investor demands for investing in the risky asset rather than the risk-free asset. *See supra* note 22.

has declined over time and is likely to continue to be low for the foreseeable future, thus implying that risk-free returns make up a significant portion of capital income.[160] Furthermore, as Zelenak notes, the only truly risk-free assets now are Treasury Inflation-Protected Securities (TIPS)—as safe as other Treasury bonds, but also free of an inflation risk.[161] According to the Federal Reserve, the average interest rate on long-term TIPS has ranged between 1.19% and 2.54% over the last ten years.[162]

Finally, Shuldiner questions whether or not inflation risk should be considered. While the normative income tax used in this article's model is presumed not to tax inflationary returns, our income tax is certainly not indexed to inflation, and, as Shuldiner has shown in other work, doing so is likely not feasible.[163] This, again, would increase the appropriate measure of the risk-free return.

This discussion of the risk-free rate fits within the framework of an income tax, where an investor borrows money or sells risk-free assets in order to fund the gross-up in risky investments. Do these same points apply to a tax only on the risk-free return? After all, under such a tax an investor does not have to borrow and gross up in order to avoid any tax on risky returns. But the same issues would apply, because there would need to be some determination by the taxing authority as to what the applicable risk-free rate is. It is not enough to just, say, levy a tax on T-Bills—the theory is that any risky investment has a risk-free and a risky element to it, even an investment that loses money ex post. However, the bifurcation between the two cannot be observed—all that can be seen are the end results. So the government must declare what the relevant risk-free rate is, and all of the same considerations mentioned above come into play—what is the relevant time period, what is the relevant benchmark interest rate, should inflation be considered or not, etc.

As of this writing, the Netherlands imposes a tax on similar grounds. In lieu of a tax on actual capital gain, the Netherlands imposes a 30% income tax on a presumed 4% return, regardless of actual returns.[164]

It is beyond the scope of this article to make the affirmative case for a particular risk-free rate. Nonetheless, there are good reasons to think that the

---

160 *See, e.g.,* Robert D. Arnott & Peter L. Bernstein, *What Risk Premium Is "Normal"?* 58 FIN. ANALYSTS J., no. 2, 2002 at 81; Eugene F. Fama & Kenneth R. French, *The Equity Premium*, 57 J. FIN. 637, 638-39 (2002) (suggesting that a historical premium of about 4% was closer to the expected premium than the more recent premium of 5-6%); Zelenak, *supra* note 10, at 888-89 (summarizing studies that suggest the risk premium may have dropped to as low as 0.7% in the current period);

161 Zelenak, *supra* note 10, at 889.

162 Federal Reserve, *Selected Interest Rates (Daily) – H. 15*, http://www.federalreserve.gov/releases/h15/data.htm. Last accessed March 9, 2012.

163 *See* Reed Shuldiner, *Indexing the Tax Code*, 48 TAX L. REV. 537, 650-51 (1993).

164 *See* Kees van Raad, 973-3rd T.M., *Business Operations in the Netherlands* § VII.B.2.a(3) (BNA 2012).

relevant rate is higher than many consumption tax advocates claim, and is at least approaching the level that would impose a real tax on risky returns.

E. Derivatives

The discussion this far has dealt only with the simple case of an idealized stock and bond—the risky and risk-free asset. But in looking at the effects of taxation on investment risk and portfolio choice we must also consider portfolios that include derivatives, i.e., financial products that can isolate certain types of risk of the underlying assets. But allowing the investor to also hold derivatives does not change the result. The reason is that the cost of entering into a derivative contract generally includes a foregone risk-free return,[165] and thus the situation is the same as if there were a tax equal to the risk-free return.

To see this, consider the simple case of a forward contract.[166] Suppose that in the absence of taxes, instead of investing $100 in the risky asset and $100 in a risk-free, our investor holds $200 in the risk-free asset, but enters into a forward contract to purchase the risky asset in one year at $105. Why a strike price of $105? Because here the long party, the investor, gets the economic return of actually owning the underlying risky asset, but without actually having to part with the money; the short party has essentially loaned the investor the $100 purchase price and will expect a time-value-of-money return on that.[167] But this higher price means that the investor has shifted the risk-free return in the underlying to the short party. The investor earns a greater risk-free return from her own portfolio, but then gives up a portion of that return to the short party.

> *Example 8A:* Investor has a portfolio of $200 in risk-free asset B and a forward contract to pay $105 for risky asset A in one year. As before, A will return either 30% with 50% probability or lose 10% with 50% probability. Thus A will be worth either $130 or $90 in one year. When Investor settles the contract, her return will be either $25 or -$15, while she earns a risk-free $10 from B. Therefore, her portfolio return is either $35 or -$5 with an expected return of $15, just as in the first part of Example 2, before the tax was imposed.

If the government imposes a 40% tax, Investor could respond by increasing the quantity under the forward contract. So instead of buying the

[165] *See, e.g.,* David M. Schizer, *Balance in the Taxation of Derivative Securities: An Agenda for Reform*, 104 COLUM. L. REV. 1886, 1902 (2004).

[166] A forward contract is a promise to purchase something at a specified price at some point in the future.

[167] *See* Shizer, *supra* note 165, at 1902.

equivalent of $100 of A, she could commit to buy the equivalent of $166.67 of A. (For simplicity, we will say that she agrees to purchase 1.667 of A at a price of $105 per unit.) It appears to be costless to gross up in this way, since she pays nothing when she enters into the contract. But she in fact increases the size of the risk-free return she transfers to the short party:

> *Example 8B:* Due to the tax, Investor increases the forward contract quantity of A to 1.667, at a strike price of $105, and continues to keep her $200 all in the risk-free asset B. If in one year A is worth $130, her pre-tax return on the forward contract will be 1.667*25 = ~$41.68, which is $25 after taxes. Similarly, if A loses, her after-tax loss will be -$15. Her pre-tax return from B is still $10, but her after-tax return from B is lowered to $6.

In Example 8B, increasing the size of the forward contract puts her right back where she was in the no-tax world *with respect to the risky asset*. But, crucially, her after-tax return on B is lowered from $10 to $6. The $4 difference is the net cost of the tax, just as it was in Example 2. That cost increases her downside risk, just as with the simple portfolio of just A and B, and if she is loss averse we are right back to same situation discussed above.

This conclusion should not be surprising, given the put-call parity theorem. That theorem holds that the combination of a put option and call option at a single strike price is equal to the underlying stock less a risk-free bond that pays the strike price:[168]

$$C_k - P_k = S - B_k$$

Where C_k is the value of a call option on stock S at strike price k, P_k is the value of a put option on stock S at strike price k, and B_k is the value of a zero-coupon risk-free bond that pays k matures at maturity.[169] But note that the combination of a call and put option at price k is equivalent to a forward contract at strike price k—either way, the option holder is paying k for the underlying.[170] So, just as shown in the example, a forward contract is equivalent to holding the underlying asset, but giving up the risk-free return.

[168] *See* Schizer, *supra* note 165, at 1929; Alvin C. Warren Jr., *Financial Contract Innovation and Income Tax Policy*, 107 HARV. L. REV. 460, 465-67 (1993).
[169] Note that in the examples, the risk-free rate is 5%, and thus the bond pays $105—which is the strike price of the forward contract.
[170] If the spot price is less than k, the counterparty will exercise the put, forcing the investor to buy at price k. If the spot price is greater than k, the investor will exercise the call and buy at price k.

Moreover, the theorem also shows that we cannot avoid giving up the risk-free return with some other combination of derivatives.[171]

V. Implications for the Debate Between an Income Tax and a Consumption Tax

The discussion thus far presents an argument for why, under plausible assumptions about investor risk preferences and stock-market behavior, a normative income tax will effectively tax risky returns. But what implications does this have on the debate between a consumption tax and an income tax? This section considers two important implications: the differential treatment of labor and capital income, and the differential treatment of winners and losers ex post. To be clear, this discussion is limited to the high-level theoretical comparisons between the two taxes. While this is generally where the discussion lies in the taxation-and-risk literature,[172] there is obviously much more that can be said about the virtues and vices of either tax and this is not intended to be a comprehensive comparison.

A. Differential Treatment of Labor and Capital

A simple cash-flow consumption tax operates by expensing—providing a full deduction for—amounts saved and invested, but then taxing the full amount of savings (plus any appreciation) as it is withdrawn for consumption—as it "flows" into cash for the taxpayer to use in consumption. The result is that amounts are only taxed if they are used for consumption, not as they are earned. As others, most notably Cary Brown and William Andrews, have shown, this structure is equivalent under certain assumptions to taxing labor income but not capital income—to a "yield exemption" consumption tax.[173]

A full explanation of the operation of a cash-flow consumption tax is beyond the scope of this article, and has been explained in detail elsewhere.[174] The key feature for purposes of this discussion, however, is that equivalence to yield exemption arises because of the same sort of grossing-up possibilities discussed above. The value of the expensing deductions can allow an investor to gross up costlessly, in a way similar to that in Example 1. However, instead

---

171 Indeed, the investor would be probably worse off if she tried to buy only the upside risk. Then $C_k = S - B_k + P_k$, i.e., owning a call option costs not only the risk-free return but also the value of the put option (in other words, the call option costs money up front). This would increase potential downside risk even further, arguably leading a risk-averse investor to gross up even less than in the examples here.

172 *See, e.g.,* Cunningham, *supra* note 4, at 17-20; Weisbach, *supra* note 4 at 1-2.

173 *See* Andrews, *supra* note 17; Brown, *supra* note 17.

174 *Id.*

of shifting assets from risk-free assets toward risky assets (or borrowing to add to the investment in risky assets), the investor can use the tax benefit from expensing to gross up *both* risk-free and risky investments without cost, and thus offset any nominal tax on the investment yield.[175] As a result, it is said that a cash-flow tax effectively taxes *neither* risk-free nor risky returns, and thus that the key difference between a cash-flow tax and an income tax is that an income tax taxes the risk-free return.

However, this conclusion neglects three key complications. First, even assuming full gross-up, an income tax would raise *more revenue* than a cash-flow tax at the same rate. The tax on the risk-free return[176] is a real tax that raises revenue under an income tax, but not under a cash-flow tax. Thus the real comparison is not between taxing the risk-free rate or not, but between taxing the risk-free rate and having a larger government on the one hand, and not taxing the risk-free rate and having a smaller government on the other.[177]

Thus, to truly compare the two tax systems independently of government size, we would have to increase the cash-flow tax rate. This would create a nominally higher tax on labor income under a cash-flow tax than under an income tax.

This leads to the second key complication, which is that the higher tax on wages under a cash-flow tax could then play a similar role, under this article's model, as the tax on the risk-free return under an income tax: it will create a wealth effect and, possibly, affect the investor's downside risk. Assuming (for a moment) homogenous taxpayers, the "extra" tax on wages would have to exactly equal the "extra" tax on the risk-free return under an income tax.[178] It would thus bring the investor the same amount closer to her downside threshold.

Using the examples from earlier, the additional tax on wages would have to raise $4 in order to have a revenue-neutral comparison to an income tax or a tax only on the risk-free return. Even though the investor can gross up her investments costlessly, and thus recreate exactly the same after-tax portfolio as

---

[175] If, as in the examples above, an investor wished to have a $200 portfolio in the no-tax world, she could invest $333.33 under a cash-flow tax to achieve the same result. This investment would generate a tax deduction worth $333.33*.4= $133.33. Thus the government would essentially be funding the gross-up from $200 to $333.33. Furthermore, the gross-up would be spread pro rata among all the investments in the portfolio. Instead of $100 in A and $100 in B, the investor would have $166.67 in A and $166.67 in B (in contrast to Example 2, where the investor had $166.67 in A and $33.33 in B).

[176] Recall that even under full gross-up and ignoring the changes in wealth and market risk, a income tax still raises the same revenue as a tax on the risk-free return.

[177] I am grateful to Louis Kaplow for this observation.

[178] As noted *supra* note 46, the comparison to a non-tax world is problematic. One could instead consider starting in a world that had only a nominal wage tax, and then compare the move to an income tax on the one hand and a cash-flow consumption tax on the other. It is in that sense that the taxes on the risk-free return and the higher tax on wages, respectively, could be seen as "extra."

in a no-tax world, she will nonetheless be $4 poorer than otherwise. Assuming the same risk preferences as before, she may wish to change her portfolio allocation in order to offset that additional risk of crossing her downside threshold. Therefore, she may not fully gross up after all, and thus would face the same sort of tax on risky returns as under an income tax.

At a first cut, this is essentially an extension of the point made by William Gentry and Glenn Hubbard, and also by David Weisbach, that an income tax and a cash-flow tax have the same treatment of risk, because we would expect to see the same sort of grossing up behavior in the face of risk under either tax.[179] However, the third implication is that the *source* of the wealth effect and market risk is important. Under a cash-flow tax, the effects arise because of a higher tax on wages; under an income tax they arise because of an effective tax on capital income. This difference has important distributional consequences.

Instead of homogenous taxpayers, imagine two taxpayers, one with exclusively wage income and one with exclusively capital income.[180] Under an income tax, the risk-free return of the capital-earner would be taxed, generating a wealth effect that would further tax risky returns. Under a cash-flow tax, the wage-earner would face a greater tax on wages than under an income tax. But the capital-earner would not. No matter what the cash-flow tax rate, the capital-earner could offset it by grossing up, without any risk of losing more wealth. Because the extra tax is borne by a different taxpayer, there would be no effect on the capital-earner's portfolio.[181]

Therefore, under plausible assumptions about the distribution of labor and capital income, the size of the risk-free rate, and investor risk preferences, the major difference between an idealized, normative cash-flow consumption tax and an idealized, normative income tax is not merely the tax on the risk-free return. Rather, the difference is a higher tax on wages under a cash-flow tax, and a higher tax on capital—both risk-free and risky—under an income tax.

---

179 *See* Gentry & Hubbard, *supra* note 6, at 8; Weisbach, *supra* note 4, at 7. Gentry, Hubbard, and Weisbach discuss the role of the government in providing sufficient risk to the market to allow for the grossing up, and not wealth effects *per se*. But the same point holds. The limits to full gross-up—whether because of limited supply of risky assets or because of wealth effects—should be the same under either tax.

180 This is not entirely farfetched. In 2008 roughly 50% of the AGI of taxpayers earning more than $200,000 a year was in the form of capital gain, dividend, interest, and business income. *See* Tax Policy Center, *High Income Return Details, 2000-2008*, *available at* http://www.taxpolicycenter.org/taxfacts/displayafact.cfm?Docid=396. For the highest 400 returns, those items made up more than 93% of AGI. *See* Tax Policy Center, *Returns of Taxpayers with the Top 400 Adjust Gross Income, 1992-2008, available at* http://www.taxpolicycenter.org/taxfacts/displayafact.cfm?Docid=260.

181 To the degree that the wealth effect itself also generates additional revenue under an income tax, as suggested by Example 6 in Part IV.A.2, the cash-flow tax rate would have to be even higher in order to maintain revenue neutrality, since the capital-earner does not face the tax. This would require increasing yet more the tax on the wage-earner.

At one level, this is not a surprising result. Most policy discussions of a consumption tax essentially conclude that there would be distributional implications of a shift from an income tax to a consumption tax.[182] But among tax law scholars, it has become close to conventional wisdom that, at least in a pure idealized world, there would actually be little to no difference at all, or that the difference is limited to the treatment of the risk-free return.[183] This analysis suggests that this is not the case.

B. Differential Treatment of Winners and Losers

One line of defense for an income tax is its different treatment of winners and losers.[184] Those who win their risky bets are better off, and thus ought to face higher taxes; those who lose are worse off and ought to be able to reduce their tax accordingly. The typical treatment of the taxation-and-risk question has challenged whether this is possible. If an investor would always fully gross up and thus avoid the tax on risky returns, that it would not be possible to treat winners and losers differently.

The same reasoning applies to the equivalence between a cash-flow consumption tax and a yield-exemption consumption tax. The latter simply ignores ex post results, but it is nonetheless equivalent to the former, which nominally does include ex post results in the tax base. The equivalence is, again, because if an investor fully grosses up, the increase in his gains would wipe out the tax on those gains, while the increase in losses would wipe out the value of the deduction of those losses.

But what about the case where the investor does not fully gross up? Consider the partial gross up described in Example 5. In that case, if the Investor "wins" and A returns 30% ex post, he will have pre-tax gains of \$40.56 (\$36.67 from A + \$3.89 from B). After tax, this is reduced to \$24.34. Recall that in the no-tax world, Investor would have earned \$35 if A's return was positive. Thus, he effectively faces a tax of \$10.66. Because, again, \$4 is the tax on the risk-free return, that means a \$6.66 effective tax on risk—higher than the \$1.33 expected ex ante tax on risky returns. This is in contrast to the

---

[182] *See, e.g.,* Michael J. Graetz, *Implementing a Progressive Consumption Tax*, 92 HARV. L. REV. 1575, 1581-82 (1979); John K. McNulty, *Flat Tax, Consumption Tax, Consumption-Type Income Tax Proposals in the United States: A Tax Policy Discussion of Fundamental Tax Reform*, 88 CAL. L. REV. 2095, 2129-30 (2000) (citing Treasury estimates of a particular consumption tax proposal); Shaviro, *supra* note 4, at 97. *But see* President's Advisory Panel on Federal Tax Reform, SIMPLE, FAIR, AND PRO-GROWTH: PROPOSALS TO FIX AMERICAN'S TAX SYSTEM 153 ("A pure income tax and a "postpaid" consumption tax … differ only in their treatment of the return to waiting.").

[183] *See, e.g.,* Bankman & Fried, *supra* note 4, at 542. [MORE (PANEL?)]

[184] *See, e.g.*, Graetz, *supra* note 182, at 1601 ("Circumstances should be considered as similar only after results are known; lucky gamblers are not the same as unlucky gamblers."); Warren, *Consumption Tax, supra* note 2, at 1098 ("[F]airness in taxation should depend on outcomes, not expectations.").

full-gross up example where the positive return would have been \$31, \$4 less than the no-tax positive return, exhibiting no tax on risky returns.

Similarly, if the bet "loses," then Investor would be down \$5 after tax. This is (by assumption) the same as in the no-tax world. But a nominal \$4 tax should apply in the after-tax world. Thus Investor receives the equivalence of a \$4 deduction (canceling out the \$4 in tax).[185] A 50% chance of "paying" \$6.66 plus a 50% chance of "deducting" \$4 gives us an expected tax ex ante of \$1.33, just as Example 5 concluded. But we now have differential treatment of winners and losers ex post.[186] Furthermore, the immediately preceding section implies that we would not have such treatment ex post under a cash-flow tax, when there are differences between wage-earners and capital-earners; in that case, the capital-earner would continue to fully gross up and offset whatever tax or deduction might apply ex post. Thus, unlike under an income tax, there would continue to be no differential treatment of winners and losers under a normative cash-flow consumption tax.

VI. Conclusion

This article presented an argument for how and why an income tax taxes capital income. While that result is perhaps not surprising to many readers, it is nonetheless contrary to the majority of the legal literature addressing the taxation-and-risk question, and the related question of the theoretical differences between an income tax and a consumption tax. I have argued herein that much of the legal literature makes mistaken assumptions about investment risk and portfolio optimization, and thus neglects or understates the resulting tax on risky returns.

There is no question that this is a theoretical result. We do not have a pure, normative Haig-Simons income tax, nor, arguably, should we. We also do not have the complete capital markets that the Domar-Musgrave result requires, and so on. This paper is not arguing that capital income is effectively taxed only because of the effects I describe here. In fact, capital income does face a real and material tax under our current income tax system.[187]

Nonetheless, theory matters. As David Weisbach has argued, if we dislike the way that our current tax system deviates from a normative Haig-Simons income tax, then it is relevant to look at such a normative income tax for guidance on what a more ideal tax system might look like and what effects it

---

185 It is no coincidence that the value of the effective deduction equals the nominal tax imposed under full gross up—since, by design, the investor was altering his portfolio precisely to offset the downside exposure that tax created.

186 I am grateful to Dan Halperin for suggesting this conclusion.

187 *See, e.g.,* JOEL SLEMROD, *Does the United States Tax Capital Income?* in TAXING CAPITAL INCOME 3 (Henry J. Aaron, Leonard E. Burman & C. Eugene Steurerle eds., 2007) (summarizing literature finding a positive effective tax rate on capital income).

might have.[188] Pointing to the Domar-Musgrave result, Weisbach argues that a normative income would tax so little capital income as to be vanishingly close to a consumption tax. Thus, he argues, supporters of a Haig-Simons income tax ought to in fact prefer a consumption tax to our imperfect tax system.[189]

Yet, as I have argued here, that conclusion only follows if an investor is no more risk-averse after the tax, and if her fully grossed-up portfolio is no riskier. As this article demonstrates, neither is true where there is a positive risk-free rate. In particular, the fact that the portfolio is actually riskier—has a chance of greater loss—has not been clearly identified before now, and this additional effect adds to the effective tax on risky returns under a normative income tax.

If this is the case, then a normative income tax is actually materially different from a consumption tax—returns to capital are likely to face a materially higher tax under an income tax than under a consumption tax, even under the idealized model used here. If capital markets are perfect, we would see little to no tax on capital under a cash-flow tax. But even if they are not (e.g., because the government does not actively manage its portfolio), the tax on capital under an income tax would remain higher than that under a cash-flow tax.[190]

The magnitude of the tax would depend on the relevant risk-free rate and the nature and degree of investor risk-aversion, which are ultimately empirical questions. Under this model, the effective tax rate on capital is still lower than the nominal tax rate, and that would present complications. But capital is taxed nonetheless.

The default treatment of taxation-and-risk issue in most of the legal literature is that an investor would fully gross up to offset the tax. Only after that is presented, do some commentators present the wealth effect, as a complication to that default treatment.[191] As my discussion shows here, however, there is no theory of investor behavior that would lead to an investor fully grossing up—fully grossing up is not consistent even with orthodox portfolio theory, much less with the further criticisms and approaches to portfolio management that I present here. Tax law scholars should thus avoid presenting the Domar-Musgrave result as the non-taxation of risky returns; as long as the risk-free rate is positive, there will be a tax on risky returns under a normative income tax.

As I have stated throughout this article, my argument does not disrupt the underlying theorem of the equivalence of a normative income tax and a tax on

188 Weisbach, *supra* note 4, at 35-38.
189 *Id.*
190 If we relax the assumption regarding the government's active portfolio policy, some tax on risky returns would appear. *See* text at notes 66-69. But that tax would be in addition to the effective tax described in this article, which arises from the tax on the risk-free return. Thus the difference between a consumption tax an income tax would continue to be the effective tax on risk-free and risky returns described herein.
191 *See, e.g.,* Weisbach, *supra* note 4, at 18.

wages plus the risk-free return to capital, as demonstrated by Kaplow. Thus, while it provides an argument in the debate between a consumption tax and an income tax, it is indifferent between a normative income tax and tax on the risk-free return. However, once we enter the real world again, the debate is not so clear. The ways in which our actual income tax system deviates from a normative Haig-Simons tax may have distributional consequences. Even in this article's model, investors are still making portfolio shifts, and if the abilities of investors to do so are not equitably distributed, the effective tax on capital will differ among investors. Deborah Schenk has argued therefore for replacing the tax on capital income with a wealth tax, which is essentially the same thing as an income tax levied on a presumed return to wealth.[192] What this article shows is that even if such a tax targeted only a risk-free return, it could still reach risky returns, which is, I believe, an appropriate result.

On the other hand, a tax only on the risk-free return would be unlikely to reach inframarginal returns—returns to rent-seeking, asymmetric information, or other unequal investment opportunities that appear as returns to capital.[193] An income tax base would capture these returns ex post, while a tax on only the risk-free return would not. However, if the tax were to be imposed on some imputed return, there is no inherent reason why it must be pegged at the risk-free rate. A higher implied rate could be used as a crude approximation of inframarginal returns and disguised labor income,[194] for example, though this would have horizontal equity implications.[195]

Ultimately, then, the policy choice would depend on weighing these different approaches along with the costs of transition. But income tax supporters need not give up the idea of taxing capital. Even assuming the most idealized normative Haig-Simons income and the most rational investors, returns from investment risk-taking are taxed.

---

[192] *Schenk*, *supra* note 5.

[193] *See supra* note 22.

[194] Such as in the Netherlands. *See* text at *supra* note 164.

[195] Assuming that inframarginal returns are not distributed pro rata among taxpayers, a tax rate that targeted average inframarginal returns would undertax those who did have inframarginal gains and overtax those who did not.