Redistribution under Super-majority Rule

Giri Parameswaran*
Haverford College
gparames@haverford.edu

Hunter Rendleman
Harvard University
hrendleman@g.harvard.edu

August 17, 2018

Abstract

The median voter theorem provides a simple and unique characterization of equilibrium redistribution under majority rule. However, its analogue under super-majority rule – the core – contains infinitely many policies. We use a bargaining approach to select a unique robust policy from within the core. Our refinement is characterized by bilateral asymmetric Nash Bargaining between two players whose identities depend on the super-majority rule. The bargaining weights are determined endogenously and depend on the full income profile. We show that, under standard conditions, increasing the required super-majority will reduce the level of taxation and redistribution.

Key Words: Redistribution, Super-majority Rules, Core, Endogenous Factions.
JEL Codes: C7, D7, H2

*Corresponding Author. Department of Economics, Haverford College, 370 Lancaster Ave, Haverford PA 19041.

1 Introduction

Beginning with the seminal contributions of Romer (1975), Roberts (1977), and Meltzer and Richard (1981), a long literature has developed that explains the features of redistribution schemes, as a consequence of the preferences of the median voter (see Epple and Romer (1991), Gans and Smart (1996), Besley and Coate (1997), Persson and Tabellini (1999), Moene and Wallerstein (2001), amongst many others). However, the logic of the median voter theorem assumes, amongst other things, that decisions be made by simple majority rule, and this logic fails under super-majority rule. Nonetheless, super-majority requirements for fiscal policy measures are not uncommon. Indeed, 14 state governments require a legislative super-majority either to raise taxes, or in some cases, to pass a budget at all.[1] Additionally, proposals to institutionalize super-majority budget rules on the federal level via a constitutional amendment have been repeatedly introduced into Congress (albeit unsuccessfully).[2]

In this paper, we provide an analysis of redistribution under a super-majority voting rule. Our approach retains the spirit of the logic of the median voter theorem in that we associate the equilibrium with a policy in the *core*.[3] However, unlike the case of simple majority, where the core is uniquely the median voter's ideal policy, under super-majority rule, the core generically contains (infinitely) many policies. Moreover, amongst this multiplicity, the core will generically contain policies that are both above and below the median voter's ideal policy – providing little guidance as to even the qualitative effect of super-majority requirements.

This paper makes two main contributions. First, we use a bargaining approach to select a unique robust policy from the multiplicity within the core. Whereas the core is an equilib-

[1] See Rueben and Randall (2017)

[2] See *H. J. Res. 111* (1998), *H. J. Res. 41* (2001)

[3] The *core* is the set of unbeatable policies; i.e. those for which there does not exist some other policy that is strictly preferred by a super-majority.

rium concept arising out of cooperative game theory, we use the seminal non-cooperative bargaining model of Baron and Ferejohn (1989) to justify a particular core refinement. Furthermore, we demonstrate that this refinement has a simple and elegant characterization – it is the result of bilateral asymmetric Nash Bargaining between two voters whose identities depend on the super-majority rule, with endogenous bargaining weights that depend on the full profile of income. Second, we perform comparative static analysis on super-majority size, and show that under standard conditions, taxation and redistribution are monotonically decreasing in the size of the required super-majority.

We present a simple public finance model in which there is a continuum of agents distinguished by their income. Redistribution is via a linear income tax (a proportional tax coupled with a uniform transfer to all agents), akin to the simplest model of a Universal Basic Income. For simplicity, we capture the dead-weight losses from taxation in reduced form by assuming that redistribution is via a 'leaky-bucket' (see Dixit and Londregan (1998), Moene and Wallerstein (2001), amongst others). In section 5.2, we micro-found these dead-weight losses in a model with elastic labor supply. Since the benefits of taxation are shared equally, but the costs fall more heavily on higher income earners, voters' most preferred income tax rates are decreasing in their incomes.

Bargaining in the Baron and Ferejohn (1989) framework proceeds as follows: There are potentially infinitely many rounds of bargaining. In a given round, a voter is randomly recognized to propose a redistributive policy. The policy is accepted if it receives the assent of at least a measure $q \geq \frac{1}{2}$ of voters. If so, it is implemented in all successive periods and bargaining ends. Else, a reversion (or 'status quo') policy is implemented in the current period, and the process repeats in the following period. Bargaining continues until a proposal is accepted. Agents discount the future at common rate $\delta \in [0, 1)$, which implies that delay is costly. Thus, δ parameterizes the (implicit) cost from rejecting a proposal and waiting for a better counter-proposal.

For any $\delta < 1$, we show (analogously to Cardona and Ponsati (2011)) that the bargaining game admits a unique no-delay equilibrium in stationary strategies. The equilibrium characterization depends crucially on the preferences of two agents, those at the $(1-q)^{th}$ and q^{th} quantiles of the income distribution, whom we call the *left* and *right decisive voters*, respectively. We show that, since, to be accepted, a proposal must receive the support of a measure q of agents, it suffices to receive the support of each of the decisive agents. The set of proposals accepted in equilibrium, then, is an interval, whose boundaries are pinned down by the preferences of the decisive agents. Following Predtetchinski (2011), we show that, as $\delta \to 1$, this set of acceptable proposals shrinks to a unique limit policy. Intuitively, when δ is low, and the cost of delay is high, voters are less wont to reject proposals, and so the proposer has a greater ability to propose a policy closer to her ideal. As $\delta \to 1$, this proposer advantage disappears, since more patient voters will be more willing to hold out for better offers.

The left and right decisive voters also determine the core. Indeed, the core corresponds to the set of policies between the ideal tax rates of the right and left decisive voters. We define the *core*$^+$ as the subset set of core policies that are weakly preferred to the reversion policy by both decisive voters. If the reversion policy is sufficiently extreme, the core and core$^+$ will coincide. By contrast, if the reversion policy is in the core, then the core$^+$ is simply the reversion policy.

There is a relationship between the equilibria of our bargaining game and the core+. When $\delta = 1$, the bargaining game admits multiple equilibria, each associated with a given policy that is proposed and accepted by all players (see Banks and Duggan (2000). In fact, the set of policies that can be sustained as an equilibrium exactly coincides with the core$^+$. This is intuitive. When the cost of delay goes to zero, the friction that enables 'sub-optimal' policies to survive, disappears. If a policy outside the core were proposed, a super-majority can costlessly reject the proposal and wait for a more desirable policy in the core to be counter-proposed.

This motivates our equilibrium refinement. Although the bargaining game admits multiple equilibria coinciding with the core$^+$ when $\delta = 1$, it admits a unique equilibrium for every $\delta < 1$. The equilibrium correspondence exhibits a failure of lower hemi-continuity at $\delta = 1$. Taking the limit as $\delta \to 1$, then, selects the unique core policy that is robust to introducing small costs to making counter-proposals. Our approach is analogous to Cho and Duggan (2009), who show that, under simple majority rule, this limit selects the median voter's ideal policy – thus providing bargaining micro-foundations for the median voter theorem. However, in contrast to Cho and Duggan (2009), we show that, under super-majority rule, the selected policy generically does not coincide with the median. Rather, the selected policy is chosen as if by bilateral asymmetric Nash Bargaining between the left and right decisive voters, with endogenous weights that depend on the entire income distribution.

Our endogenous Nash Bargaining characterization admits the following interpretation: Although voters are heterogeneous in their preferences, they separate into two cohesive factions led by the left and right decisive voters. Policy is chosen as a consequence of asymmetric Nash Bargaining between the factional leaders, and their bargaining strengths depend on the sizes of their respective coalitions. Voters, in turn understand that their factional choice affects the policy that results, and choose which faction to join, accordingly. The equilibrium coalitions are stable, in the sense that, given the policy that results, no voter would want to switch factions. Our result can thus be interpreted as a super-majority analogue to Duverger's Law, in that, it micro-founds the emergence of two cohesive factions or 'parties'. However, these endogenous factions need not be equally sized (although neither faction will contain the required super-majority), and will have non-median factional leaders.

A different way to interpret our Nash Bargaining result is that provides a procedure for identifying the 'pivotal' voter.[4] Meltzer and Richard (1981) argue that the level of redistri-

[4]We distinguish the 'pivotal' voter from the (left and right) 'decisive' voters. The former is the agent

bution in any polity should reflect the ideal level of the pivotal voter. In the case of simple majority rule, they associate the pivotal voter with the median income earner. By contrast, under super-majority rule, we show that the pivotal voter will generically not be the median. Instead – understanding the Nash Bargaining procedure as selecting an outcome that is an 'average' of those preferred by the parties to the bargain – we show that the pivotal agent is the voter whose income is a particular weighted generalized mean of the incomes of the left and right decisive agents. Naturally, this voter is the one who is indifferent between joining either faction.

Having characterized our refinement, we turn our attention to comparative statics on the size of the super-majority requirement and the location of the reversion policy. As the super-majority requirement increases, we show that the level of redistribution decreases monotonically, provided that the income distribution is right-skewed and the reversion policy is 'low'. We decompose this result into two complementary effects. First, if the income distribution is right-skewed (as is the case empirically), then for a given increase in the super-majority requirement, the income of the right decisive voter increases by more than the income of the left decisive voter decreases. This causes the income of the decisive voter – i.e. the 'average' of the incomes of the left and right decisive voters – to be larger, resulting in less redistribution.

Second, even if the income distribution were not skewed, increasing the super-majority voting rule tilts bargaining power in favor of the right decisive (i.e. richer) voter. If the reversion policy is low, then as q increases, it becomes more favorable from the perspective of the (now richer) right decisive voter, and less favorable from the perspective of the (now poorer) left decisive voter. Disagreement is made less costly for the right decisive voter, which improves her bargaining power. More formally, the potential gains from the bargain are made smaller for right decisive voter and larger for the left decisive voter. But since Nash Bargaining

who is indifferent between joining either faction; he is the agent for whom the chosen policy is ideal. The latter agents determine the set of acceptable equilibrium offers.

maximizes the (weighted) product of these gains, this requires that the right decisive voter now realize a larger fraction of his potential gains, which tilts the bargaining dynamic in his favor.

Our second comparative static result concerns the effect of changing the reversion policy. We show that the status quo policy generically affects our equilibrium refinement. When the status quo policy is contained in the core, then it is the natural focal policy that is selected. This is the well known result that super-majority rules exhibits a status quo bias, by making it harder to build a coalition around replacing it. Additionally, when the status quo policy lies outside the core, the equilibrium policy becomes more moderate, in the sense of being closer to the middle of the core, as the status quo becomes more extreme. Hence, we document an important role for the status quo, even in cases where a coalition can be found to replace the status quo. Unlike the case of simple majority, where the equilibrium policy is independent of the status quo, equilibria are generically sensitive to the status quo, under super-majority rule.

Our theoretical results find support in the empirical literature. Knight (2000), Besley and Case (2003), and Lee (2014) find that super-majority budget rules are associated with a reduction in tax rates. Bradbury and Johnson (2006) and Bails and Tieslau (2000) find a similar results with regard to lower public welfare transfers and state expenditures, respectively. In a slightly different context, Heckelman and Dougherty (2010) demonstrate that there is an inverse relationship between the size of the majority requirement and tax rates on cigarettes and distilled spirits.

Notwithstanding these empirical studies, there has been little formal analysis on the effect of super-majority rules on redistribution. The most relevant paper to ours is Gradstein (1999), who studies a public finance model, similar to ours, except that his focus is on public goods provision rather than redistribution. (We show in section 5.1 that our results continue to hold in this alternative setting.) However, his paper takes a distinctly different approach to collective decision making, and this generates a different equilibrium selection criterion. The procedure is as follows: Starting from zero taxation and redistribution, the voters consider a series of incremental proposals, each requiring a super-majority, to raise the tax rate by ϵ. This process continues until a super-majority cannot be found to raise taxes any further. This incremental procedure is intended to capture the informal argument in Buchanan and Tullock (1962), who themselves build on insights in Wicksell (1896). The procedure is also found in Dal Bo (2006) and Riboni and Ruge-Murcia (2010).

We think our approach is an improvement over this procedure for two reasons. First, the status quo policy plays an out-sized role in the incremental procedure. Indeed, the equilibrium policy is simply the policy within the core that is closest to the status quo. If the status quo policy is zero redistribution, this will be the ideal policy of the right decisive voter. (Of course, if the status quo policy is high, the procedure will choose the ideal policy of the left decisive voter.) We could recast the procedure as selecting a policy via Nash Bargaining, however with exogenous bargaining weights that confer all bargaining power on the right faction. By contrast, under our approach, bargaining weights are determined endogenously, and depend on the preferences and influence of all voters.

Second, the incremental procedure functions more like an ascending English auction[5] than a true bargaining game, in which agents would be free to propose policies. Citing Baron (1996)[6], Gradstein claims that the incremental procedure coincides with the long-run policy that would emerge in a dynamic model of bargaining, where today's selected policy becomes tomorrow's status quo. In this, we think Gradstein is mistaken. If players are impatient (i.e. $\delta < 1$), then although the system will converge to a long-run policy, the policy will not be unique, and the *ex ante* distribution over long-run outcomes will be non-degenerate.[7]

[5] Unlike an English auction, which continues until all but one player has dropped out of the bidding, this 'auction' stops once a measure $1 - q$ of agents drop out.

[6] Baron (1996) shows that under simple majority rule, in a dynamic game, the policy will converge to the median voter's ideal, in the long-run.

[7] For example, if players are not too impatient, then the acceptance set will generically be a subset of the core. In the first round, the proposer – whomever they are – will propose the most favorable policy in the

If agents are not too impatient, then the support of this distribution over long-run policies will *not* include the 'auction outcome'. Finally, as players become perfectly patient ($\delta \to 1$), this distribution will indeed become degenerate, but the steady-state policy will coincide precisely with the endogenous Nash Bargaining solution selected by our method.[8]

This paper contributes to more broadly to the literature on bargaining in uni-dimensional policy spaces. Banks and Duggan (2006), in a framework that influences our own, study bargaining games when disagreement entails reversion to a status quo. They show that such games always admit equilibria in no-delay, and that equilibrium proposals coincide with the status quo whenever the status quo is in the core. Cardona and Ponsati (2011) provide conditions (which Parameswaran and Murray (2018) generalize) for the equilibrium to be unique. Predtetchinski (2011) shows that equilibrium proposals converge as $\delta \to 1$ and shows that the limit policy is the generalized root of a characteristic function. Under simple majority rule, Cho and Duggan (2009) show that this limit policy is simply the ideal policy of the median voter. In a game with finitely many players, Parameswaran and Murray (2018) provide an explicit characterization of the limit policy for any super-majority rule, showing that it is either the ideal policy of some agent (not necessarily the median) or the result of asymmetric Nash Bargaining between the left and right decisive agents, with weights depending on factional size. In this paper, we extend Parameswaran and Murray (2018) to settings with a continuum of agents, and in which disagreement results in reversion to a status quo policy.[9]

The remainder of this paper is organized as follows: Section 2 introduces the model. In Section 3, we analyze the equilibrium of the bargaining game and characterize the limit equilibrium. In Section 4, we consider the effect of changing the super-majority requirement.

---

acceptance set. This policy will be accepted, and since it is in the core, will not be replaced in subsequent rounds.

[8] We prove all of these claims on page 48 in the Appendix.

[9] Banks and Duggan (2000), Cardona and Ponsati (2011), Predtetchinski (2011), and Parameswaran and Murray (2018) all make the common assumption that disagreement is the worst possible outcome from the perspective of all players. Reversion to a status quo policy introduces the complication that the status quo may be preferred to a range of policies, for some agents.

Section 5 presents some extensions and Section 6 concludes.

2 Framework

In this section, we present the underlying model of household behavior and preferences over redistribution policies. Our approach is analogous to Meltzer and Richard (1981), with one difference. In their model, dead-weight losses from taxation arise endogenously as a consequence of households' elastic labor supply. By contrast, we assume that labor supply is inelastic, and instead introduce convex dead-weight losses in a reduced form way by assuming taxation via a leaky-bucket (see Moene and Wallerstein (2001) and Dixit and Londregan (1998), amongst others). As we previously noted, our reduced form approach is mostly for simplicity, and enables us to focus attention on the novel feature of our paper — equilibrium selection via bargaining. Additionally, whilst micro-foundations are intrinsically valuable, they also play a critical role in Meltzer and Richard (1981) in establishing that the median voter theorem holds in their setting. By contrast, since core selection in our approach is generated through the bargaining game, the need for micro-foundations is less crucial. In any case, we present a micro-founded version of the model for robustness in Section 5.2.

2.1 Preferences

There is a continuum of voters, each with income y drawn from a continuous distribution F that admits a density f. Since the number of voters is large, F also represents the empirical distribution of incomes in the economy. Let $\overline{y} = \int y dF(y) < \infty$ be the average income.

The government can levy a proportional income tax τ that finances uniform lump-sum transfers T to each individual. Agents supply their labor inelastically, so that each agent's pre-tax income is unaffected by the tax policy. However, taxes are collected via a leaky bucket, which implies dead-weight losses. Let $e(\tau) \in [0,1]$ be the effective tax rate.[10] Then, if the government levies a proportional labor tax $\tau \in [0,1]$, it actually receives only $e(\tau)$ for each dollar of income taxed. We assume that e is strictly concave, $e'(0) = 1$ and $e'(\overline{\tau}) = 0$ for some $\overline{\tau} \in (0,1)$. Together, these imply that $e(\tau) < \tau$ whenever $\tau > 0$. In the absence of dead-weight losses, a tax rate of τ would generate revenue of $\tau \int y dF(y) = \tau \overline{y}$. Instead, the government's revenue is $R(\tau) = e(\tau)\overline{y} < \tau\overline{y}$.

The revenue function $R(\tau)$ represents the Laffer curve. By construction, $\overline{\tau}$ is the tax rate that maximizes government revenue; i.e. the tax rate associated with the peak of the Laffer curve. Given the concavity of e, government revenue is increasing in the tax rate when $\tau \in (0, \overline{\tau})$ and it is decreasing when $\tau \in (\overline{\tau}, 1)$. Naturally, in equilibrium, we should never expect $\tau > \overline{\tau}$.

The consumption of a voter with pre-tax income y is $T + (1-\tau)y$. Voters have expected utility preferences over consumption, represented by a continuous and concave utility index u, where $u' > 0$ and $u'' \leq 0$.

Government transfers are financed using tax revenues, and the government's budget is assumed to be in balance. (Accordingly, we ignore spending by the government on public goods, or the possibility of debt financing. In section 5.1, we show that our results continue to hold in a model with public goods.) Given the government budget constraint, and since there is a unit measure of agents, the government's revenue is also the size of the transfer T that the government can give each voter. Hence, at tax rate τ, a voter with pre-tax income y has consumption $c(\tau, y) = e(\tau)\overline{y} + (1-\tau)y$.

Let $v(\tau, y)$ represent the preferences over tax policies of an agent with income y, taking the

[10] In section 5.2 we connect the reduced form dead-weight losses to those arising from a micro-founded model with endogenous labor supply. Briefly, let $\epsilon_\tau(\tau, \overline{y}) = \frac{\partial \overline{y}}{\partial \tau} \cdot \frac{\tau}{\overline{y}}$ denote the tax elasticity of average income in a model with elastic labor supply. Then setting $e(\tau) = \tau + \int_0^\tau \epsilon_\tau(t, \overline{y}) dt$ causes the dead-weight loss to have the same marginal behavior in both the structural and reduced-form models.

government's budget constraint as given. We have:

$$v(\tau, y) = u\left(e(\tau)\overline{y} + (1-\tau)y\right).$$

Since e is strictly concave, then $c(\tau, y)$ is strictly concave in τ for each y. Then, since u is increasing and concave, $v(\tau, y)$ is strictly concave in τ for each y. We have:

$$v_\tau(\tau, y) = [e'(\tau)\overline{y} - y]u'(c) \tag{1}$$

and

$$v_{\tau\tau}(\tau, y) = e''(\tau)\overline{y}u'(c) + [e'(\tau)\overline{y} - y]^2 u''(c) < 0.$$

Strict concavity implies that preferences are single-peaked. Let $\tau(y) = \arg\max_{\tau \in [0,1]} v(\tau, y)$ denote the most preferred tax rate of an agent with income y. Since v is strictly concave, this most preferred policy satisfies the first order condition:

$$e'(\tau)\overline{y} - y \leq 0 \tag{2}$$

where the condition holds with equality unless the optimum is at $\tau = 0$. Notice that equation (2) is analogous to equation (13) in Meltzer and Richard (1981), which defines the optimal tax rate for agents in their setting. We can explicitly characterize the optimal tax rate for a given voter by[11]:

$$\tau(y) = \begin{cases} [e']^{-1}\left(\frac{y}{\overline{y}}\right) & y \leq \overline{y} \\ 0 & y > \overline{y} \end{cases}$$

A marginal increase in taxes has two effects; it increases the size of the transfer from the government whilst reducing the voter's take home pay. The ideal tax rate for a given agent is the one that appropriately balances these competing effects. An agent with zero income

[11] $e'(\cdot)^{-1}$ is well defined since, by the strict concavity of e, e' is strictly decreasing.

would ideally have taxation at the peak of the Laffer curve (i.e. $\tau = \overline{\tau}$), since this maximizes the transfer received at no cost to their post-tax income. As income increases, the lost earnings become more salient and so the voter's ideal tax rate decreases. Indeed, by the implicit function theorem, we have: $\frac{\partial \tau}{\partial y} = \frac{1}{e''(\tau(y))} < 0$. Moreover, all voters with income above the mean would ideally prefer zero redistribution.

The concavity assumption disciplined each voter's preferences over consumption/tax plans. We supplement this with an additional requirement that disciplines the behavior of preferences across individuals. Formally, we require:

$$v_{\tau y}(\tau, y) = -u'(c) + \left[e'(\tau)\overline{y} - y\right](1-\tau)u''(c) \leq 0$$

. This is the Spence-Mirrlees condition. It requires that, for any policy, the marginal utility of taxation is monotone in agents' income. The Spence-Mirrlees condition implies that, if an agent approves of a particular tax increase, so will all agents with strictly lower income, and if an agent disapproves of that tax increase, so will all agents with strictly higher income.[12]

Notice that, by the concavity of u, the Spence-Mirrlees condition is automatically satisfied whenever $y \leq e'(\tau)\overline{y}$. Hence, adding this requirement only potentially has bite if $y > e'(\tau)\overline{y}$. Let $R(c) = -c\frac{u''(c)}{u'(c)}$ be the coefficient of relative risk aversion. It is easily verified that the following assumption on preferences over consumption guarantees that the Spence-Mirrlees condition holds:

Assumption 1.

$$R(c^*) \leq \frac{(1-t)\,y + e(t)\,\overline{y}}{(1-t)\,y - (1-t)\,e'(t)\,\overline{y}}$$

for every $\tau \in [0, \overline{\tau}]$ *and every* y *for which* $y > e'(\tau)\overline{y}$ *(i.e. for which the denominator is positive).*

[12] To see this, take any two redistributive policies τ_1 and τ_2 with $\tau_1 < \tau_2$. Then: $\frac{\partial}{\partial y}[v(\tau_2, y) - v(\tau_1, y)] = \frac{\partial}{\partial y}\int_{\tau_1}^{\tau_2} v_\tau(\tau, y)d\tau = \int_{\tau_1}^{\tau_2} v_{\tau y}(\tau, y)d\tau \leq 0$.

It suffices that the degree of risk aversion of higher income agents is not too large. The condition is satisfied for commonly used classes of preferences. For example, if preferences over consumption are CRRA (with coefficient of relative risk aversion θ), then the condition is satisfied for any $\theta \leq 1$, which includes log utility as a special case.

Finally, as we establish in more detail in section 5, in a model with endogenous labor supply, assuming the Spence-Mirrlees condition holds is equivalent to assuming that no agent increases her work effort following a tax increase. As we demonstrate in section 5, agents with $y \leq e'(\tau)\overline{y}$ would never do so, because the substitution and wealth effects associated with a tax increase (including the increased governmental transfer) both push in the direction of deterring work effort. By contrast, if $y > e'(\tau)\overline{y}$, the marginal loss in earned income is larger than the marginal gain from the transfer, and so the wealth effect pushes in the direction of working more. The Spence-Mirrlees condition ensures that this wealth effect does not overwhelm the substitution effect.[13]

2.2 The Core

Suppose redistributive policies are chosen by (super)-majority rule, such that to be implemented, a policy requires the assent of at least a measure $q \in [\frac{1}{2}, 1]$ of agents. The *core* is the set of unbeatable policies; i.e. the set of policies for which there does not exist another policy that is strictly preferred by the required measure of agents. Under simple majority rule ($q = \frac{1}{2}$), we know that the core is uniquely the ideal policy of the median voter τ_{med} (see Black (1948) and Downs (1957)).

Suppose there is a strict super-majority rule $q > \frac{1}{2}$. Let $y_L = F^{-1}(1-q)$ and $y_R = F^{-1}(q)$ be the incomes of the agents at the $(1-q)^{th}$ and q^{th} quantiles of the income distribution.[14]

[13]The assumption does not rule out a backward-bending labor supply curve. However, it insists that labor supply cannot bend back too far. Put differently, whilst the wage elasticity of labor supply may be negative, the tax elasticity of labor supply may not be positive.

[14]y_L and y_R are obviously functions of q, although we suppress this dependence in the notation.

For reasons that will become clear, we refer to the voters with incomes y_L and y_R as the *left* and *right decisive voters*, respectively.[15] Let $\tau_L = \tau(y_L)$ and $\tau_R = \tau(y_R)$ be the ideal tax rates for the left and right decisive players, respectively. Note that $y_L < y_R$ and so $\tau_L > \tau_R$.

Under super-majority requirement q, the core is the set of policies in the interval $[\tau_R, \tau_L]$. Hence, under super-majority rule, there are a multiplicity of possible 'equilibrium' policies. Furthermore, $\tau_R < \tau_{med} < \tau_L$, and so the core contains policies that are both higher and lower than the median voter's ideal policy. Absent a selection criterion, we cannot predict which policy will prevail in super-majority regimes. Moreover, from a comparative static perspective, it is indeterminate whether super-majority rule will cause redistribution to go up or down. In the next section, we use a bargaining approach to select a focal equilibrium policy from the core.

As we previously noted, our results will relate to the notion of the *core*$^+$, which we define as the set of core policies that are weakly preferred by both decisive voters to the 'status quo'. The distinction between the core and core$^+$ is analogous to the distinction between the set of Pareto optima and the contract curve; the latter is the subset of Pareto optima that are also Pareto improvements given the agents' endowments. If the status quo is sufficiently extreme, then the core and core$^+$ will coincide. By contrast, if the status quo is contained within the core, then the core$^+$ is a singleton set containing only the status quo.

3 Bargaining

In this section, we analyze the redistributive outcomes that would result if chosen as a consequence of bargaining within a committee or legislature. We will subsequently demonstrate that we can make a compelling selection from amongst the set of core$^+$ policies by taking a particular limit equilibrium of the bargaining game.

[15] Cardona and Ponsati (2011) refer to them as the left and right *boundary* players.

3.1 The Bargaining Protocol

The bargaining protocol is the standard procedure in Baron and Ferejohn (1989) and Banks and Duggan (2000). There are potentially infinitely many bargaining rounds. In a given round of bargaining, a voter is randomly recognized to propose a policy. Let $P(y)$ be a distribution function that describes the probability that the recognized proposer has income less than y, and suppose $P(y)$ admits a density $p(y)$. In the special case that $P(y) = F(y)$, voters are recognized to propose with equal probability. However, the framework easily accommodates unequal recognition probabilities, which might, for example, capture the idea that richer voters tend to exert more influence in policy-making that poorer voters (see Benabou (2000)). After observing the proposal, all players simultaneously vote to either accept or reject the proposal. Acceptance requires that the proposal receive the assent of at least a measure $q \geq \frac{1}{2}$ of agents. If so, the policy is implemented, and the bargaining game ends. In the event of disagreement, a 'status quo' (or, reversion) policy τ_{sq} is implemented in the current period, and the players reconvene for another round of bargaining in the following period. This process continues until agreement is reached. Players discount the future at a common rate $\delta \in [0, 1)$.[16]

Although we use the term 'status quo' (as does the literature, more generally), we stress that the reversion policy need not be the policy most recently in effect. For example, disagreement may trigger spending cuts, as was the case in 2013 budget sequester. Moreover, to the extent that redistribution and public goods spending are discretionary items in the budget, disagreement may result in zero spending, for example in the event of a government shutdown. In section 4.1 we pay particular attention to equilibrium behavior when the reversion policy is 'low' (although we assume the policy is at least 0).

A strategy for a voter with income y is a pair $(t(y), A(y))$, where t is the tax rate proposed

[16]The bargaining framework admits an alternative interpretation in which, following disagreement, the bargaining game exogenously terminates with probability $1 - \delta$. The discount factor, then, captures the likelihood of there being additional opportunities for negotiation.

whenever a type y voter is recognized,[17] and $A \subset [0,1]$ is the set of tax rates that such a voter will accept. We solve for stationary sub-game perfect equilibria in weakly undominated strategies. The weak undominance requirement implies that each agent votes as if they were pivotal (i.e. they only support proposals that they weakly prefer to the continuation game).[18]

We say an equilibrium is in *no-delay* if there will be immediate agreement in equilibrium, regardless of the identity of the proposer. We say that an equilibrium is *static* if the implemented policy is unchanging across periods, even if immediate agreement is not reached. A static equilibrium will obtain, for example, if the only policy that is socially acceptable is the status quo. If so, the implemented policy will be the same whether the status quo policy is proposed and accepted, or some other policy is proposed and rejected. Banks and Duggan (2006) establish (see Theorems 1,4 and 7), that there always exists an equilibrium in no-delay, and that under certain conditions, equilibria must be in no-delay. Furthermore, they show that whenever there is an equilibrium with delay, it must be a static equilibrium. If so, the policy implemented in any equilibrium must coincide with the policy chosen in a no-delay equilibrium. Hence, we focus on no-delay equilibria, and this is without important loss of generality.

3.2 Equilibrium

In the analysis that follows, we identify each agent with their income y. Let $t(y)$ be the equilibrium proposal of a type y agent. The expected equilibrium utility of a type y agent is $V(y) = \int_0^\infty v(t(z), y)\, dP(z)$. Hence, y will accept a proposal t if:

[17] To clarify the notation: $\tau(y)$ is the ideal tax rate of a type-y voter, whereas $t(y)$ is the equilibrium tax proposal for that voter.

[18] Since every voter is an atom, no agent's vote can sway the outcome of an election. If so, any voting strategy can be sustained as an equilibrium, since no single agent's vote matters. The weak-dominance refinement rules out perverse equilibria of this sort, by requiring agents to vote for their preferred alternative, even though no agent's choice, in isolation, can affect the policy outcome.

$$v(t,y) \geq (1-\delta)\, v(\tau_{sq};y) + \delta \int_0^\infty v(t(z),y)\, dP(z)$$

Since v is strictly quasi-concave in τ for each y, then the acceptance sets for each agent must be an interval. Let $A(y) = \left[\underline{t}(y), \bar{t}(y)\right]$, where $\underline{t}(y)$ and $\bar{t}(y)$ are the left- and right certainty equivalents given the lottery over policies induced by the continuation game. For each y, we have:

$$\underline{t}(y) = \min\left\{t \geq 0 \middle|\, v(t,y) \geq (1-\delta)v(\tau_{sq},y) + \delta V(y)\right\}$$
$$\bar{t}(y) = \max\left\{t \leq 1 \middle|\, v(t,y) \geq (1-\delta)v(\tau_{sq},y) + \delta V(y)\right\}$$

Let $E[t] = (1-\delta)\tau_{sq} + \delta \int_0^\infty t(y) dP(z)$ be the expected policy in the continuation game. Since v is concave in τ, $v(E[t],y) \geq (1-\delta)v(\tau_{sq},y) + \delta V(y)$ for all y, and so $\underline{t}(y) \leq E[t] \leq \bar{t}(y)$.

Let $\mathcal{C} = \{C \subset \mathbb{R}_+ |\, \#C \geq q\}$, where $\#C$ is the probability measure over $\mathbb{R}$ consistent with the distribution function F. Any $C \in \mathcal{C}$ is a decisive coalition. Let $A_C = \cap_{y\in C} A(y)$ be the set of policies that will be accepted by decisive coalition C, and let $A = \cup_{C\in\mathcal{C}} A_C$ be the set of policies that will be accepted by *some* decisive coalition. Since $E[t] \in A(y)\, \forall y$, then each A_C is non-empty. Furthermore, since each $A(y)$ is an interval, so must be each A_C. Finally, the A_C's cannot be disjoint, since each contains $E[t]$. Hence A is also an interval. We have $A = \left[\underline{t}, \bar{t}\right]$.

Given the Spence-Mirrlees condition, we can show that any proposal not accepted by the income earner at the q^{th} income quantile (i.e. the right decisive voter) will not be accepted by any player whose income is even higher. Similarly, any proposal not accepted by the income earner at the $(1-q)^{th}$ income quantile (i.e. the left decisive voter) will not be accepted by any voter whose income is even lower. Since equilibrium proposals must receive the support of a measure q of agents, it suffices for every proposal to be acceptable to both the left and right decisive agents. Hence, the social acceptance set is $A = [\underline{t}(y_L), \bar{t}(y_R)]$. The set of socially acceptable proposals are those that are not too low from the perspective of the left decisive voter and not too high from the perspective of the right decisive voter. With this discussion in mind, we have the following result:

Proposition 1. *The bargaining game admits a unique stationary equilibrium in no-delay, characterized by a pair of thresholds* $\underline{t}(y_L)$ *and* $\bar{t}(y_R)$*, where:*

1. *Equilibrium proposals are given by*[19]*:*

$$t(y) = \begin{cases} \bar{t}(y_R) & y \leq \tau^{-1}\left(\bar{t}(y_R)\right) \\ \tau(y) & y \in \left(\tau^{-1}\left(\underline{t}(y_L)\right), \tau^{-1}\left(\bar{t}(y_R)\right)\right) \\ \underline{t}(y_L) & y \geq \tau^{-1}\left(\underline{t}(y_L)\right) \end{cases}$$

2. *The acceptance sets are given by:* $A(y) = \left[\underline{t}(y), \bar{t}(y)\right]$*, where:*

$$\underline{t}(y) = \min\left\{t \geq 0 | \, v(t, y) \geq (1-\delta)\, v(\tau_{sq}, y) + \delta V\left(\underline{t}(y_L), \bar{t}(y_R), y\right)\right\}$$
$$\bar{t}(y) = \max\left\{t \leq 1 | \, v(t, y) \geq (1-\delta)\, v(\tau_{sq}, y) + \delta V\left(\underline{t}(y_L), \bar{t}(y_R), y\right)\right\}$$

and $V\left(\underline{t}(y_L), \bar{t}(y_R), y\right) = F\left(\tau^{-1}\left(\bar{t}(y_R)\right)\right) v\left(\bar{t}(y_R), y\right) + \int_{\tau^{-1}\left(\bar{t}(y_R)\right)}^{\tau^{-1}\left(\underline{t}(y_L)\right)} v\left(\tau(z), y\right) dF(z) + \left[1 - F\left(\tau^{-1}\left(\underline{t}(y_L)\right)\right)\right] v\left(\underline{t}(y_L), y\right)$.

Proposition 1 is analogous to Proposition 2 in Cardona and Ponsati (2011) and Proposition 1 in Parameswaran and Murray (2018). Any agent whose ideal policy lies in the social acceptance set will simply propose their ideal policy whenever they are recognized to propose. All remaining agents will propose one of the end-point policies — whichever is closest to their ideal policy. Furthermore, Proposition 1 pins down the boundaries of the social acceptance set as a consequence of the preferences of the left and right decisive voters.

[19] $\tau^{-1}(t)$ is well defined for any $t \in (0, \bar{\tau}]$. For $t = 0$, define $\tau^{-1}(t) = \inf\{y \geq 0 | \tau(y) = 0\} = \bar{y}$

The properties of equilibrium proposals depends crucially on whether the reversion policy is contained in the core or not. We explore each of these possibilities in turn, starting with the latter.

Corollary 1. *Suppose $\tau_{sq} \notin [\tau_R, \tau_L]$. Then, for any $\delta < 1$, the equilibrium is stationary and in no-delay. Moreover, $\underline{t} < \bar{t}$.*

Corollary 1 asserts that, whenever the status quo is not in the core, the bargaining game admits a unique equilibrium, and that this equilibrium must be in no-delay. Additionally, the Corollary shows that the social acceptance set is (generically) an interval, and so the equilibrium policy that arises will be proposer-dependent.

To build intuition for Proposition 1, note that, per the preceding discussion, if the expected policy, $E[t]$, is proposed, then it will receive unanimous support. Since the reversion policy is outside the core, the continuation lottery must be non-degenerate, which implies by strict concavity (and since $\delta < 1$) that there must be some neighborhood about $E[t]$ in which proposals will also be unanimously accepted. Consider any voter y whose ideal tax rate is above the expected proposal. Starting from $E[t]$, when recognized as the proposer, she can offer a slightly higher policy without losing the unanimous support of the legislature. As she increases the policy further, she will eventually lose the support of some voters, starting with the highest income earner, and proceeding down the income schedule. Since she needs the support of the right decisive voter, she can afford to increase the proposed tax rate either until she arrives at her ideal policy, or the support of the right decisive voter would be lost. In the latter case, she would simply propose the highest tax policy that the right decisive voter would accept.

Hence, when the reversion policy lies outside the core, the social acceptance set is generically an interval, and so there is a range of policies that are consistent with equilibrium, depending on the identity of the proposer. By contrast, when the status quo policy lies within the core, the social acceptance set collapses to a singleton.

Corollary 2. *Suppose* $\tau_{sq} \in [\tau_R, \tau_L]$. *Then, every equilibrium of the bargaining game is static, and the only policy that is socially acceptable is the status quo. Formally,* $A = \{\tau_{sq}\}$ *and so* $t(y) = \tau_{sq}\, \forall y$.

Corollary 2 follows straightforwardly from Theorem 7 in Banks and Duggan (2006). Intuitively, if the status quo policy is in the core, then for *any* other policy, there is some blocking coalition that prefers the status quo to it. Hence, it is impossible to move the policy away from the status quo. Corollary 2 reflects the well known result that super-majority rules exhibit a status-quo bias. Whenever the reversion policy is in the core, the policy that results from bargaining is immediate. By contrast, characterizing equilibrium outcomes when the status quo lies outside the core is more complicated. Hence, whilst the analysis in the following subsections applies equally to both cases, the implications will be much more interesting in the case when the reversion policy is not in the core.

3.3 Limit Equilibria

The discount factor δ played an important role in determining the equilibrium of the bargaining game, above. When $\delta < 1$, delay is costly, and this has two important implications for the nature of equilibrium proposals. First, it is costly for voters to reject the current proposal and wait for a better proposal in the continuation game. This means that proposals that lie outside of the core (i.e. for which there is a different policy that is strictly preferred by a decisive coalition) may nevertheless be accepted and implemented. Second, costly delay empowered the proposer to pull the policy away from the mean and towards her preferred policy. This agenda-setting power resulted in a range of policies being potentially implemented in equilibrium, depending on the identity of the proposer.

In this section, we focus on the limit equilibrium of the bargaining game as the cost of delay is made arbitrarily small (i.e. as $\delta \to 1$). Taking this limit allows us to focus on situations where the cost of making counter-proposals is very small, so that non-core policies will not survive. This makes for a fairer comparison between our bargaining equilibrium and other equilibrium notions such as the core. Furthermore, as the next proposition shows, in this limit, the proposer's agenda-setting power disappears. The policy proposed in the limit equilibrium is independent of the identity of the proposer. Hence, in the limit, we can sensibly talk about a single equilibrium policy, rather than a menu of proposer-dependent policies.

Proposition 2. *The equilibrium proposals converge as the discount factor goes to* 1*. Formally,* $A \to [t^*, t^*]$ *as* $\delta \to 1$ *(i.e.* $\lim_{\delta \to 1} \underline{t}(y_L) = t^* = \lim_{\delta \to 1} \bar{t}(y_R)$*).*

Proposition 2 is analogous to Theorem 3.6 from Predtetchinski (2011) and Proposition 2 in Parameswaran and Murray (2018). The intuition is precisely as in the discussion above. Proposers are able to exercise a degree of agenda control in the bargaining game to the extent that costly delay creates a disincentive for other players to reject proposals not at the mean, in favor of the continuation game. As delay becomes costless, a blocking coalition can always be found who would rather face the continuation game than implement any non-mean proposal. This forces all proposers to make identical proposals.

3.4 Costless Bargaining

In the previous subsections, we studied games with a positive cost of delay. We showed that these games admit a unique equilibrium, and that in the limit as delay became costless, all agents make the same equilibrium proposal.

Suppose we, instead, took the case where delay was exactly costless (i.e. $\delta = 1$). In this world, there are, in fact, potentially multiple equilibria. Since delay is costless, there will now be many equilibria with delay, in addition to no-delay equilibria. Moreover, even if we limited attention to no-delay equilibria, the bargaining game now (generically) admits a continuum of no-delay equilibria, with each equilibrium being characterized by a single policy that is proposed and accepted by all agents. In fact, the set of equilibrium policies that can be sustained exactly coincides with the core$^+$, thereby establishing an equivalence between the core$^+$ and the equilibria of bargaining games when $\delta = 1$.[20]

Notice that, although when $\delta = 1$, there are potentially a continuum of equilibria, for any $\delta < 1$, there is a unique equilibrium, and these equilibria converge to a unique policy as $\delta \to 1$. The equilibrium correspondence exhibits a failure of lower-hemicontinuity at $\delta = 1$, and this failure presents a natural candidate for an equilibrium refinement. Although every policy in the core$^+$ can be sustained as an equilibrium when $\delta = 1$, only one of these continues to be equilibrium consistent for δ slightly below 1. The limit equilibrium that we identified in the previous subsection is the unique robust policy in the core that survives the introduction of small positive costs to making counter-proposals. We take this to be a focal policy amongst the many within the core.

3.5 Limit Equilibrium and the Nash Bargaining Solution

We seek an explicit characterization for our refinement. For any $\delta < 1$, let $\left[\underline{t}_L(\delta), \overline{t}_R(\delta)\right]$ be the associated social acceptance set. Then all agents with $y \leq \tau^{-1}\left(\overline{t}_R(\delta)\right)$ will propose $\overline{t}_R(\delta)$, all agents with $y \geq \tau^{-1}\left(\underline{t}_L(\delta)\right)$ will propose $\underline{t}_L(\delta)$, and all remaining agents will propose their ideal policies. Hence, a measure $P\left(\tau^{-1}\left(\overline{t}_R(\delta)\right)\right)$ will behave as a faction and propose $\overline{t}_R(\delta)$, and a measure $1 - P\left(\tau^{-1}\left(\underline{t}_L(\delta)\right)\right)$ will behave as a faction and propose $\underline{t}_L(\delta)$. Now, as $\delta \to 1$, since $\overline{t}_R(\delta) - \underline{t}_L(\delta) \to 0$, the measure of agents who propose their own ideal policy is squeezed to zero. In the limit, it is as if there are simply two cohesive factions that bargain over the equilibrium policy. Moreover, the policies proposed by each faction are jointly determined

[20]Banks and Duggan (2000) study bargaining games under a common alternative framework where disagreement is uniformly bad. In that setting, they show that, when delay is costless, the set of bargaining equilibrium coincides with the entire core.

by the preferences of either the left or right decisive voter. With this insight in mind, we are ready to state the first significant result of this paper.

First, some notation. Denote by $\Delta v(t, y) = v(t, y) - v(\tau_{sq}, y)$ the utility improvement for voter y of any policy t over the reversion policy. Take any $\phi \in [0, 1]$. Let $B(\phi, y_L, y_R)$ denote the solution to the following asymmetric bilateral Nash Bargaining problem between the left and right decisive voters:

$$B(\phi, y_L, y_R) = \arg\max_t [\Delta v(t, y_L)]^{\phi} [\Delta v(t, y_R)]^{1-\phi}$$

B is defined by the first order condition:

$$\phi \frac{v_\tau(B, y_L)}{\Delta v(B, y_L)} + (1 - \phi) \frac{v_\tau(B, y_R)}{\Delta v(B, y_R)} = 0$$

It is straightforward to show that B is strictly increasing in ϕ whenever $y_L < y_R$. Notice that, for any ϕ, $B(\phi, y_L, y_R) \in core^+$, since Nash Bargaining always selects Pareto optimal outcomes that are improvements for both players. In particular, if $\tau_{sq} \in [y_L, y_R]$, then the only candidate solution is τ_{sq}, since any other policy would be worse than the status quo for at least one of the decisive voters.

In what follows, we make explicit the dependence of the equilibrium and limit policies on the super-majority rule q. In particular, we write $y_L = F^{-1}(1 - q)$ and $y_R = F^{-1}(q)$.

Proposition 3. *The limit equilibrium policy is characterized by the following system:*

$$t^* = B\left(\phi^*, F^{-1}(1 - q), F^{-1}(q)\right)$$
$$\phi^* = P\left(\tau^{-1}(t^*)\right)$$

Proposition 3 states that, in the limit, the equilibrium tax rate is a consequence of asymmetric Nash Bargaining between the left and right decisive voters, with endogenous bargaining

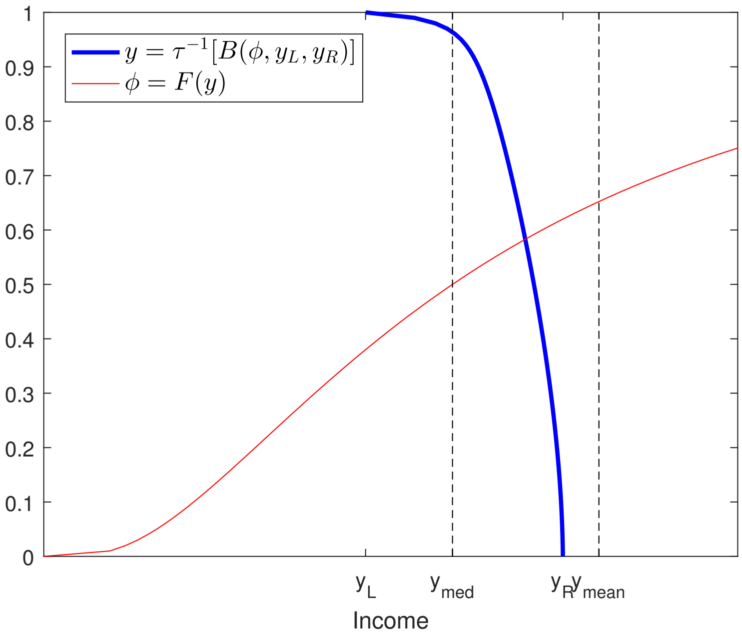

Figure 1: Income of the Pivotal Voter. Voters are recognized to propose with equal probability. The income distribution is assumed log-normal, with variance calibrated to the U.S. Gini coefficient. The thick line represents the identity of the decisive voter for different value of ϕ under a 60 percent super-majority rule. In equilibrium, the decisive voter is richer than the median, and the left faction represents more than half of voters.

weights that depend on the distribution of recognition probabilities. In the limit, it as if the voters separate into two distinct factions, and that the bargaining outcome is the result of Nash bargaining between the factional leaders. Moreover, since the relative likelihood that a proposal emanates from a given faction is determined by the number of agents in that faction, the bargaining weights are proportional to the recognition probability weighted size of each faction.

We see this in Figure 1. The think line shows the Nash Bargaining outcome for arbitrary bargaining weight ϕ. (In fact, it displays the income of the 'pivotal' voter – the agent for whom the chosen policy is optimal.) As ϕ increases from 0 to 1, the Nash Bargaining policy increases from R's ideal tax rate to L's, which implies that the 'pivotal' voter becomes poorer. The thin line is the cumulative distribution of income, assumed log-normal and with variance calibrated to the U.S. Gini coefficient. We assume that voters are recognized to propose with equal probability (i.e. $P(y) = F(y)$). As the diagram makes clear, the pivotal voter need not be the median income earner, and the equilibrium coalitions need not be equally sized.

A different way to conceive of this result is to consider the voters' decisions about which faction to join. Take a (connected) ϵ measure of agents. Those agents understand that joining one faction over the other increases the bargaining weight of the former, which pushes the equilibrium policy towards the ideal policy of the leader of the faction joined. Voters rationally make their factional choices, anticipating the policy that will follow. Hence, the equilibrium policy and coalitions are both determined endogenously. The policy identified in Proposition 3 is the unique policy that is equilibrium consistent, in the sense that the policy induces voters to separate into particular factions, and the bargaining weights implied by those factions cause the bargaining between factional leaders to select the equilibrium policy. In equilibrium, no (ϵ mass of) voters could do better by switching factions.

Our result suggests an interesting analogue to Duverger's Law, that under plurality rule, the stable number of parties is two. Along with median voter logic, this suggests that voters should divide into two equally sized factions that both advocate for the policy preferred by the median voter (either because of electoral incentives, or in a citizen-candidate model Besley and Coate (1997), because only the median can be credibly elected to be a factional leader). Under super-majority rule, our results suggest that Duverger's Law should continue to hold – the electorate will continue to divide into two factions. However, these factions need no longer be equally sized. As we will show in Section 4, we may reasonably expect two parties: a larger party of the poor and a smaller party of the rich. Moreover, these parties will be credibly led by agents who themselves have non-median preferences. However, because policy-making requires a super-majority, which neither party will have on its own, neither factional leader can implement their ideal policy unilaterally, and the equilibrium policy is the result of bargaining between factional leaders.

Meltzer and Richard (1981) argue that the equilibrium policy should coincide with the ideal policy of the pivotal voter, and that, under simple majority rule, the pivotal voter will be the median income earner. Given the optimal policy t^* in Proposition 3, we can directly identify the pivotal voter as the one whose ideal policy coincided with the equilibrium policy (i.e. $y^* = \tau^{-1}(t^*)$.) Such a voter is pivotal in the sense of being indifferent between joining either faction (much as the median voter would be in the case of simple majority).

However, given our Nash Bargaining framework, we have the following alternative interpretation for the income of the pivotal voter: The pivotal voter is the one whose income is a particular 'average' of the incomes of the left and right decisive voters. To see this, note that, by construction, $v_\tau(t^*, y^*) = 0$ and so:

$$(1-\phi^*)\frac{v_\tau\left(t^*, y_L\right)}{\Delta v\left(t^*, y_L\right)} + \phi^*\frac{v_\tau\left(t^*, y_R\right)}{\Delta v\left(t^*, y_R\right)} = \frac{v_\tau\left(t^*, y_d\right)}{\Delta v\left(t^*, y^*\right)}$$

Then, letting $g(\cdot) = \frac{v_\tau(t^*,\cdot)}{\Delta v(t^*,\cdot)}$, demonstrates that y^* is the generalized-g weighted average of y_L and y_R.[21] Hence, under super-majority rule, rather than being the median income, the pivotal voter is the one whose income is a (generalized) weighted average of the incomes of the leaders of each faction, where the weights depend on the sizes of each faction.

4 Comparative Statics

We now turn our attention to comparative statics. We first consider the effect of changing the required super-majority, and then consider the effect of changing the status quo policy.

[21] For any increasing function g, the generalized-g weighted-mean of two numbers x_1 and x_2, with weights ϕ and $1-\phi$ is a number x_m satisfying $g(x_m) = \phi g(x_1) + (1-\phi)g(x_2)$. Several examples of generalized means are familiar. Setting $g(x) = x$ gives the arithmetic mean; setting $g(x) = \log x$ gives the geometric mean, and likewise $g(x) = \frac{1}{x}$ gives the harmonic mean. Moreover, if $g(x) = u(x)$ where u is an expected utility index, then the generalized mean is a certainty equivalent.

4.1 Changing the Super-majority Requirement

It has been long recognized (see Romer (1975), Meltzer and Richard (1981)) that the extent of redistribution is affected by the shape of the income distribution, and in particular, its skewness. As conventionally defined, a distribution's skewness depends on the properties of its third moment. We introduce a different notion of skewness, similar to Boshnakov (2007) and Critchley and Jones (2008), that depends on the properties of the density function, directly. Our alternative notion, which we call *skewness**, is stronger than the conventional notion, but we think it reasonably applies to typical income profiles.

Let $\pi \in \left(\frac{1}{2}, 1\right)$. We say a distribution F is *right-skewed* at* π if $f(F^{-1}(\pi)) < f(F^{-1}(1-\pi))$. Moreover, for any interval $I \subset \left(\frac{1}{2}, 1\right)$, we say a distribution F is *right-skewed* on* I if it is right-skewed* for every $\pi \in I$, and it is *right-skewed** if it is right-skewed* on $\left(\frac{1}{2}, 1\right)$. *Left-skewness** is defined analogously, reversing the sign of the inequality.

To understand the implications of our concept, suppose F is right-skewed* on interval I, and take any $p, p' \in I$ satisfying $\frac{1}{2} < p < p' < 1$. Then the distance traversed moving from quantile p to p' is larger than the distance traversed moving from quantile $1-p$ to $1-p'$. Formally, $F^{-1}(p') - F^{-1}(p) \geq F^{-1}(1-p) - F^{-1}(1-p')$. Setting $p = \frac{1}{2}$ and $p' = q$, it is immediate that right-skewness* implies that, for any super-majority requirement q the income of the right decisive voter ($y_R = F^{-1}(q)$) is further from the median income than the income of the left decisive voter ($y_L = F^{-1}(1-q)$). In fact, more strongly, whenever the super-majority requirement increases, the income of the right decisive voter moves farther than the income of the left decisive voter.

Although the skewness* concept is stronger than regular skewness, the property holds for commonly used families of distributions. For example, the log-normal, and Pareto distributions are both right-skewed*, as is the Gamma distribution (which includes the Chi-Square distribution as a special case). The Weibull distribution is right-skewed* provided the shape parameter κ is not too large.[22] In particular, the exponential distribution, which is the special case of $\kappa = 1$ is right-skewed*.

With this new concept, we are ready to state our second main result. For any super-majority requirement $q \geq \frac{1}{2}$, let $t^*(q)$ be the equilibrium tax rate, and $y^*(q) = \tau^{-1}(t^*(q))$ be the income of the pivotal voter. Additionally, let $y_{sq} = \tau^{-1}(\tau_{sq})$ be the income of the voter whose ideal policy coincides with the status quo.[23]

Proposition 4. *Suppose $\tau_{sq} < \tau_{med}$ so that $y_{sq} > y_{med}$. The equilibrium tax rate has the following properties:*

- *If $q = \frac{1}{2}$, then the median voter is decisive and $t^* = \tau_{med}$.*
- *If $q \geq F(y_{sq})$, then $t^*(q) = \tau_{sq}$*
- *If $q \in \left(\frac{1}{2}, F(y_{sq})\right)$, then $t^*(q) > \tau_{sq}$. Furthermore, if F is right-skewed* on $\left(\frac{1}{2}, F(y_{sq})\right)$, then $t^*(q)$ is strictly decreasing in q.*

Proposition 4 demonstrates the effect of varying the super-majority requirement on the equilibrium level of taxation and redistribution, assuming a reversion policy below the median voter's ideal. Several features are worth noting. First, our results are consistent with Meltzer and Richard (1981) in that, under simple-majority rule, the equilibrium tax rate coincides with the ideal policy of the median voter. The median voter theorem obtains. Second, we know that the size of core is increasing in the required super-majority. For q sufficiently large (i.e. for $q \geq F(y_{sq})$), the core will have expanded sufficiently to include the status-quo policy. If so, by Proposition 1, the equilibrium policy is simply the status quo.

The more interesting case arises when q takes an intermediate value. If so, we know that $t^* > \tau_{sq}$, since the Nash Bargaining solution must be contained in the core$^+$. Moreover, if

[22] Formally, there exists $\overline{\kappa} > 1$ such that the Weibull distribution is right-skewed* whenever $\kappa < \overline{\kappa}$. Moreover, for every $\kappa > \overline{\kappa}$, there exists $q(\kappa) > \frac{1}{2}$ such that the distribution is right-skewed* on $\left[\frac{1}{2}, q(\kappa)\right]$.

[23] If $\tau_{sq} = 0$, then it is ideal for every voter with income $y > \overline{y}$. We take the lowest income earner amongst these, i.e. $\overline{y}$.

the income distribution is right-skewed*, the equilibrium tax rate decreases monotonically from τ_{med} to τ_{sq} as q increases from $\frac{1}{2}$ (simple majority rule) to $F(y_{sq})$.

There are two effects at play that cause this result to be true. First, increasing the super-majority requirement causes both the left and right decisive voters to become more 'extreme' (in the sense that the left decisive voter becomes poorer and therefore demands even more redistribution, and the right decisive voter becomes richer and demands even less redistribution). But right-skewness* implies that, for a given increase in q, the income of the right decisive voter increases by more than the income of the left decisive voter decreases. Since Nash Bargaining selects an 'average' policy between the ideal policies of the left and right decisive voters, this skews the policy in right decisive voter's favored direction; the equilibrium tax decreases.

Additionally, there is a second effect that would be present even if the income distribution were not skewed. For argument's sake, suppose the income distribution was symmetric, so that the incomes of the left and right decisive voters change by the same amount. Since the right decisive voter's ideal policy is now closer to the reversion policy, disagreement is less painful to her. By contrast, since the left decisive voter's ideal policy is now even further from the reversion policy, and so disagreement becomes costlier. Intuitively, this will increase the bargaining strength of the right decisive voter. One way to see this is to note that Nash Bargaining maximizes the (weighted) product of the decisive agents' gains over the status quo. Since the potential gains for R are now smaller, and the potential gains for L are larger, the solution must realize a larger share of R's potential gains. Increasing the super-majority rule gives an in-built advantage to the player whose ideal policy is closer to the status quo.

We note that this second effect would work in the opposite direction (i.e. it would skew taxes in favor of poorer agents) if the status quo policy were 'high'. However, as long as the income distribution is right-skewed*, the first effect will continue to privilege richer agents. Hence, the overall effect will be ambiguous.

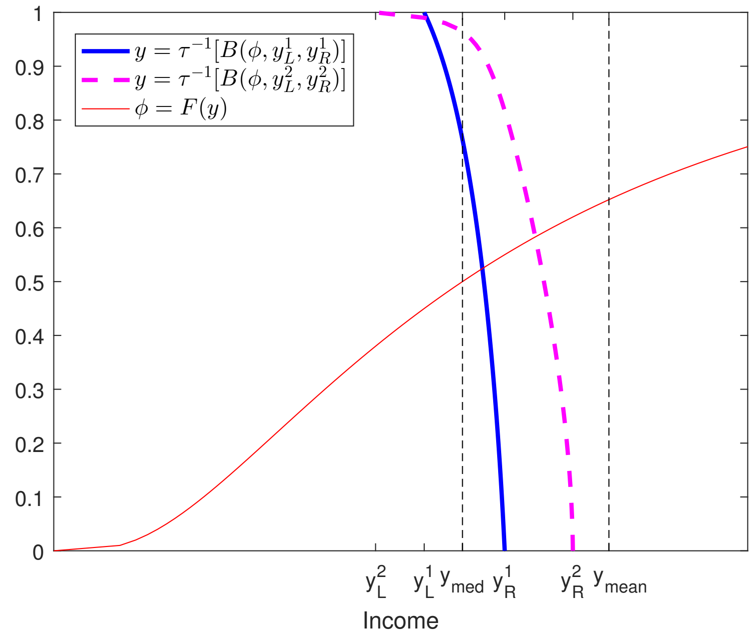

Figure 2: Effect of Changing the Required Super-Majority. The income distribution is the same as in Figure 1. The thick solid line corresponds to the pivotal voter's income for different bargaining weights when $q = 0.55$. The thick dashed line represents the case of $q = 0.6$

We see the two effects, described above, in Figure 2 which shows the effect of an increase in the super-majority requirement. As is evident, the income of the right decisive voter increases by more than the income of the left decisive voter decreases. This reflects the right-skewness* of the income distribution. Additionally, the downward sloping lines become more bowed, reflecting the greater bargaining power of the right decisive voter. As the super-majority requirement increases, for any bargaining weight, the implied income of the pivotal voter becomes closer to the right decisive voter's income.

Proposition 4 and Figure 2 also demonstrate the effect of changing the super-majority rule on the size and composition of the equilibrium factions. As the required super-majority increases, the pivotal voter becomes richer, which implies a larger left faction and smaller right faction. This, in turn, changes the equilibrium bargaining weights. In fact, although the overall outcome favors the R faction, the bargaining weights are now more favorable to the L faction. The dynamic that causes the equilibrium tax rate to fall, also increases the bargaining power of the left faction, *ceteris paribus*, which partially counter-acts the decrease in the tax rate. Hence, a dynamic, akin to Le Chatelier's Principle, obtains.

4.2 Changing the Status Quo

We now ask how the equilibrium tax rate changes with the status quo policy, taking the super-majority rule q, and thus the incomes of the right and left decisive voters, as given. To do so, we must introduce one additional assumption.

Let $\Delta v(\tau, y) = v(\tau, y) - v(\tau_s q, y)$ denote the utility improvement for a y-type voter from policy τ over the reversion policy. We say preference improvements are log-submodular if $y' > y$ and $\tau' > \tau$ implies $\frac{\Delta v(\tau', y')}{\Delta v(\tau, y')} < \frac{\Delta v(\tau', y)}{\Delta v(\tau, y)}$. Roughly speaking, log-supermodularity of utility improvements implies that, the coefficient of absolute risk aversion of the indirect utility function $v(\tau, y)$ is increasing in agent's income.[24] [25] Log-submodularity implies that when policy moves in a direction that improves utility, the coefficient of absolute risk aversion decreases.

Proposition 5. *Fix any super-majority requirement q. Let $t^*(\tau_{sq})$ be the equilibrium tax rate, given status quo policy τ_{sq}.*

- *For $\tau_{sq} \in [\tau_R, \tau_L]$, then $\frac{\partial t^*}{\partial \tau_{sq}} = 1$*
- *For $\tau_{sq} \notin [\tau_R, \tau_L]$, if preferences exhibit log-supermodular improvements, then $\frac{\partial t^*}{\partial \tau_{sq}} < 0$*

[24] When utility is increasing, the coefficient of absolute risk aversion is defined by $A(\tau, y) = -\frac{v_{\tau\tau}(\tau, y)}{v_\tau(\tau, y)}$ which takes a positive value if the agent is risk averse over tax policy. The appropriate analogue when utility is decreasing is: $A(\tau, y) = \frac{v_{\tau\tau}(\tau, y)}{v_\tau(\tau, y)}$.

[25] To see this, suppose without loss of generality, that policy $\tau > \tau_{sq}$ is an improvement over the reversion policy. By the mean value theorem, there exists $\gamma \in (\tau_{sq}, \tau)$ such that $\Delta v(\tau, y) = (\tau - \tau_{sq}) \cdot v_\tau(\gamma, y)$. Then $\frac{\partial}{\partial \tau} \log \Delta v(\tau, y) = \frac{v_{\tau\tau}(\gamma, y)}{v_\tau(\gamma, y)} \cdot \frac{\partial \gamma}{\partial \tau} + \frac{1}{\tau - \tau sq}$. It is easily verified that $\frac{\partial \gamma}{\partial \tau} > 0$. Hence, $\frac{\partial^2}{\partial y \partial \tau} < 0$ provided that $-\frac{v_{\tau\tau}(\gamma)}{v'_\tau(\gamma)}$ is increasing in y.

Proposition 5, illustrated graphically in 3, demonstrates the effect of the location of the status quo policy on the equilibrium tax rate. First, and trivially, when the status quo is in the core, the equilibrium policy selects the status quo. Again, this follows from Banks and Duggan (2006). Hence, changes in the status quo will be matched by identical changes in the equilibrium policy.

Second, if the status quo policy is not in the core, we find that as the status quo becomes more extreme (or, it increases), the equilibrium policy becomes more moderate. This echoes Banks and Duggan (2006) comparative static result that a reversion policy closer to the core will result in equilibrium policies close to the status quo. The intuition is similar to that of the second effect described above. As the status quo policy approaches the core, the cost of a legislative breakdown in the bargaining game becomes smaller for both decisive voters. However, the cost is much smaller for the decisive voter whose ideal policy is closer to the status quo. His bargaining strength is greater than his opponent's, and enables him to pull the equilibrium policy closer to his ideal.

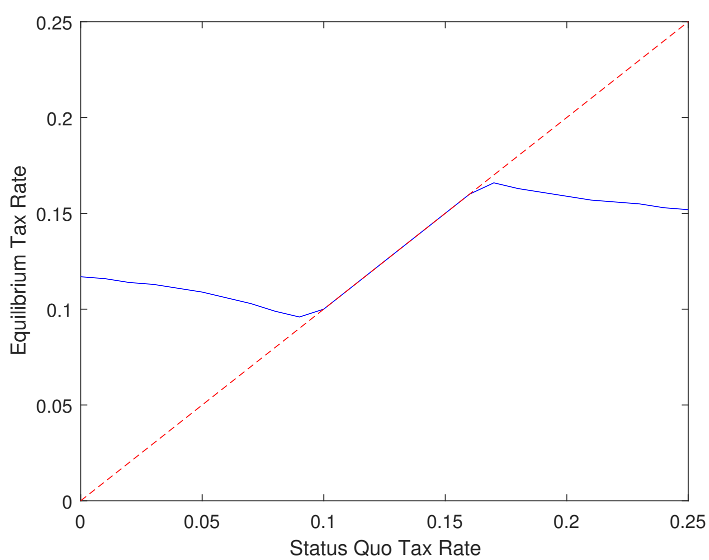

Figure 3: Effect of Changing the Status Quo. The income distribution is the same as in Figure 1, and the super-majority requirement is $q = 0.55$.

We noted above that policy selection under super-majority voting rules exhibits a status quo bias, in that the status quo is selected in equilibrium whenever it is in the core. This effect is well known. Our results go even further and imply that even if the status quo is outside the core, it will impart a sort of gravitational effect on the policy selected from the core+. By contrast, under simple-majority rule, the reversion policy was inconsequential to determining the equilibrium outcome policy because the core is generically a singleton. Thus, no selection procedure needs to take place.

5 Extensions

The subsequent sections extend upon our analysis. We consider two variant formulations of the preferences in our model, and show that our results are robust to these changes. In doing so, we note that micro-foundations will affect the results only insofar as they change salient properties of the indirect utility function $v(\tau, x)$. Absent such changes, all of the results from sections 3 and 4 will continue to hold. Hence, it suffices to check the properties of $v(\tau, y)$.

5.1 Public Goods

In our first extension, we extend our baseline model to the case considered by Gradstein (1999), where taxation finances the provision of a public good. Voters have preferences $u(c) + w(g)$ defined separably over consumption c and public goods g. The function u is as defined in our baseline framework, and w is similarly increasing $w' > 0$ and concave $w'' \leq 0$. The government's budget must be in balance, and so $g = e(\tau)\overline{y}$.

Again, we let $v(\tau, y)$ represent the preferences over tax policies of an agent with income y, taking the government's budget constraint as given. We have:

$$v(\tau, y) = u[(1-\tau)y] + w\left(\overline{y}e(\tau)\right).$$

Following the same argument as the above model framework, v is strictly concave in τ for each y. We have:

$$v_\tau\left(\tau, y\right) = \overline{y}e'\left(\tau\right)w'\left(g\right) - yu'\left(c^*\right)$$

The most preferred tax policy of a type-y agent is the solution to:

$$e'(\tau)\overline{y}w'(e(\tau)\overline{y}) - yu'((1-\tau)y) \leq 0$$

. Furthermore, by the implicit function theorem, these preferred tax rates are monotone in agents' income $-\frac{\tau}{y} < 0$.

Finally, $v_{\tau y} = -u'(c)0 - (1-\tau)yu''(c)$, where $c = (1-\tau)y$. Clearly $v_{\tau y} \leq 0$ provided that

$$R(c) = -\frac{u''(c)}{u'(c)}c \leq 1$$

This condition is analogous to Assumption 1, and is identical to a condition in Gradstein (1999). Since the indirect utility function retains all of its salient properties, it is evident that the results presented in sections 3 and 4 will continue to hold under this alternate set-up

.

5.2 Endogenous Labor Supply

In this section, we consider a micro-founded version of the model, analogous to Meltzer and Richard (1981), in which agents supply labor elastically, and dead-weight losses arise endogenously as a consequence of labor market distortions.

There is a unit mass of agents. Each agent is characterized by their productivity x which is an i.i.d. draw from some distribution F. From herein, we refer to an agent's productivity as their type. Agents have preferences $u(c) + w(l)$ defined separably over consumption c and leisure l. Utility is increasing in both consumption and leisure ($u' > 0$ and $w' > 0$), and u and w both are concave, with at least one strictly concave. Agents are endowed with one unit of time, which they may allocate between leisure and work effort n. For simplicity, we assume $\lim_{l \to 0} w'(l) = \infty$, which rules out the corner solution in which some agent spends all of her time working. Agents supply their labor in competitive labor markets, and earn a wage equal to their productivity. Hence, the income of an agent with productivity x is $y = xn$.

The government levies a proportional tax τ on labor income that finances a lump sum transfer T to each agent. Given the government policy (τ, T), the consumption of a type-x agent is: $c = T + (1-\tau)\, xn$. The agent's problem is to choose the quantity of labor to supply to maximize:

$$\max_{n \in [0,1]} u\left(T + (1-\tau)\, xn\right) + w\left(1-n\right)$$

Given that preferences are strictly concave, the problem has a unique maximizer $\hat{n}\left(\tau, T; x\right)$. The maximizer is the solution to the first order condition:

$$(1-\tau)\, xu'\left(T + (1-\tau)\, xn\right) - w'\left(1-n\right) \leq 0$$

with strict equality unless $\hat{n} = 0$. This will occur if $(1-\tau)\, xu'\left(T\right) - w\left(1\right) < 0$, which implies that:

$$x < \frac{1}{1-\tau} \cdot \frac{w'\left(1\right)}{u'\left(T\right)} = x_0\left(\tau, T\right)$$

Hence, all but the least productive agents will work. It is easily verified that work-effort is decreasing in the size of the transfer (i.e. $\frac{\partial \hat{n}}{\partial T} \leq 0$, with strict inequality whenever $x > x_0$[26]), which is implies that leisure is a normal good. Let $\hat{y}\left(\tau, T; x\right) = x\hat{n}\left(\tau, T; x\right)$ denote the

[26] To see this, let $D = (1-\tau)^2\, x^2 u''\left(\hat{c}\right) + w''\left(\hat{l}\right)$. By the strict concavity of preferences, $D < 0$. Applying the implicit function theorem to the first order condition gives: $\frac{\partial n}{\partial T} = \frac{(1-\tau)xu''(\hat{c})}{-D} < 0$.

income of a type-x agent. Notice that $\frac{\partial \hat{y}}{\partial x} = \frac{(1-\tau)xu'(\hat{c}) - nw''\left(\hat{l}\right)}{-D} > 0$, and so agents' incomes are monotone in their productivity.

The average income in the economy is: $\overline{y}(\tau, T) = \int_0^\infty \hat{y}(\tau, T; x) dF(x)$. Since the government policy must be feasible, we have $T = \tau \overline{y}(\tau, T)$. Intuitively, the government budget constraint establishes a feasible level of transfers $T(\tau)$ for each level of taxes τ.[27] Hence, the government's redistribution policy amounts to the choice of a tax rate τ. Moreover, we assume that households understand that government policy is subject to its budget constraint; there is no fiscal illusion. Accordingly, let $n(\tau; x) = \hat{n}\left(\tau, T(\tau); x\right)$ and $y(\tau; x) = \hat{y}\left(\tau, T(\tau); x\right)$ be the labor supply and income of a type-x agent, given tax rate τ and the associated transfer $T(\tau)$. Similarly, let $\overline{y}(\tau) = \int_0^\infty y(\tau; x) dF(x)$ denote the average income, given tax rate τ and the associated transfer $T(\tau)$.

Let $v(\tau, x)$ denote the indirect utility function of a type x agent. We have:

$$v(\tau, x) = u\left(\tau \overline{y}(\tau) + (1-\tau) x n(\tau, x)\right) + w\left(1 - n(\tau, x)\right)$$

We seek to establish the parallels between the properties of the indirect utility functions from the structural and reduced-form approaches. By the envelope theorem, $v_\tau(\tau, x) = \left[\frac{\partial \tau \overline{y}}{\partial \tau} - y(\tau; x)\right] u'\left(c(\tau, x)\right)$. This is directly analogous to the correponding expression (equation 1) in the reduced-form model.[28]

The indirect utility function does not generically inherit the curvature properties of the direct utility function. In particular, v need not be concave in τ. However, we have the following

[27] To see this formally, fix any $\tau \in [0, 1]$. Define the function $\psi(T; \tau) = \tau \overline{y}(\tau, T) - T$. Notice that $\psi(0; \tau) > 0$ and $\lim_{T \to \infty} \psi(T; \tau) < 0$ and $\frac{\partial \psi}{\partial T} = \tau \frac{\partial \overline{y}}{\partial \tau} - 1 < 0$, since $\frac{d\overline{y}}{\partial T} < 0$. The result follows by the intermediate value theorem.

[28] In fact, if the reduced-form function $e(\tau)$ satisfies $e'(\tau) = 1 + \frac{\partial \overline{y}}{\partial \tau} \cdot \frac{\tau}{\overline{y}} = 1 + \varepsilon(\tau)$, where $\varepsilon(\tau)$ is the average tax elasticity of labor supply, then the expressions for the marginal utility of taxation are identical across the two models. For example, if preferences are given by $c - \frac{\theta}{1+\theta}(1-l)^{\frac{1+\theta}{\theta}}$ (as in Greenwood, Hercowitz and Huffman (1988) and Correia, Neves and Rebelo (1995), amongst others) then the tax elasticity of labor supply is a constant $\varepsilon(\tau) = -\theta$, and so the implied reduced-form dead-weight loss function is $e(\tau) = (1+\tau)\theta + \theta \ln(1-\tau)$.

result:

Lemma 1. *The indirect utility function $v(\tau, x)$ is pseudo-concave in τ for each x.*

The pseudo-concavity of $v(\dot{\tau}, x)$ guarantees that the first order conditions characterize the optimal tax rate for each voter. Analogous to the reduced form case, any voter with above-average income where there is zero taxation will prefer zero taxation, and all other agents will demand a positive level of redistribution, with the optimal tax rate satisfying: $\overline{y}+\tau\frac{\partial\overline{y}}{\partial\tau} - y(\tau; x) = 0$. This condition is identical to equation (13) in Meltzer and Richard (1981), which defines the optimal tax rate in their framework.[29]

Finally, we establish that a condition, analogous to Assumption 1 ensures that agents' preferences over tax policies satisfy the Spence-Mirrlees condition. First note that:

$$v_{\tau x} = \left[(1-\tau)\left(\frac{\partial\tau\overline{y}}{\partial\tau} - y(\tau,x)\right)u''(c(\tau,x)) - u'(c(\tau,x))\right]\frac{\partial y(\tau,x)}{\partial x}$$

Since $\frac{\partial y(\tau,x)}{\partial x} > 0$, the sign of $v_{\tau x}$ depends on the sign of the term in square brackets. Now, as in the reduced form case, the condition is guaranteed to be satisfied for agents with incomes are sufficiently low (i.e. if $y(\tau,x) < \frac{\partial\tau\overline{y}}{\partial\tau}$). For agents with larger incomes, the Spence-Mirrlees condition is satisfied provided that:

$$R(c^{*}) < \frac{(1-\tau)\,y(\tau,x) + \tau\overline{y}\,(\tau)}{(1-\tau)\,y^{(}\tau,x) - (1-\tau)\,\frac{\partial(\tau y)}{\partial\tau}}$$

which is analogous to Assumption 1. The marginal utility of taxation is montone in agents' incomes provided that the coefficient of relative risk aversion is not too large for high productivity agents.

It is easily shown that this assumption is equivalent to the assuming that $\frac{\partial n}{\partial\tau} < 0$ for all τ and

[29] Pseudo-concavity does not guarantee that the problem admits a unique maximizer, although the set of optimizers is guaranteed to be convex. We follow Meltzer and Richard (1981) in assuming a unique solution.

all x.[30] As we briefly noted in section 2, when labor supply is elastic, imposing the Spence-Mirrlees condition is equivalent to assuming that taxation deters work effort. To make sense of this, note that increasing labor taxes (whilst simultaneously increasing transfers) has two effects. The substitution effect unambiguously deters work effort, whilst the sign of the wealth effect is ambiguous. Since a tax increase is combined with an increase in transfers, the wealth effect (further) deters work effort whenever $y(\tau, x) < \frac{\partial \tau \overline{y}}{\partial \tau}$, and stimulates it otherwise.[31] The Spence Mirrlees condition implies that, for high productivity agents, the wealth effect cannot be so large as to overwhelm the substitution effect. Following an increase in taxes, all agents work less.

Thus, we have shown, with the exception of concavity, all salient features of the indirect utility function are implied by micro-foundations. The fact that $v(\tau, x)$ is not guaranteed to be strictly concave is unfortunate, but not fatal. We have already shown that the optimal tax rate is characterized by the first order conditions, notwithstanding the failure of concavity. The other main role played by concavity was in guaranteeing that the bargaining game was guaranteed equilibrium in no delay, and that the social acceptance set was an interval. And, to achieve this, concavity was sufficient, but not necessary. For example, the existence of a no-delay equilibrium requires that, for every agent, there is some decisive coalition C including that agent, for which the associated coalitional acceptance set A_C is non-empty. Concavity of preferences guaranteed that *every* coalitional acceptance set is non-empty – which is clearly stronger that necessary.[32] Hence, we may plausibly expect no-delay equilibria to continue to exist, even when some agents' preferences are non-concave, provided that they are not too convex.

[30] To see this, note that $\frac{\partial n}{\partial \tau} = x \frac{\left[(1-\tau)\left(\overline{y}+\tau \frac{\partial \overline{y}}{\partial \tau} - y(\tau,x)\right)u''(c^*) - u'(c^*)\right]}{-D}$. Hence $v_{\tau x} = \frac{(-D)}{x} \cdot \frac{\partial y}{\partial x} \cdot \frac{\partial n}{\partial \tau}$. Then since $D < 0$ and $\frac{\partial y}{\partial x} > 0$, then $sign\,(v_{\tau x}) = sign\left(\frac{\partial n}{\partial \tau}\right)$.

[31] To see this, first note that absent the increase in transfers, the Slutsky Equation implies: $\frac{\partial n}{\partial \tau} = \frac{\partial n^h}{\partial \tau} - y \frac{\partial n}{\partial T}$. Adding the increased transfers gives an overall effect: $\frac{\partial n^h}{\partial \tau} - \left(y - \frac{\partial \tau \overline{y}}{\partial \tau}\right) \frac{\partial n^*}{\partial T}$.

[32] Similarly, connectedness of the social acceptance set requires that, for any pair of coalitions C and C', we can find a finite chain $C, C_1, .., C_k, C'$, such that every adjacent pair of coalitions has non-empty intersection, even if C and C' do not. Concavity of preferences, by contrast, guarantees that every pair of coalitions has non-empty intersection.

6 Conclusion

Despite its ubiquity in theoretical research on redistribution, the median voter theorem does not provide us with what we should expect in a variety of legislative settings. This paper provides some guidance in the case of a legislature with a super-majority voting rule. We employ a bargaining approach to select a unique robust policy from the generically non-singleton core and demonstrate that this policy is the result of bilateral asymmetric Nash Bargaining between two decisive voters determined by the voting rule.

We show that the equilibrium tax rate selected by our bargaining game is decreasing in the voting rule. While the result is broadly the same as Gradstein (1999), our findings are distinct in that equilibrium tax rate not only depends on the voting rule itself but also depends on the shape of the income distribution. The results are robust to an as assuming a micro-founded model functional form analogous to that of Meltzer and Richard (1981). Further, the results indicate that the equilibrium policy is decreasing in the status quo policy so long as the status quo does not fall inside the core. This result, which is akin to that of Banks and Duggan (2006), suggests that the status quo imparts a gravitational affect on the equilibrium tax rate selected. Additionally, this paper serves to support Duverger's Law. Our model does not presume the existence of party affiliations prior to the start of the game and yet the bargaining game leads to the selection of two 'party' leaders in the form of the left and right-decisive voters. This is in contrast to other theoretical papers that show the Law breaks down when there are voter-agents with heterogeneous policy preferences.

Our findings ultimately have implications for how we interpret the effect of imposing a super-majority voting rule. Beyond making the formation of coalitions more difficult and preventing increases to existing redistributive policies, this paper shows that super-majority rules in fact impart preferential bargaining power on the 'richer' side of the income distribution, adding an extra dimension to the argument of Buchanan and Tullock (1962) as well as anecdotal reasons for imposing such electoral systems.

7 Appendix

Proof of Proposition 1. The proof follows the strategy in Cardona and Ponsati (2011) and Parameswaran and Murray (2018). Suppose the equilibrium social acceptance set is $\left[\underline{t}, \overline{t}\right]$. It is immediate that the equilibrium proposals are given by:

$$t(y) = \begin{cases} \underline{t} & y > \tau^{-1}(\underline{t}) \\ \tau(y) & y \in \left[\tau^{-1}(\underline{t}), \tau^{-1}(\overline{t})\right] \\ \overline{t} & y < \tau^{-1}(\overline{t}) \end{cases}$$

Let $V(\underline{t}, \overline{t}; y) = (1 - P(\tau^{-1}(\underline{t})))v(\underline{t}, y) + \int_{\tau^{-1}(\overline{t})}^{\tau^{-1}(\underline{t})} v(\tau(z), y)dP(z) + P(\tau^{-1}(\overline{t}))v(\overline{t}, y)$ be the expected utility of a type-y agent in the continuation game. Let the individual acceptance sets $A(y) = [\underline{t}(y), \overline{t}(y)]$ be defined as in the statement of the proposition.

Step 1. We first show that in any equilibrium, $\underline{t} = \underline{t}_L$ and $\overline{t} = \overline{t}_R$. Take any y, y' with $y' < y$. The following claims are true:

1. Suppose $v(\underline{t}, y) \leq (1 - \delta)v(\tau_{sq}, y) + \delta V(\underline{t}, \overline{t}; y)$. Then $v(\underline{t}, y') \leq (1 - \delta)v(\tau_{sq}, y') + \delta V(\underline{t}, \overline{t}; y')$.
2. Suppose $v(\overline{t}, y') \leq (1 - \delta)v(\tau_{sq}, y') + \delta V(\underline{t}, \overline{t}; y')$. Then $v(\overline{t}, y) \leq (1 - \delta)v(\tau_{sq}, y) + \delta V(\underline{t}, \overline{t}; y)$.

We prove (1), and note that (2) is proved analogously. For notational simplicity, define $\Delta v(\tau, y) = v(\tau, y) - v(\tau_{sq}, y)$ and $\Delta V(\underline{t}, \overline{t}; y) = V(\underline{t}, \overline{t}; y) - v(\tau_{sq}, y)$. Suppose (1) is not true. Then there exists some $y' < y$ s.t. $\Delta v(\underline{t}, y) \leq \delta \Delta V(\underline{t}, \overline{t}; y)$ and $\Delta v(\underline{t}, y') > \delta \Delta V(\underline{t}, \overline{t}; y')$. This implies:

$$\Delta v(\underline{t}, y) - \Delta v(\underline{t}, y') < \delta \left[\Delta V(\underline{t}, \overline{t}; y) - \Delta V(\underline{t}, \overline{t}; y')\right]$$

Now, by the Spence-Mirlees condition, $\frac{\partial}{\partial \tau}\left[\Delta v(\tau, y)-\Delta v(\tau, y')\right]=\int_{y'}^{y} v_{\tau y}(\tau, z) dz \leq 0$. Hence:

$$
\begin{aligned}
\Delta V(\underline{t}, \bar{t}; y)-\Delta V(\underline{t}, \bar{t}; y') = & \left(1-P(\tau^{-1}(\underline{t}))\right)\left[\Delta v(\underline{t}, y)-\Delta v(\underline{t}, y')\right]+ \\
& \int_{\tau^{-1}(\bar{t})}^{\tau^{-1}(\underline{t})}\left[\Delta v(\tau(z), y)-\Delta v(\tau z, y')\right] dP(z)+P(\tau^{-1}(\bar{t}))\left[\Delta v(\bar{t}, y)-\Delta v(\bar{t}, y')\right] \\
\leq & \Delta v(\underline{t}, y)-\Delta v(\underline{t}, y')
\end{aligned}
$$

Then $\Delta v(\underline{t}, y)-\Delta v(\underline{t}, y')<\delta\left[\Delta V(\underline{t}, \bar{t}; y)-V(\underline{t}, \bar{t}; y')\right]<\delta\left[\Delta v(\underline{t}, y)-\Delta v(\underline{t}, y')\right]$, which cannot be, since $\delta<1$. Hence, the claim is true.

Now, suppose $\underline{t}<\underline{t}_L$. Then, $\Delta v(\underline{t}, y_L)<\delta \Delta V(\underline{t}, \bar{t}, y_L)$, and so, by (1), $\Delta v(\underline{t}, y)<\delta \Delta V(\underline{t}, \bar{t}, y)$ for all $y<y_L$. But since $F(y_L)=1-q$, this means fewer than q agents support $\underline{t}$, which cannot be. Hence $\underline{t} \geq \underline{t}_L$. Next, since $\Delta v(\underline{t}_L, y_L) \geq \delta \Delta V(\underline{t}, \bar{t}, y_L)$, taking the contrapositive of (1), $\Delta v(\underline{t}_L, y)>\delta \Delta V(\underline{t}, \bar{t}, y)$ for all $y>y_L$. This implies that there is a coalition that would accept $\underline{t}_L$, and so $\underline{t} \leq \underline{t}_L$. Hence $\underline{t}=\underline{t}_L$. By a similar argument, we can show that $\bar{t}=\bar{t}_R$.

Step 2. Next, we show that the bargaining game admits a unique equilibrium. Let $\bar{\zeta}_R(\theta)=\max\left\{\tau \in[0,1] \mid \Delta v(\tau, y_R) \geq \delta \Delta V(\theta, \tau; y_R)\right\}$ and $\underline{\zeta}_L(\theta)=\min\left\{\tau \in[0,1] \mid \Delta v(\tau, y_L) \geq \delta \Delta V(\tau, \theta; y_L)\right\}$. Naturally, if $(\underline{t}, \bar{t})$ are a pair of equilibrium thresholds, we must have: $\underline{t}=\underline{\zeta}_L(\bar{t})$ and $\bar{t}=\bar{\zeta}_R(\underline{t})$. Let $H(\underline{t})=\underline{\zeta}_L\left(\bar{\zeta}_R(\underline{t})\right)$. Then $(\underline{t}, \bar{t})$ is an equilibrium if $\underline{t}$ is a fixed point of H and $\bar{t}=\bar{\zeta}_R(\underline{t})$. Since Δv is continuous, so are $\bar{\zeta}_R$ and $\underline{\zeta}_L$. Hence, by Brouwer's fixed point theorem, H admits a fixed point.

We need to show that this fixed point is unique. Implicitly differentiating the function that defines $\bar{\zeta}_R(\theta)$, we have:

$$
\bar{\zeta}_R'(\theta)=\frac{\delta(1-P\left(\tau^{-1}(\theta)\right))}{1-\delta P\left(\tau^{-1}(\bar{\zeta}_R(\theta))\right)} \cdot \frac{v_\tau(\theta, y_R)}{v_\tau(\bar{\zeta}_R, y_R)}
$$

Similarly, we have:

$$\underline{\zeta}_L'(\theta) = \begin{cases} \frac{\delta P\left(\tau^{-1}(\theta)\right)}{1-\delta+\delta P\left(\tau^{-1}(\underline{\zeta}_L(\theta))\right)} \cdot \frac{v_\tau(\theta,y_L)}{v_\tau(\underline{\zeta}_L,y_L)} & \underline{\zeta}_L(\theta) > 0 \\ 0 & \underline{\zeta}_L(\theta) = 0 \end{cases}$$

Let $(\underline{t}, \bar{t})$ be equilibrium thresholds (which implies $\bar{\zeta}_R(\underline{t}) = \bar{t}$ and $\underline{\zeta}_L(\bar{t}) = \underline{t}$). Then:

$$H'\left(\underline{t}\right) = \begin{cases} \frac{\delta(1-P\left(\tau^{-1}(\underline{t})\right)}{1-\delta P\left(\tau^{-1}(\bar{t})\right)} \cdot \frac{\delta P\left(\tau^{-1}(\bar{t})\right)}{1-\delta+\delta P(\tau^{-1}(\underline{t}))} \cdot \frac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)} \cdot \frac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)} & \underline{t} > 0 \\ 0 & \underline{t} = 0 \end{cases}$$

We seek to show that $H'(\underline{t}) < 1$ at any fixed point $\underline{t}$. Notice that this is immediate if $\underline{t} = 0$. Suppose $\underline{t} > 0$. Then $H'\left(\underline{t}\right)$ is the product of 4 terms, the first two of which are positive and less than 1. It suffices then to show that the product of the third and fourth terms is also less than 1.

Suppose $H\left(\underline{x}\right) \geq 1$. Then at least one of $\left|\frac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| > 1$ or $\left|\frac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)}\right| > 1$. There are several cases to consider. First, suppose $\left|\frac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| > 1$. Since $\bar{t} > \tau(y_R)$ then $v_\tau(\bar{t}, y_R) < 0$ by the concavity of v. If $\tau(y_R) \leq \underline{t} < \bar{t}$, then concavity implies $v_\tau(\bar{t}) < v_\tau(\underline{t}) \leq 0$, which contradicts $\left|\frac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| > 1$. Hence $\underline{t} < \tau(y_R) < \bar{t}$, and so $v_\tau(\underline{t}) > 0$. Suppose additionally $\tau(y_L) \geq \bar{t} > \underline{t}$. Then $v_\tau(\underline{t}, y_L) > 0$ and $v_\tau(\bar{t}, y_L) \geq 0$. Hence $\frac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)} < 0$, and $\frac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)} > 0$, and so $H < 0$, which cannot be. Hence $\underline{t} < \tau(y_R) < \tau(y_L) < \bar{t}$. Then, by the Spence-Mirrlees condition, $0 < v_\tau(\underline{t}, y_R) < v_\tau(\underline{t}, y_L)$ and $v_\tau(\bar{t}, y_R) < v_\tau(\bar{t}, y_L) < 0$, and so:

$$\frac{v_\tau(\underline{t}, y_R)}{v_\tau(\bar{t}, y_R)} \cdot \frac{v_\tau(\bar{t}, y_L)}{v_\tau(\underline{t}, y_L)} = \frac{v_\tau(\underline{t}, y_R)}{v_\tau(\underline{t}, y_L)} \cdot \frac{v_\tau(\bar{t}, y_L)}{v_\tau(\bar{t}, y_R)} < 1$$

Hence $H < 1$, which cannot be, and so $\left|\frac{v_\tau(\underline{t},y_R)}{v_\tau(\bar{t},y_R)}\right| \leq 1$.

By a similar logic, we show that $\left|\frac{v_\tau(\bar{t},y_L)}{v_\tau(\underline{t},y_L)}\right| \leq 1$. Hence our initial supposition was wrong; $H'(\underline{t}) \ngeq 1$. Hence, $H' < 1$ and so H admits a unique fixed point. $\square$

Proof of Proposition 2. Recall, the acceptance set is $[\underline{t}, \overline{t}]$, where $\Delta v(\underline{t}, y_L) = \delta \Delta V(\underline{t}, \overline{t}, y_L)$ and $\Delta v(\overline{t}, y_R) = \delta \Delta V(\underline{t}, \overline{t}, y_R)$. Now, by construction, $\Delta v(\overline{t}, y_L) \geq \delta \Delta V(\underline{t}, \overline{t}, y_L)$ since y_L will accept $\overline{t}$. Then, since $\Delta v(\tau, y_R)$ is strictly quasi-concave in τ for each y, $\Delta v(t, y_L) \geq \Delta v(\underline{t}, y_L)$ for every $t \in int[\underline{t}, \overline{t}]$. Similarly, $\Delta v(\overline{t}, y_R) \geq \delta \Delta V(\underline{t}, \overline{t}, y_R)$ and so $\Delta v(t, y_R) \geq \Delta V(\overline{t}, y_R)$ for every $t \in int[\underline{t}, \overline{t}]$. Hence $\Delta V(\underline{t}, \overline{t}, y_L) > \Delta v(\underline{t}, y_L)$ and $\Delta V(\underline{t}, \overline{t}, y_R) > \Delta v(\overline{t}, y_R)$ whenever $\underline{t} < \overline{t}$. Now, for every $\delta < 1$, $\frac{\Delta v(\underline{t}, y_L)}{\Delta V(\underline{t}, \overline{t}, y_L)} = \delta = \frac{\Delta v(\overline{t}, y_R)}{\Delta V(\underline{t}, \overline{t}, x_R)}$ and so as $\delta \to 1$, we need $\Delta V(\underline{t}, \overline{t}, y_L) - \Delta v(\underline{t}, y_L) \to 0$ and $\Delta V(\underline{t}, \overline{t}, y_R) - \Delta v(\overline{t}, y_R) \to 0$. But this requires $\overline{t} - \underline{t} \to 0$. Hence $A \to [t^*, t^*]$ as $\delta \to 1$. □

Proof of Proposition 3. For notational convenience, denote: $\Delta v_i(\tau) = v(\tau, y_i)$ for $i \in \{L, R\}$, and denote $\underline{t}(y_L) = \underline{t}_L$ and $\overline{t}(y_R) = \overline{t}_R$. For every $\delta < 1$, we know that $\underline{t}_L$ and $\overline{t}_R$ are defined by the system:

$$\Delta v_R\left(\overline{t}_R\right) = \delta \left[F\left(\tau^{-1}(\overline{t}_R)\right) \Delta v_R(\overline{t}_R) + \int_{\tau^{-1}(\overline{t}_R)}^{\tau^{-1}(\underline{t}_L)} \Delta v_R\left(\tau(z)\right) dF(z) + \left(1 - F\left(\tau^{-1}(\underline{t}_L)\right)\right) \Delta v_R(\underline{t}_L) \right]$$

$$\Delta v_L\left(\overline{t}_L\right) = \delta \left[F\left(\tau^{-1}(\overline{t}_R)\right) \Delta v_L(\overline{t}_R) + \int_{\tau^{-1}(\overline{t}_R)}^{\tau^{-1}(\underline{t}_L)} \Delta v_L\left(\tau(z)\right) dF(z) + \left(1 - F\left(\tau^{-1}(\underline{t}_L)\right)\right) \Delta v_L(\underline{t}_L) \right]$$

Fix some $\varphi \in [0, 1]$. Let $E = (1 - \varphi)\overline{t}_R + \varphi \underline{t}_L$, and let $\varepsilon = E - \underline{t}_L$. Note that $(\overline{t}_R, \underline{t}_L)$ is fully characterized by (E, ε), and that these are implicitly functions of δ. (Indeed, $\underline{t}_L = E - \varepsilon$ and $\overline{t}_R = E + \frac{\varphi}{1-\varphi}\varepsilon$, and $\varepsilon \to 0$ as $\delta \to 1$.) Affect this change of variables. We have:

$$\left[1 - \delta P\left(\tau^{-1}\left(E + \frac{\varphi}{1-\varphi}\varepsilon\right)\right)\right] \Delta v_R\left(E + \frac{\varphi}{1-\varphi}\varepsilon\right) = \delta \int_{\tau^{-1}\left(E + \frac{\varphi}{1-\varphi}\varepsilon\right)}^{\tau^{-1}(E-\varepsilon)} \Delta v_R\left(\tau(z)\right) dP(z) + \\ + \delta\left(1 - P\left(\tau^{-1}(E - \varepsilon)\right)\right) \Delta v_R(E - \varepsilon) \tag{3}$$

$$\left[1-\delta\left(1-P\left(\tau^{-1}\left(E-\varepsilon\right)\right)\right)\right]\Delta v_L\left(E-\varepsilon\right)=\delta\int_{\tau^{-1}(E+\frac{\varphi}{1-\varphi}\varepsilon)}^{\tau^{-1}(E-\varepsilon)}\Delta v_L\left(\tau(z)\right)dP(z)+$$
$$+\,\delta P\left(\tau^{-1}(E+\frac{\varphi}{1-\varphi}\varepsilon)\right)\Delta v_L\left(E+\frac{\varphi}{1-\varphi}\varepsilon\right) \quad (4)$$

Totally differentiating (3) w.r.t δ gives:

$$\begin{aligned}
&\left\{\left[1-\delta P\left(\tau^{-1}\left(\bar{t}_R\right)\right)\right]\frac{\partial\Delta v_R\left(\bar{t}_R\right)}{\partial\tau}-\delta\left(1-P\left(\tau^{-1}\left(\underline{t}_L\right)\right)\right)\frac{\partial\Delta v_R\left(\underline{t}_L\right)}{\partial\tau}\right\}\frac{\partial E}{\partial\delta}\\
&=P\left(\tau^{-1}\left(\bar{t}_R\right)\right)\Delta v_R\left(\bar{t}_R\right)\\
&\quad+\left\{\frac{\varphi}{1-\varphi}\left[1-\delta P\left(\tau^{-1}\left(\bar{t}_R\right)\right)\right]\frac{\partial\Delta v_R\left(\bar{t}_R\right)}{\partial\tau}+\delta\left(1-P\left(\tau^{-1}\left(\underline{t}_L\right)\right)\right)\frac{\partial\Delta v_R\left(\underline{t}_L\right)}{\partial\tau}\right\}\frac{\partial\varepsilon}{\partial\delta}\\
&\quad+\int_{\tau^{-1}\left(\bar{t}_R\right)}^{\tau^{-1}\left(\underline{t}_L\right)}\Delta v_R\left(\tau\left(z\right)\right)dP\left(z\right)+\left(1-P\left(\tau^{-1}\left(\underline{t}_L\right)\right)\right)\Delta v_R\left(\underline{t}_L\right)
\end{aligned} \quad (5)$$

Taking $\delta\to1$ gives:

$$\Delta v_R\left(t^*\right)=\frac{1}{1-\varphi}\left(1-P\left(\tau^{-1}\left(t^*\right)\right)\right)\frac{\partial\Delta v_R\left(t^*\right)}{\partial\tau}\lim_{\delta\to1}\frac{\partial\varepsilon}{\partial\delta}$$

which implies:

$$\frac{1}{\lim_{\delta\to1}\frac{\partial\varepsilon}{\partial\delta}}=\frac{1}{1-\varphi}\left(1-P\left(\tau^{-1}\left(t^*\right)\right)\right)\frac{\frac{\partial\Delta v_R(t^*)}{\partial\tau}}{\Delta v_R\left(t^*\right)}$$

Similarly differentiating (4) w.r.t. δ, and taking the limit as $\delta\to1$ gives:

$$\frac{1}{\lim_{\delta\to1}\frac{\partial\varepsilon}{\partial\delta}}=\frac{1}{1-\varphi}P\left(\tau^{-1}\left(t^*\right)\right)\frac{\frac{\partial\Delta v_L(t^*)}{\partial\tau}}{\Delta v_L\left(t^*\right)}$$

It follows that:

$$P\left(\tau^{-1}\left(t^*\right)\right)\frac{\frac{\partial\Delta v_L(t^*)}{\partial\tau}}{\Delta v_L\left(t^*\right)}+\left(1-P\left(\tau^{-1}\left(t^*\right)\right)\right)\frac{\frac{\partial\Delta v_R(t^*)}{\partial\tau}}{\Delta v_R\left(t^*\right)}=0$$

But this is precisely the Nash Bargaining solution when $\phi=P\left(\tau^{-1}\left(t^*\right)\right)$. $\square$

Proof of Proposition 4. Let $(t^*(q), \phi^*(q))$ be the equilibrium policy under supermajority rule q, i.e.

$$t^*(q) = \arg\max_{t\in[0,1]} \left[\Delta v(t, y_L(q))\right]^{\phi^*(q)} \left[\Delta v(t, y_R(q))\right]^{1-\phi^*(q)}$$

and $\phi^* = P(\tau^{-1}(t^*))$, where $y_L(q) = F^{-1}(1-q)$, and $y_R(q) = F^{-1}(q)$.

Let us first dispose of the boundary cases. First suppose $q = \frac{1}{2}$. Then $y_L(q) = y_R(q) = y_{med}$, and so $B(\phi, y_L, y_R) = \tau(y_{med})$ for all $\phi \in [0,1]$. Hence $t^*\left(\frac{1}{2}\right) = \tau(y_{med})$. (Moreover, $\phi^*\left(\frac{1}{2}\right) = P(F^{-1}\left(\frac{1}{2}\right))$.) Second, suppose $q \geq (y_{sq})$, so that $y_R \geq_{sq}$. Then $y_L < y_{sq} < y_R$. Notice that it is a necessary condition that $\Delta v(t^*, y_i) \geq 0$ for each $i \in \{L, R\}$. Since $y_L < y_{sq}$, then $\Delta v(\tau, y_L) < 0$ whenever $\tau < \tau_{sq}$. Similarly, since $y_R \geq y_{sq}$, then $\Delta v(\tau, y_R) < 0$ for all $\tau > \tau_{sq}$. Hence, for any $\phi \in [0,1)$, the only possible solution is $B(\phi, y_L, y_R) = \tau_{sq}$. And so $t^*(q) = \tau_{sq}$ for all $q \geq (y_{sq})$. (Moreover, $\phi^*(q) = P(y_{sq})$.)

We now consider the most interesting case. Suppose $q \in \left(\frac{1}{2}, F(y_{sq})\right)$. Then $\tau_L > \tau_R > \tau_{sq}$, which implies that there is a range of $t > \tau_R$ for which $\Delta v(t, y_R) > 0$. Hence, the optimizer $t^* \in (0,1)$, and so must satisfy the first and second order conditions. Let $\psi(t, \phi, q) = \phi \frac{v_\tau(t, y_L)}{\Delta v(t, y_L)} + (1-\phi)\frac{v_\tau(t, y_R)}{\Delta v(t, y_R)}$, where $y_L = F^{-1}(1-q)$ and $y_R = F^{-1}(q)$. Then, $\psi(t^*(q), \phi^*(q), q) = 0$ for every $q \in \left[\frac{1}{2}, F(y_{sq})\right]$, and $\psi_t(t^*, \phi^*, q) < 0$, by the second order conditions.

In what follows, we simplify notation by denoting $\Delta v^i = \Delta v(t^*, y_i)$. Take $q \in \left(\frac{1}{2}, F(y_{sq})\right)$, and suppose that $\frac{\partial \phi^*}{\partial q} \leq 0$. Since $\phi^* = P(\tau^{-1}(t^*))$, we know that $\frac{\partial \phi^*}{\partial q} = \frac{p\left(\tau^{-1}(t^*)\right)}{\tau'(t^*)}\frac{\partial t^*}{q}$. We know that $\tau'(t) < 0$ for every t, since higher taxes preferred by lower income earners. Hence $\frac{\partial \phi^*}{\partial q} \cdot \frac{\partial t^*}{\partial q} < 0$, and so our assumption implies $\frac{\partial t^*}{\partial q} > 0$.

Totally differentiate ψ w.r.t. q gives:

$$\psi_t(t^*, \phi^*, q)\frac{\partial t^*}{\partial q} + \psi_\phi \frac{\partial \phi^*}{\partial q} + \psi_q = 0$$

Now, $\psi_\phi = \frac{v_\tau(t,y_L)}{\Delta v(t,y_L)} - \frac{v_\tau(t,y_R)}{\Delta v(t,y_R)} > 0$, since $\Delta v^i > 0$ for each $i \in \{L, R\}$, and $v_\tau(t^*, y_L) > 0 > v_\tau(t^*, y_R)$. Additionally:

$$\begin{aligned}
\psi_q &= \frac{(1-\phi^*)}{f(y_R)} \cdot \frac{\Delta v^R \cdot v^R_{\tau y} - \Delta v^R_y \cdot v^R_\tau}{(\Delta v^R)^2} - \frac{\phi^*}{f(Y_L)} \cdot \frac{\Delta v^L \cdot v^L_{\tau y} - \Delta v^L_y \cdot v^L_\tau}{(\Delta v^L)^2} \\
&= \frac{(1-\phi^*)}{f(y_R)} \frac{v^R_\tau}{\Delta v^R} \left[\frac{v^R_{\tau y}}{v^R_\tau} - \frac{\Delta v^R_y}{\Delta v^R} \right] - \frac{\phi^*}{f(x_L)} \frac{v^L_\tau}{\Delta v^L} \left[\frac{v^L_{\tau y}}{v^L_\tau} - \frac{\Delta v^L_y}{\Delta v^L} \right]
\end{aligned}$$

Using the fact that $\psi(t^*, \phi^*, q) = 0$, we know that $\phi^* \frac{v^L_\tau}{\Delta v^L} = -(1-\phi^*) \frac{v^R_\tau}{\Delta v^R}$, we have:

$$\psi_q = \frac{(1-\phi^*)}{f(y_R)} \frac{v^R_\tau}{\Delta v^R} \left[\left(\frac{v^R_{\tau y}}{v^R_\tau} - \frac{\Delta v^R_y}{\Delta v^R} \right) + \left(\frac{v^L_{\tau y}}{v^L_\tau} - \frac{\Delta v^L_x}{\Delta v^L} \right) \cdot \frac{f(y_R)}{f(y_L)} \right]$$

Now, since F is right-skewed* on $\left(\frac{1}{2}, F(y_{sq})\right)$, then $\frac{f(y_R)}{f(y_L)} < 1$. This guarantees that the term in the square bracket is positive. (The first term is guaranteed to be positive, and the second cannot be too negative.) Then, since $\frac{v^R_\tau}{\Delta v^R} < 0$, $\psi_q < 0$. Hence,

$$\frac{\partial t^*}{\partial q} = -\frac{\psi_\phi(t^*, \phi^*, q) \cdot \frac{\partial \phi^*}{\partial q} + \psi_q(t^*, \phi^*, q)}{\psi_t(t^*, \phi^*, q)}$$

Now, we have shown that the numerator is negative, and the denominator is negative by the second order conditions, and so $\frac{\partial t^*}{\partial q} < 0$. But this contradicts $\frac{\partial t^*}{\partial q} > 0$. Hence $\frac{\partial \phi^*}{\partial q} > 0$ and $\frac{\partial t^*}{\partial q} < 0$. □

Proof of Proposition 5. Fix a super-majority requirement q. Suppose $\tau_{sq} \notin [\tau_R, \tau_L]$. We seek to show that $\frac{\partial t^*}{\partial \tau_{sq}} < 0$. First, let $\Delta v(\tau, y; \tau_{sq}) = v(\tau, y) - v(\tau_{sq}, y)$. Define ψ as in the proof of Proposition 4. I.e.:

$$\psi(\tau; \tau_{sq}, \phi) = \phi \frac{v_\tau(\tau, y_L)}{\Delta v(\tau, y_L; \tau_{sq})} + (1-\phi) \frac{v_\tau(\tau, y_R)}{\Delta v(\tau, y_R; \tau_{sq})}$$

and

$$\psi_{\tau_{sq}} = \phi \frac{v_\tau(\tau, y_L)}{\Delta v(\tau, y_L; \tau_{sq})} \cdot \frac{v_\tau(\tau_{sq}, y_L)}{\Delta v(\tau, y_L; \tau_{sq})} + (1-\phi) \frac{v_\tau(\tau, y_R)}{\Delta v(\tau, y_R; \tau_{sq})} \cdot \frac{v_\tau(\tau_{sq}, y_R)}{\Delta v(\tau, y_R; \tau_{sq})}$$

By construction, we know that $\psi(t^*(\tau_{sq}); \tau_{sq}, \phi^*(\tau_{sq})) = 0$ and that $\psi_\tau(t^*; \tau_{sq}, \phi^*) < 0$. Additionally, since $\phi^* = P\left(\tau^{-1}(t^*)\right)$, we know that $\frac{\partial \phi^*}{\partial \tau_{sq}} = \frac{p\left(\tau^{-1}(t^*)\right)}{\tau'(t^*)} \frac{\partial t^*}{\partial \tau_{sq}}$. Totally differentiating $\psi(t^*(\tau_{sq}); \tau_{sq}, \phi^*(\tau_{sq})) = 0$ with respect to the status quo gives:

$$\psi_{\tau_{sq}}(t^*; \tau_{sq}, \phi^*) + \left[\psi_\tau(t^*; \tau_{sq}, \phi^*) + \psi_\phi(t^*; \tau_{sq}, \phi^*) \frac{p\left(\tau^{-1}(t^*)\right)}{\tau'\left(t^*\right)}\right] \frac{\partial t^*}{\partial \tau_{sq}} = 0$$

It follows that:

$$\frac{\partial t^*}{\partial \tau_{sq}} = -\frac{\psi_{\tau_{sq}}}{\psi_\tau + \psi_\phi \frac{p(\tau^{-1}(t^*))}{\tau'(t^*)}}$$

Since $\psi_\tau < 0$, $\psi_\phi > 0$ (which we verified in the proof of Proposition 4) and $\tau' < 0$, the denominator must be negative. Hence, $\frac{\partial t*}{\partial \tau_{sq}}$ has the same sign as $\psi_{y, x_{sq}}$. Furthermore, since

$$(1-\phi) \frac{v_{\tau_{sq}}(t^*, y_R)}{\Delta v(t^*; \tau_{sq}, y_R)} = -\phi \frac{v_{\tau_{sq}}(t^*, y_L)}{\Delta v(t^*; \tau_{sq}, y_L)}$$

we have:

$$\psi_{\tau_{sq}}(t^*; \tau_{sq}, \phi^*) = \phi^* \frac{v_\tau(t^*, y_L)}{\Delta v(t^*; \tau_{sq}, y_L)} \left[\frac{v_\tau(\tau_{sq}, y_R)}{\Delta v(t^*; \tau_{sq}, y_R)} - \frac{v_\tau(\tau_{sq}, y_L)}{\Delta v(t^*; \tau_{sq}, y_L)}\right] < 0$$

, where the inequality follows from the fact that Δu is log-submodular and $y_R > y_L$. $\square$

Proof of Lemma 1. To prove the pseudo-concavity of the indirect utility function it suffices to show that, for $\tau \in (0,1)$, if $v_\tau(\tau^*, x) = 0$ then v_τ achieves a maximum at τ^*. Suppose $v_\tau(\tau^*, x) = 0$ for some $x > 0$. Recall $v_\tau(\tau, x) = \left[\frac{\partial \tau \overline{y}}{\partial \tau} - y(\tau; x)\right] u'\left(c(\tau, x)\right)$ and that income y is monotonically increasing in productivity x. Thus, whenever $x' < x$, then $v_\tau(\tau^*, x') > 0$. Similarly, whenever $x'' > x$, $v_\tau(\tau^*, x'') < 0$.

Takes some small $\epsilon > 0$. Since v is continuously differentiable, it follows that $v_\tau(\tau^* - \epsilon, x') > 0$ for ever $x' < x$. By continuity, this implies that $v_\tau(\tau^* - \epsilon, x) \geq 0$. Similarly, $v_\tau(\tau^* + \epsilon, x) \leq 0$. Together, these rule out τ^* as a minimizer (which would require $v_\tau(\tau^* - \epsilon, x) < 0$) or as a saddle point (which would require that $v_\tau(\tau^* - \epsilon, x)$ and $v_\tau(\tau^* + \epsilon, x)$ have the same sign. □

Outcomes in a Model with Repeated Bargaining. Consider a dynamic variant of our bargaining model à la Baron (1996) in which the status quo evolves through the game – i.e. the policy chosen today becomes tomorrow's status quo. Assume all other features of the game are unchanged. Following Baron (1996), we solve for Markov Perfect equilibria.

Let $W(\tau_{sq}, y)$ be the value function of a type-y agent when the status quo is τ_{sq}. First, note that if the core contains the status quo $\tau_{sq} \in (\tau_R, \tau_L)$, then subsequent proposals can never change the status quo. To see this, consider any deviation policy τ' and suppose it is accepted. Given the equilibrium strategies, let $\sigma_s(t)$ be a probability distribution over the policies implemented s periods into the future. Let $E = (1-\delta)\tau' + (1-\delta)\sum_{s=1}^{\infty} \delta^s \int t d\sigma_s(t)$. The deviation utility of a type-y agent is:

$$\hat{W}(\tau', y) = (1-\delta)v(\tau', y) + (1-\delta)\sum_{s=1}^{\infty} \delta^s \int v(t, y) d\sigma_s(t) \leq v(E, y)$$

where the inequality follows by the concavity of v, and will be strict unless $\sigma_s(\tau') = 1$ for all $s \geq 1$. By contrast, the utility from not deviating for a type-y agent is: $v(\tau_{sq}, y)$.

Without loss of generality, suppose $E \leq \tau_{sq}$. If $E < \tau_{sq}$, then $v(\tau_{sq}, y) > v(E, y) \geq \hat{W}(\tau', y)$ for every $y < y_{sq}$. Since τ_{sq} is in the core, then $1 - q < F(y_{sq}) < q$, and this means at least a measure $1 - q$ of agents are made strictly worse off by deviating. Hence a blocking coalition exists that would prevent the deviation. (The same argument applies if $E = \tau_{sq}$, except that now, $v(E, y) > \hat{W}(\tau', y)$, since the continuation policies cannot be unchanging.)

Now, by the same arguments as in the proof of Proposition 1, we know that the social acceptance set is given by an interval $A(\tau_{sq})$, and that the width of this interval shrinks as

δ increases. Then, for any τ_{sq}, there exists some $\underline{\delta} \geq 0$ s.t. $A(\tau_{sq}, \delta) \subset (\tau_L, \tau_R)$ whenever $\delta > \underline{\delta}$. Hence, for any $\delta > \underline{\delta}$, after the first round of bargaining, the policy will definitely be in the interior of the core, and it will not change any further. Moreover, this policy may potentially be any policy in $A(\tau_{sq}, \delta)$ depending on the income of the proposer. Then, *ex ante*, there will be a distribution over long-run policies, rather than a unique long-run policy, and the support of this distribution will not include τ_R – the policy selected by the 'auction procedure'.

Finally, note that, as $\delta \to 1$, Proposition 2 implies that $A(\tau_{sq}, \delta)$ will converge to a singleton element, and once this policy is implemented it will remain in effect for the remainder of the game. In equilibrium, the status quo does not evolve after this first period. But this is precisely the game that we study. Hence, by Proposition 3, in the limit as $\delta \to 1$, the long-run policy must exactly coincide with the policy we select. $\square$

References

Bails, Dale and Margie A Tieslau. 2000. "The impact of fiscal constitutions on state and local expenditures." *Cato J.* 20:255.

Banks, J. and J. Duggan. 2006. "A general bargaining model of legislative policy-making." *Quarterly Journal of Political Science* 1(1):49–85.

Banks, Jeffrey S. and John Duggan. 2000. "A Bargaining Model of Collective Choice." *The American Political Science Review* 94(1):pp. 73–88.

Baron, David P. 1996. "A dynamic theory of collective goods programs." *American Political Science Review* 90(2):316–330.

Baron, David P. and John A. Ferejohn. 1989. "Bargaining in Legislatures." *The American Political Science Review* 83(4):pp. 1181–1206.

Benabou, Roland. 2000. “Unequal societies: Income distribution and the social contract.” *American Economic Review* 90(1):96–129.

Besley, Timothy and Anne Case. 2003. “Political institutions and policy choices: evidence from the United States.” *Journal of Economic Literature* 41(1):7–73.

Besley, Timothy and Stephen Coate. 1997. “An economic model of representative democracy.” *The Quarterly Journal of Economics* 112(1):85–114.

Black, D. 1948. “On the rationale of group decision-making.” *The Journal of Political Economy* 56(1):23–34.

Boshnakov, Georgi N. 2007. “Some measures for asymmetry of distributions.” *Statistics & probability letters* 77(11):1111–1116.

Bradbury, John Charles and Joseph M Johnson. 2006. “Do supermajority rules limit or enhance majority tyranny? evidence from the US States, 1960–1997.” *Public Choice* 127(3-4):429–441.

Buchanan, James M. and Gordon Tullock. 1962. *The calculus of consent, logical foundations of constitutional democracy [by] James M. Buchanan and Gordon Tullock*. University of Michigan Press Ann Arbor.

Cardona, Daniel and Clara Ponsati. 2011. “Uniqueness of stationary equilibria in bargaining one-dimensional policies under (super) majority rules.” *Games and Economic Behavior* 73(1):65–75.

Cho, Seok-ju and John Duggan. 2009. “Bargaining foundations of the median voter theorem.” *Journal of Economic Theory* 144(2):851–868.

Correia, Isabel, Joao C Neves and Sergio Rebelo. 1995. “Business cycles in a small open economy.” *European Economic Review* 39(6):1089–1113.

Critchley, Frank and MC Jones. 2008. "Asymmetry and Gradient Asymmetry Functions: Density-Based Skewness and Kurtosis." *Scandinavian Journal of Statistics* 35(3):415–437.

Dal Bo, Ernesto. 2006. "Committees with supermajority voting yield commitment with flexibility." *Journal of Public Economics* 90(4-5):573–599.

Dixit, Avinash and John Londregan. 1998. "Ideology, tactics, and efficiency in redistributive politics." *The Quarterly Journal of Economics* 113(2):497–529.

Downs, A. 1957. "An economic theory of political action in a democracy." *The Journal of Political Economy* 65(2):135–150.

Epple, Dennis and Thomas Romer. 1991. "Mobility and redistribution." *Journal of Political economy* 99(4):828–858.

Gans, Joshua S and Michael Smart. 1996. "Majority voting with single-crossing preferences." *Journal of Public Economics* 59(2):219–237.

Gradstein, Mark. 1999. "Optimal taxation and fiscal constitution." *Journal of Public Economics* 72(3):471–485.

Greenwood, Jeremy, Zvi Hercowitz and Gregory W Huffman. 1988. "Investment, capacity utilization, and the real business cycle." *The American Economic Review* pp. 402–417.

Heckelman, Jac C. and Keith L. Dougherty. 2010. "Majority Rule versus Supermajority Rules: Their Effects on Narrow and Broad Taxes." *Public Finance Review* 38(6):738–761.

H. J. Res. 111. 1998.

H. J. Res. 41. 2001.

Knight, Brian G. 2000. "Supermajority voting requirements for tax increases: evidence from the states." *Journal of Public Economics* 76(1):41–67.

Lee, Soomi. 2014. "The Effect of Supermajority Vote Requirements for Tax Increase in California: A Synthetic Control Method Approach." *State Politics & Policy Quarterly* 14(4):414–436.

Meltzer, Allan H and Scott F Richard. 1981. "A rational theory of the size of government." *The Journal of Political Economy* pp. 914–927.

Moene, Karle Ove and Michael Wallerstein. 2001. "Inequality, social insurance, and redistribution." *American Political Science Review* 95(04):859–874.

Parameswaran, Giri and Jacob P Murray. 2018. "Limit Equilibria of Bargaining Games under Super-Majority Rule: A Core Refinement.".

Persson, T. and G. Tabellini. 1999. "The size and scope of government::: Comparative politics with rational politicians." *European Economic Review* 43(4-6):699–735.

Predtetchinski, Arkadi. 2011. "One-dimensional bargaining." *Games and Economic Behavior* 72(2):526–543.

Riboni, Alessandro and Francisco J Ruge-Murcia. 2010. "Monetary policy by committee: consensus, chairman dominance, or simple majority?" *The Quarterly Journal of Economics* 125(1):363–416.

Roberts, Kevin WS. 1977. "Voting over income tax schedules." *Journal of public Economics* 8(3):329–340.

Romer, Thomas. 1975. "Individual welfare, majority voting, and the properties of a linear income tax." *Journal of Public Economics* 4(2):163–185.

Rueben, Kim S. and Megan Randall. 2017. "Supermajority Budget and Tax Rules.".

Wicksell, Knut. 1896. *finanztheoretische untersuchungen: Nebst Darstellung und kritik des steuerwesens schwedens.* G. Fischer.