Sovereign Debt and Moral Hazard: The Role of Contractual Ambiguity and Asymmetric Information

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Abstract

The pari passu clause (PPC) in sovereign debt contracts has long puzzled commentators by its ambiguous phrasing. This paper shows that the confluence of an ambiguous PPC and asymmetric information on a sovereign borrower's (Country) asset value engenders strategic rivalry among Creditors. This rivalry in turn hinders debt restructuring when Country cannot meet its payment obligations. By varying the PPC's ambiguity level, parties can induce an (ex ante) optimal probability of costly default, thereby reducing moral hazard. As information asymmetry decreases, however, a PPC becomes a coarser instrument for configuring Creditors' rivalry. An ambiguous PPC is thus potentially optimal only if both information asymmetry and moral hazard are sufficiently high.

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Babylonian Talmud

1 Introduction

The pari passu clause is one of the most common, most commented upon, and most controversial clauses in sovereign debt contracts. The clause, a version of which is included in the vast majority of unsecured sovereign bonds (Gulati & Scott, 2013; p. 187), provides that the bonds, and/or the sovereign's payment obligations, shall rank equally or pari passu with other unsecured debt of the sovereign. According to some interpretations, the clause prohibits a sovereign borrower from selectively paying one group of creditors and not paying others. Recently, the pari passu clause has drawn considerable attention in the wake of litigation brought by creditors against the Republic of Argentina.¹

Although corporate debt agreements occasionally include similar provisions, the *pari* passu clause has special significance in sovereign bond contracts. Payment obligations of sovereign states are notoriously difficult to enforce: sovereign states cannot be forced into bankruptcy and often hold few non-domestic assets that creditors can attach. Thus, even if a creditor's right to payment is clear, the creditor may face difficulty collecting its debt (Bulow and Rogoff, 1989a).

When a sovereign debtor is unable to pay its creditors, it often tries to exchange its outstanding debt for new debt with less onerous payment terms (such as a reduced principal amount or interest rate). Whereas some creditors fearing an imminent default may agree to reduce their debt ("consenting creditors"), other creditors may hold out by retaining their original bonds ("holdout creditors"). In this restructuring context, a sovereign debtor might threaten to pay nothing to holdout creditors to induce creditors to consent to reduce their debt.²

Arguably, the purpose of the *pari passu* clause is to prohibit such discrimination between consenting and holdout creditors. If the *pari passu* clause prohibited discrimination, sovereign debtors would be barred from paying consenting creditors their renegotiated (reduced) claims without also paying holdout creditors their original (full) claims.³ If

¹Sovereign Debt: Hold-Outs Upheld, The Economist, Nov. 3, 2012.

²Older bonds contain clauses that make it difficult to change payment terms for all creditors by majority consent. More recent sovereign bonds include so-called collective action clauses to permit such changes. Collective action clauses usually require a supermajority of bonds of any issue to bind all holders of that issue. But since it is often possible for holdout creditors to accumulate a blocking minority in one or more bond issues, collective action clauses do not fully resolve the holdout problem. See Weidemaier, 2013; Buchheit et al., 2013.

³The potency of a pari passu clause to prohibit discrimination (when it does) lies in its enforceability. As noted, contractual obligations under a sovereign debt contract cannot be effectively enforced against the sovereign. To pay consenting creditors in violation of a pari passu clause, however, a sovereign debtor often needs the assistance of other entities which are subject to a court's enforcement powers (such as banks and trustees that act as intermediaries in processing the payment). Because a court can order such entities not to assist the sovereign debtor in violating the pari passu clause, and because such entities are likely to follow a court's orders even if the sovereign debtor is not, a pari passu clause is more easily enforceable than the sovereign's underlying payment obligations.

creditors' claims had to be treated equally, therefore, more creditors may hold out in the hope of recovering their entire debt and it may become more difficult for distressed countries to effect a restructuring.

Whether the pari passu clause in fact prohibits such discrimination is subject to substantial controversy. Many commentators have argued, based on historical and policy grounds, that the clause merely prohibits sovereigns from creating a class of creditors that is legally senior (e.g., Buchheit and Pam, 2004; Gulati and Klee, 2001). Others have presented arguments that the clause should also apply to de facto discrimination among creditors (Semkow, 1984; Bratton, 2004). Most commentators, including many who have taken a positive or normative position on the interpretation of the clause, concede that its meaning is not clear. For example, Buchheit and Pam (2004, p. 871) described the clause as possessing a "measure of opacity"; Montelore (2013) described it as "obscure"; and Weidemaier, Scott, and Gulati (2011) remarked that "it is fair to say that no one really knows what the pari passu clause means, something that even eminent practitioners have long acknowledged." Moreover, several commentators puzzled about the persistence of ambiguously-phrased clauses and the failure of contracting parties to clarify them despite ongoing disputes over their scope and meaning (e.g., Gulati and Scott, 2013; Goss, 2014; Buchheit and Martos, 2014).

As if to compound the confusion, the pari passu clause comes in at least three different versions, each entailing a different likelihood of being interpreted "broadly" to prohibit de facto discrimination (Gulati and Scott, 2013, p. 187; Weidemaier, 2013). Some clauses specify only that the bonds must rank equally or pari passu with other debt; other clauses specifically provide that the payment obligations rank equally or pari passu with other debt; and a third version further requires that the bonds be paid in accordance with their equal or pari passu ranking.

Two court rulings that sided with holdout creditors—one by a Court of Appeals in Belgium in 2000 and another by the Second Circuit Court of Appeals in New York in 2012—dispelled the notion, professed by some commentators, that the clause is universally understood to permit *de facto* discrimination. Yet because they involved idiosyncratic circumstances, these rulings provide only limited guidance for the resolution of future disputes (see, e.g., Gulati and Klee, 2001; Gulati and Klee, 2001, Alfaro, 2015; Ku, 2014).⁴

In this paper, we present a three-period model of sovereign debt which sheds light on the persistent ambiguity of pari passu clauses. We consider a sovereign (Country) and a continuum of Creditors that negotiate in the first period a loan amount and a probability that a pari passu clause would be interpreted to prohibit de facto discrimination. In the second period, Country must choose a policy from a set of policies, where each policy entails a different probability of failure and a riskier policy yields a higher payout upon success. In the third period, the payout of Country's chosen policy is realized. If the policy succeeds, its payout is sufficient to pay off Creditor's debt. If the policy fails,

⁴The 2000 decision was preliminary, rendered on an ex parte motion, and involved a foreign court interpreting New York Law. The 2012 decision was based, in part, on equitable considerations and on Argentina's enactment of the Lock Law which may have violated even a narrow interpretation of the pari passu clause. The Second Circuit Court of Appeals specifically noted that it did not decide whether "any non-payment that is coupled with payment on other debt" would breach the pari passu clause. NML Capital. 246 F.3rd at 264, fn. 16.

by contrast, the policy's realized payout falls short of Country's debt. The realized payout upon failure is private information to Country; Creditors only know the payout distribution. Restructuring negotiations under asymmetric information then follow in the shadow of the probability that the *pari passu* clause would be interpreted broadly.

This setup is intended to capture the effect of restructuring negotiations on Country's choice of policy and thereby on the optimal design of a loan agreement. In particular, the first-period loan negotiations are conducted in the shadow of Country's anticipated choice of policy in the second period, which is shaped by the expected outcome of the restructuring negotiations in the final period. Thus, in the presence of weak contractual enforcement of Country's payment obligations, the design of restructuring negotiations through a pari passu clause is a key instrument for curbing moral hazard.

To model the equilibrium outcome of the restructuring negotiations following a policy failure, we assume that a sufficiently large fraction of Creditors (consenting creditors) can make Country a take-it-or-leave-it restructuring demand requiring that Country pays them a reduced debt amount. We call the fraction of consenting creditors out of all creditors the "participation rate." The complementary fraction of Creditors (holdout creditors) retains their original debt. Country cannot pay Creditors more than its realized policy payout and, if the pari passu clause is interpreted broadly, may not pay consenting creditors their restructured debt without paying holdout creditors their debt in full.

If Country fails to pay off Creditors according to the realized interpretation of the pari passu clause, Country defaults and incurs default costs and Creditors receive nothing. More specifically, Country can avoid incurring default costs iff (i) the pari passu clause is interpreted narrowly and Country pays consenting creditors their demand or (ii) the pari passu clause is interpreted broadly and Country pays both consenting creditors their demand and holdout creditors their claim in full.

We solve the game using the notion of restructuring equilibrium. For a given a probability that the pari passu clause would be interpreted broadly and a given level of asymmetric information between Country and Creditors, a restructuring equilibrium consists of a restructuring demand and a participation rate that satisfy two conditions: (i) consenting creditors' restructuring demand maximizes their recovery given the participation rate; and (ii) no creditor has incentives to deviate from his equilibrium strategy (consent or hold out) given the participation rate and consenting creditors' restructuring demand.

Our model brings to the fore the significance of uncertainty embedded in the pari passu clause for the outcome of restructuring negotiations. The more likely a pari passu clause is to be interpreted broadly, the lower the equilibrium participation rate and the higher the associated probability of Country's default. By varying the probability that a pari passu clause would be interpreted broadly, Country and Creditors can therefore affect the ex ante probability of costly default in the event of a policy failure. A higher probability of costly default given a policy failure in turn induces Country to choose a safer policy. A pari passu clause improves social welfare if the increase in the policy payout as a result of a safer policy outweighs Country's expected default costs. Moreover, a pari passu clause that increases social welfare also allows Country to borrow more from Creditors.

To see how a pari passu clause shapes Creditors' incentives during restructuring negotiations and thereby affects the probability of Country's default, observe that a stronger

clause (i.e., one that is more likely to be interpreted broadly) produces stronger incentives to hold out. As more creditors hold out and the participation rate decreases, consenting creditors (weakly) lower their restructuring demand. In equilibrium, the decrease in consenting creditors' restructuring demand is lower than the corresponding increase in holdout creditors' claim. A stronger clause therefore results in a higher aggregate claim by consenting and holdout creditors given a broad interpretation of the clause. Thus, as the pari passu clause becomes stronger, Country's probability of default given a policy failure increases both because a broad interpretation of the clause is more likely and because Country is less likely to meet its payment obligations to consenting and holdout creditors when the clause is interpreted broadly.

Asymmetric information is essential for the pari passu clause to produce this dynamic. If the degree of information asymmetry is sufficiently low, Consenting Creditors' optimal restructuring demand is one where Country either never defaults irrespective of the interpretation of the clause or always defaults given a broad interpretation of the clause (depending on the participation rate). An equilibrium in which Country never default is of no avail because it doesn't constrain Country's moral hazard. On the other hand, a demand that induces certain default cannot be sustained in equilibrium for it would leave Holdout Creditors with no recovery. It consequently takes a sufficiently high degree of information asymmetry for the pari passu clause to give rise to equilibria involving an interior probability of Country's default.

Our analysis has implications both for the optimal strength of the *pari passu* clause and for the effect of asymmetric information on the potential benefit of the clause. First, a *pari passu* clause with a positive probability of being interpreted broadly can induce Country to pursue a more efficient policy at the cost of a deadweight loss from default should the policy fail and restructuring negotiations break down. The optimal strength of a *pari passu* clause accordingly depends on the benefits from reducing Country's moral hazard versus the associated default costs.

Second, a decrease in the level of asymmetric information during restructuring negotiations has two opposite effects on the potential benefit of a pari passu clause. On the one hand, Creditors' recovery given a policy failure increases as information asymmetry decreases, making Country internalize a higher portion of the cost of a policy failure. As asymmetric information decreases, therefore, there is less of a need to curb Country incentives through a pari passu clause. On the other hand, a pari passu clause might be more effective in resolving moral hazard the higher the level of information asymmetry about Country's ability to repay Creditors at the renegotiation stage.

Prior Literature:

There is an extensive literature on sovereign debt. One branch of this literature revolves around mechanisms for ensuring repayment of sovereign debt that substitute for direct legal enforcement. Eaton and Gersovitz (1981) present a model of sovereign debt with post-default penalties that provide sovereigns with an incentive to pay even when the payment obligations cannot be directly enforced. Subsequent papers have proposed different theories on the sources of default penalties (see Eaton and Gersovits, 1981; Grossman and Van Huyck, 1988; Eaton, 1996; Cole and Kehoe, 1995; Bulow and Rogoff (1989b); Fernandez and Rosenthal, 1990; Panizza et al., 2009, Chabot and Santarosa, 2017).

Bulow and Rogoff (1989b) incorporate the possibility of renegotiations (without holdouts) into a model of sovereign lending. Fernandez and Fernandez (2007) consider the case in which debt can be renegotiated and countries may engage in strategic default. Atkeson (1991), Boot and Kanatas (1995), and Schwartz and Zurita (1992) present models of sovereign borrowing that include debtor moral hazard.⁵

The holdout problem in the sovereign debt context and the associated difficulties in renegotiating debt have been noted, among others, by Buchheit and Gulati (2000) and by Schwarcz (2000). Bolton and Jeanne (2007) present a model where competition among lenders can result in sovereign debt that is excessively difficult to renegotiate. Ghosal and Thampanishvon (2013) argue that strengthening collective action away from unanimity can reduce the holdout problem but aggravate moral hazard.

Most of the literature on pari passu clauses has centered on the interpretation of the clause (Bratton, 2004; Buchheit and Pam, 2004; Gulati and Klee, 2001; Semkow, 1984; Wood, 1995), on discussions of the case law (Alfaro, 2015; Gulati and Klee, 2001; Ku, 2014; Montelore, 2013; Tsang, 2015; Weidemaier, 2013), and on an empirical investigation of the prevalence of various versions of the clause (Gulati and Scott, 2013; Weidemaier, Scott and Gulati, 2011). Prior work has noted that a broad reading of the pari passu clause that prohibits de facto discrimination can inhibit consensual restructurings (Gulati and Klee, 2001) and may reduce moral hazard (Bratton, 2004), but hasn't provided a formal account of these effects. Our paper explicates the strategic consequences of pari passu clauses and thereby explains their prolonged and seemingly-deliberate ambiguous phrasing.

The paper is organized as follows. Section 2 sets up the model and introduces sovereigns' moral hazard. Section 3 presents and classify restructuring equilibria. Section 4 derives the equilibrium outcomes and Section 5 characterizes the optimal strength of a pari passu clause. Section 6 concludes.

2 Model

2.1 Setup

Consider a three-period game between a country (Country) and a unit mass of identical competitive creditors (Creditors). Both Country and Creditors are risk-neutral and the market interest rate is zero.

In **period 0**, Creditors lend Country $k \leq 1$ in exchange of Country's commitment to repay Creditors 1 in period 2. As will be clear soon, Country cannot commit to pay back more than 1. The loan amount, k, is determined such that Creditors break even in expectation. The loan includes a pari passu clause (PPC) along with a probability $w \in [0, 1]$ that the clause would be interpreted "broadly" to prohibit de facto discrimination. If the PPC

⁵In addition to the classic moral hazard problem relating to a debtor's choice of policy that affects the probability of distress, other related forms of moral hazard include a debtor's decision to seek a restructuring and creditors' decision to lend given the possibility of bailout by international institutions. Buchheit et al., 2013.

is interpreted broadly, then in period 2 Country may not pay some Creditors but not others.

In **period 1**, Country must choose a policy $p \in (0, 1]$. We think of a policy as any fiscal or monetary measure designed to produce a long-term fiscal payout such as tax, pension or currency reform. A policy p succeeds with probability p and fails with the complementary probability. If a policy p succeeds it yields a payout of r(p), where r(1) > 1, r'(p) < 0, and $r''(p) \le 0$. Thus, a successful policy yields a payout greater than (or equal to) 1, where the policy payout decreases at a decreasing rate with the policy's probability of success. If the policy fails, its payout (independent of p) is a uniform random variable with mean $\mu = \frac{3}{4}$ and Country's specific support $[\underline{s}, \overline{s}]$, where $s \equiv \overline{s} - \underline{s} \in (0, \frac{1}{2}]$.

In **period 2**, the observable outcome - success or failure - of Country's chosen policy is realized. The realized value of the policy payout, however, is private information to Country. If Country pays Creditors 1, the game ends. If Country does not pay Creditors, a restructuring phase follows, in which Creditors and Country negotiate a reduction of Country's debt. Country cannot pay Creditors more than its policy payout. If Country and Creditors fail to reach a restructuring agreement, Country incurs default costs of 1 and Creditors obtain 0.⁶

This setup is designed to capture two essential features of sovereign debt. The first is that Country can affect the probability of default through its choice of fiscal and monetary policies. The probability p in our model accordingly reflects a risk-return trade-off: a lower p represents a policy that is more likely to fail but that yields a higher payout if it succeeds. The second essential feature of sovereign debt that our model captures is that if Country doesn't meet its payment obligations, Creditors have incomplete information about the maximum amount Country is able or willing to pay to satisfy Creditors' debt, an amount which depends on Country's political and economic constraints. This information asymmetry can be interpreted as a measure of economic and political transparency. The more transparent Country is, the less uncertainty Creditors face about the maximum amount Country is able or willing to pay to avoid default costs.

2.2 The restructuring phase

The restructuring phase proceeds in two stages. In stage 1, a mass $\alpha \geq a_{\min}$ of Creditors, called Consenting Creditors (CC), may make Country a take-it-or-leave-it demand of $d \in [0, \alpha]$, where α represents Creditors' participation rate (i.e., the fraction of CC out of all Creditors) and a_{\min} stands for Creditors' participation constraint. The participation constraint is the minimum fraction of Creditors that must consent and be paid according to a restructuring plan for Country to avoid the (political and economic) costs from defaulting on its payments to Creditors. If Creditors do not make a demand, Country has to pay Creditors in full to avoid default costs. If Creditors make a demand, the non-consenting creditors, called Holdout Creditors (HC), retain their original debt with an aggregate claim of $1 - \alpha$.

In stage 2, the interpretation of the PPC is determined. If the PPC is interpreted

⁶We assume that Country would rather pay Creditors if it is indifferent between incurring default costs and paying Creditors.

narrowly to permit de facto discrimination, Country may avoid default costs by paying d to CC, without paying anything to HC. If the PPC is interpreted broadly, by contrast, Country may not discriminate between CC and HC; to avoid default costs, Country must pay d to CC and $1 - \alpha$ to HC.

If the policy succeeds, the realized policy payout is greater than Country's debt of 1. Because Country's default costs are equal to its debt, Country will pay off Creditors in full (whether or not Creditors make a demand to Country). If the policy fails, by contrast, Country will default if (i) its realized policy payout is lower than d irrespective of the realized interpretation of the PPC; or (ii) its realized policy payout is lower than $d+1-\alpha$ and the PPC is interpreted broadly.

Our modeling of the restructuring negotiations is intended to capture the fact that Country can avoid (or reduce) the penalty associated with a sovereign default if it reaches and implements a restructuring plan approved by a significant fraction of creditors (i.e., greater than a_{\min}). A broad interpretation of the PPC, however, prevents the implementation of a restructuring unless hold-out creditors are paid as well.

2.3 Moral Hazard

As a benchmark, consider the outcome of the restructuring negotiations when Country's policy fails in the absence of a PPC (w=0). Clearly, all Creditors would rather participate than hold out for holding out yields no recovery. Because Country always accepts a demand $d < \underline{s}$ and always rejects a demand $d \geq \overline{s}$, Creditors choose a demand $d \in [\underline{s}, \overline{s})$ to maximize

$$d \times \frac{\overline{s} - d}{s},\tag{1}$$

where $\frac{\bar{s}-d}{s} \in (0,1]$ is the probability that Country accepts Creditors' demand. Because Creditors' expected recovery decreases with d for $d \in [\underline{s}, \overline{s})$, Creditors' optimal demand is \underline{s} . The information asymmetry between Country and Creditors thus causes Creditors to make a cautious restructuring demand.

The following proposition considers the effect of the outcome of restructuring negotiations in period 2 on Country's policy choice in period 1.

Proposition 1 (moral hazard) Country chooses in period 1 a riskier policy than the socially optimal one, where Country's policy becomes safer as information asymmetry during restructuring negotiations decreases.

Let $R(p) = pr(p) + (1-p)\mu$ denote policy p's expected payout. The socially optimal (interior) policy satisfies R'(p) = 0, implying that the marginal net benefit of a policy that is safer than the socially optimal one is nil.⁸

⁷The derivative of the objective function in (1) with respect to d is $\frac{\overline{s}-2d}{s}$, which is negative for $d \in [\underline{s}, \overline{s})$ (because $\underline{s} \geq \frac{\overline{s}}{2}$).

⁸The second order condition for a maximum, R''(p) < 0, is satisfied because r'(p) < 0 and $r''(p) \le 0$.

Now, Country's interim payoff is $R(p) - [p + (1-p)\underline{s}]$, where $p + (1-p)\underline{s}$ is Country's expected repayment to Creditors (1 if the policy succeeds and \underline{s} if it fails). Differentiating Country's payoff with respect to p, equating to zero, and rearranging gives

$$R'(p) = 1 - \underline{s}. (2)$$

Country thus equates the marginal net benefit of a safer policy (right-hand side) and the corresponding marginal cost (left-hand side). Country's marginal cost of a safer policy is equal to the difference between Country's payments to Creditors upon success and failure (1 versus \underline{s}). Country thus trades off in period 1 a lower policy payout for an even lower expected repayment to Creditors. Country accordingly has stronger incentives to choose a riskier policy if Creditor's recovery given a policy failure is lower. By contrast, as information asymmetry decreases and Creditors' recovery increasers, Country internalizes a higher fraction of the cost of a policy failure and therefore chooses a safer policy.

3 Restructuring Equilibria

In this section, we turn to the case in which the probability the PPC is interpreted broadly is strictly positive. We first define and classify restructuring equilibria and then flesh out the equilibrium restructuring demand and participation rate.

Definition 1 (restructuring equilibrium) A restructuring equilibrium is a pair of a participation rate (α) and a restructuring demand (d) such that:

- (i) Consenting Creditors' demand maximizes their recovery given the participation rate;
- (ii) given Consenting Creditors' demand and the participation rate, no creditor can profitably switch position (consent or hold out).

Two types of equilibria emerge in our setup: a full-participation equilibrium, in which all Creditors participate (a=1) and make a restructuring demand of \underline{s} ; and a partial-participation equilibrium, in which some Creditors participate and others hold out (a < 1) and CC make a restructuring demand equal to or lower than \underline{s} . A partial participation equilibrium is unconstrained if Creditors' participation constraint is not binding and is constrained if this constraint is binding. To sustain a full-participation equilibrium, no mass of Creditors could profitably deviate to holding out; to support an unconstrained partial-participation equilibrium, CC and HC must obtain the same recovery rate; and to support a constrained equilibrium, the participation rate must equal \hat{a} (the participation constraint) and CC's recovery rate may not exceed HC's recovery rate.

⁹We assume that Country benefits from a higher loan amount and cannot commit to a borrowing cap. As a result, Country cannot reduce moral hazard by promising to pay back Creditors less than 1 (and borrowing less).

 $^{^{10}}$ The loss of social welfare is captured by the area under the marginal policy payout curve (R'(p)) between the privately and socially optimal policies.

In the rest of this subsection, we formally represent the two-prong equilibrium condition. To this end, consider first the probability that Country does not default given that the PPC is interpreted broadly as a function of CC's demand, the participation rate, and the degree of information asymmetry between Country and Creditors:

$$P_{nd}(a,d,s) = \begin{cases} \frac{1}{\overline{s} - [(1-\alpha) + d]} \in (0,1) & \text{if } d + 1 - \alpha \in (\underline{s}, \overline{s}] \\ 0 & \in (\overline{s}, 1] \end{cases}$$
 (3)

If the sum of Creditors' demands, $d+1-\alpha$, is less than \underline{s} (the lower bound of a failed policy payout), Country never defaults (first line). If the sum of Creditors' demands is greater than \overline{s} (the upper bound of a failed policy payout), Country defaults with certainty if the PPC is interpreted broadly (third line). Finally, if the sum of Creditors' demands is strictly between \underline{s} and \overline{s} , Country defaults with a probability strictly between 0 and 1 if the PPC is interpreted broadly, depending on Country's realized policy payout (middle line).¹¹

Now, CC cannot demand more than their claim of α . Moreover, CC's optimal demand for any w > 0 is no greater than \underline{s} because \underline{s} is CC's optimal demand for w = 0 and because CC's optimal demand (weakly) decreases with w. CC's optimal demand as a function of α is therefore:

$$\max_{d \in (0,\min\{\alpha,\underline{s}\}]} d \times [(1-w) + wP_{nd}(a,d,s)], \tag{4}$$

The first term in the square brackets, 1-w, is the probability that the PPC is interpreted narrowly, in which case Country always meets CC's demand. The second term, $wP_{nd}(a,d,s)$, is the joint probability that the PPC is interpreted broadly and that Country can meet both CC's and HC's demands.

The next definition sorts CC's demands.

Definition 2 (classification of restructuring demand) For a given participation rate (α) and a given level of asymmetric information between Country and Creditors (s), CC's demand is (i) a "default demand" if $P_{nd}(a,d,s)=0$, (ii) a "no-default demand" if $P_{nd}(a,d,s)=1$, and (iii) an "interior demand" if $P_{nd}(a,d,s)\in(0,1)$.

CC's demand is a default demand if Country always defaults given a broad interpretation of the PPC; a no-default demand if Country never defaults irrespective of the interpretation of the PPC; and an interior demand if Country's probability of default given a broad interpretation of the PPC is strictly between 0 and 1. CC's optimal demand (although not necessarily an equilibrium demand) is either a default demand of $\min\{\alpha, \underline{s}\}$, a no-default demand of $\underline{s} - (1 - \alpha)$, or an interior demand that is equal to or lower than \underline{s} .

¹¹The derivation of P_{nd} for this case is in the Appendix.

An optimal interior demand of $d < \underline{s}$ equates CC's marginal benefit of raising that demand and the corresponding marginal cost:

$$1 - w + wP_{nd}(a, d, s) = -dw \frac{\partial P_{nd}(a, d, s)}{\partial d}.$$
 (5)

CC's marginal benefit of making a higher demand (left-hand side) is equal to the probability that the PPC is interpreted narrowly (in which case Country always accepts CC's demand) plus the probability that the PPC is interpreted broadly and Country doesn't default. The corresponding marginal cost (right-hand side) is equal to the demand times the marginal decrease in the probability that Country doesn't defaults as a result of a higher demand, i.e., the probability that the PPC is interpreted broadly times the marginal decrease in the probability that Country can meet both CC's and HC's demands given a broad interpretation.¹²

Turning to the second equilibrium prong, no creditor would have an incentive to switch position under an unconstrained partial-participation equilibrium iff

$$\frac{d}{\alpha} \times \left[(1 - w) + w P_{nd}(a, d, s) \right] = w P_{nd}(a, d, s). \tag{6}$$

The left-hand side is CC's recovery rate, which is equal to CC's expected recovery divided by the participation rate. The right-hand side is HC's recovery rate, which is equal to the joint probability that the PPC is interpreted broadly and that Country doesn't default (recall that HC's demand consists of their entire claim). In any equal recovery (unconstrained) equilibrium, CC's demand and the participation rate must satisfy both (4) and (6). In any constrained equilibrium, CC's demand and the participation rate must satisfy (4) and (6) as an inequality.

4 Equilibrium Outcomes

We begin by considering a full-participation equilibrium in which all Creditors participate in a restructuring plan. In this equilibrium, given the distribution of Country's failed policy payout, Creditors make a riskless restructuring demand equal to the lower bound of Country's policy payout (\underline{s}) , which Country always accepts. Country thus never defaults and its payment to Creditors upon a policy failure is \underline{s} . The next proposition presents a necessary and sufficient condition for the existence of a full-participation equilibrium as a function of w and s.

Proposition 2 (Full-Participation Equilibrium) There exists a payoff dominant full-participation equilibrium iff $w \leq \underline{s}$ (Area A in Figure 1).

Proof. See the Appendix.

¹²An interior demand of \underline{s} is optimal iff the marginal benefit of raising any lower interior demand is less than the corresponding marginal cost; a no-default demand of $\underline{s} - (1 - \alpha)$ is optimal iff the marginal benefit of raising this demand is less than or equal to the corresponding marginal cost.

The intuition for the existence of a full-participation equilibrium for $w \leq \underline{s}$ is as follows. If all Creditors participate, their optimal restructuring demand is \underline{s} . Because Country always accepts Creditors' demand, Creditors' recovery rate is \underline{s} . By contrast, holding out yields a recovery rate no greater than w, the probability that the PPC will be accorded a broad interpretation. Because holding out yields a lower recovery rate than participating, no creditor has incentives to deviate to holding out.

A full-participating equilibrium is payoff dominant because under any partial-participation equilibrium HC's recovery rate is capped at w and CC's recovery rate cannot be higher than HC's recovery rate. Both CC and HC thus obtain a higher recovery rate under a full-participation equilibrium (where $w \leq \underline{s}$) than under a partial-participation equilibrium.¹³

There does not exist a full-participation equilibrium for $w > \underline{s}$ because, given that all other Creditors participate and make a restructuring demand of \underline{s} , an infinitesimal mass of consenting creditors can profitably deviate to holding out. In the limit, as the deviating mass approaches zero, Country's probability of not defaulting conditional on a broad interpretation of the PPC tends to 1. The deviating creditors' recovery rate would consequently be arbitrarily close to w and thus greater than their (putative) equilibrium recovery rate of \underline{s} . These holdout incentives in turn disrupt a full-participation equilibrium.

We now turn to unconstrained partial-participation equilibria in which some creditors participate in a restructuring plan while other creditors hold out. In any such equilibria, CC and HC obtain equal recovery rates; we shall accordingly call these equilibria "equal recovery equilibria." We shall assume for now that the participation constraint is not binding ($\hat{\alpha} = 0$) and will comment later on the implications of relaxing this assumption. The next proposition presents conditions for the existence of equal recovery equilibria as well as a key property of these equilibria.

Proposition 3 (Equal Recovery Equilibria) Let $\overline{w}(s) \equiv \max\{\underline{s}, \frac{s}{1-\underline{s}}\}$ (see Figure 1) and assume that the participation constraint is not binding.

- (a) For $w \in (\underline{s}, \overline{w}(s)]$ (Area B in Figure 1) there exists a unique partial participation equilibrium in which (i) Consenting Creditors make an interior demand and (ii) Consenting and Holdout Creditors obtain equal recovery rates.
- (b) For $w \in (\overline{w}(s), 1]$ (Area C in Figure 1) there does not exist an equal recovery equilibrium.

Proof. See the Appendix.

We begin by explaining why for $w > \underline{s}$ any candidate equal recovery equilibrium must involve an *interior* restructuring demand; i.e., a demand under which Country's probability of default given a broad interpretation of the PPC is strictly between 0 and 1.

 $^{^{13}}$ As we show in Proposition 2A in the Appendix, for s sufficiently small and $w < \underline{s}$ sufficiently high, there exists a partial-participation equilibrium where CC make a no-default demand and Creditors' equilibrium recovery is w.

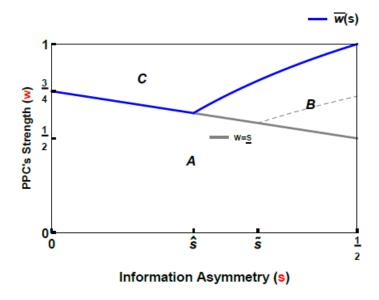


Figure 1: Equilibrium outcome regions

A default demand cannot be part of an equilibrium for any w. This is because under any putative equilibrium involving a default demand, CC's recovery rate is strictly positive but HC's recovery rate is nil. Holdout creditors can consequently profitably deviate to consenting, thereby upsetting the putative equilibrium.

On the other hand, a no-default demand cannot be part of an equilibrium for any $w > \underline{s}$. To see why, observe that HC's recovery rate under a no-default demand is w (the probability that the PPC would be interpreted broadly). If a no-default demand were part of an equal recovery equilibrium, CC's recovery rate as well as Creditors' recovery would have to be w too, which is greater than Creditors' full-participation recovery of \underline{s} for any $w > \underline{s}$. But under a no-default demand Creditors' recovery is strictly less than \underline{s} given a narrow interpretation of the PPC and \underline{s} given a broad interpretation (because the sum of HC's and CC's demands is \underline{s}), implying that Creditors' recovery under a no-default demand must be strictly less than \underline{s} . Thus, if CC made a no-default demand for any $w > \underline{s}$, HC would obtain a higher recovery rate than CC, providing the latter incentives to switch to holding out. An equilibrium for $w > \underline{s}$ therefore exists for such values of w and s for which CC's optimal restructuring demand is neither a default demand nor a no-default demand but rather an interior demand.¹⁴

To understand why there does not exist an equilibrium for $w > \overline{w}(s)$, observe that an equal recovery equilibrium exists only if CC's optimal demand given α is an interior demand rather than a default demand (of either \underline{s} or α). Now, CC's recovery rate conditional on a broad (narrow) interpretation of the PPC is higher (lower) under an interior demand than under a default demand. Because under equal recovery equilibria both CC's equilibrium recovery rate and the equilibrium participation rate decrease with w, there exists a threshold $\overline{w}(s)$ above which CC would rather maximize their recovery rate

 $^{^{14}}$ More generally, a full-participation equilibrium maximizes Creditors' recovery for any w. Under a partial-participation equilibrium, Creditors maximize their expected recovery given a broad interpretation of the PPC by having CC make a no-default demand. Because a no-default demand is strictly less than s, it fails to maximize Creditors' recovery given a narrow interpretation of the PPC.

conditional on a narrow interpretation by making a default demand and forgo recovery altogether given a broad interpretation.¹⁵ If an equilibrium does not exist, Country defaults with certainty.

More specifically, if the level of information asymmetry between Country and Creditors is low $(s \leq \hat{s})$, the incentives to hold out and consent never equilibrate for any $w > \underline{s}$. For high participation rates CC's optimal demand is a no-default demand, which gives consenting creditors incentives to hold out. The participation rate consequently drops until CC's optimal demand becomes a default demand. A default demand, conversely, gives holdout creditors incentives to consent and thereby raise the participation rate. The interchanging incentives to hold out (for high participation rates) or consent (for low participation rates) thus frustrate any potential equilibrium.

Finally, Creditors' participation constraint would suppress an equal recovery equilibrium if it were greater than the equilibrium participation rate $(\widehat{\alpha} > \alpha^*)$. In turn, the participation constraint can generate new equilibria in which the participation rate is equal to the participation constraint. Such equilibria exist if and only if, given CC's optimal demand, CC's recovery rate does not exceed HC's recovery rate.

The next proposition considers the effect of an increase in w on the equilibrium quantities.

Proposition 4 (Effects of a stronger PPC) In an equal recovery equilibrium, both the participation rate (a^*) and Consenting Creditors' demand (d^*) (weakly) decrease with w, but the sum of Creditors' demands $(d^* + (1 - a^*))$ increases with w. Consequently, given that Country's policy fails, (i) Country's probability of default increases with w, and (ii) the sum of Country's payments to Creditors and default costs increases with (and is equal to) w.

The effect of an increase in w on the equilibrium participation rate and restructuring demand (in Area B) follows from the corresponding effect on CC's and HC's recovery rates. Other things being equal, an increase in w increases HC's recovery rate and decreases CC's recovery rate. More creditors consequently hold out, thereby lowering the participation rate. The drop in the participation rate in turn (weakly) lowers CC's optimal demand, which decreases the participation rate further and so on. The resulting equilibrium accordingly involves a lower participation rate and a (weakly) lower CC's demand. Furthermore, as we explain below, the sum of Creditors' equilibrium demands, $d^* + (1 - a^*)$, increases with w and therefore Country's equilibrium probability of default increases with w as well.

To see why $d^* + (1 - a^*)$ increases with w, suppose that this sum remained unchanged or decreased as w increases. Since an increase in w lowers the participation rate and (weakly) lowers CC's optimal demand, the decline in CC's optimal demand would have to be equal to or greater than the corresponding drop in the participation rate for the sum of Creditors' demands to remain unchanged or to decrease.

¹⁵As we show in the proof of Proposition 3, if the equilibrium restructuring demand is greater than half the equilibrium participation rate $(d^* \ge \alpha^*/2)$, CC's optimal demand is an interior demand. The threshold $\overline{w}(s)$ is accordingly reached when $d^* = \alpha^*/2$. As \underline{s} increases (and thus s decreases), the threshold $\overline{w}(s)$ decreases, reaching \underline{s} at $s = \hat{s}$ (≈ 0.23).

If CC's optimal demand declined by as much as or by more than the drop in the participation rate, then Country's probability of no-default conditional on a broad interpretation of the PPC would either not change or increase with w. HC's recovery rate, $wP_{nd}(a,d,s)$, would consequently increase with w. By contrast, CC's recovery rate conditional on a narrow interpretation (d/a) would decrease with w and their recovery rate conditional on a broad interpretation $(d/a \times P_{nd}(a,d,s))$ would either decrease or increase with w (depending on whether the decrease in d/a is less than or greater than the increase in $P_{nd}(a,d,s)$). But CC' recovery rate conditional on a broad interpretation (and a fortiori CC's overall recovery rate) could not increase by more than HC's recovery rate as the driving force for CC's higher recovery rate—a higher probability of Country not defaulting—affects HC's recovery rate more than CC's recovery rate. HC's recovery rate would consequently be higher than CC's recovery rate, in violation of the equilibrium condition of equal recovery rates.

It follows that for HC and CC to obtain equal recovery rates as w increases, the decline in CC's optimal demand must be smaller than the drop in the participation rate. But if CC's optimal demand declines by less than the drop in the participation rate, the sum of Creditors' equilibrium demands $(d^* + (1 - a^*))$ must increase with w.

The sum of Country's payments to Creditors and default costs increases with w because Country's probability of default given a policy failure increases with w and because a default is more costly than a restructuring (under either a broad or narrow interpretation). More specifically, observe that under an equal recovery equilibrium Creditors' expected recovery, and therefore Country's expected payment to Creditors upon a policy failure, is $wP_{nd}(a,d,s)$. Because Country's expected default costs are $w(1-P_{nd}(a,d,s))$, Country's total expected costs of a policy failure sum up to w.

5 Optimal Strength of a Pari Passu Clause

In this section, we present the trade-off associated with a higher probability of Country's default given a policy failure as a result of a stronger PPC (higher w) and consider the effect of asymmetric information on the potential benefit of a PPC.

The potential value of a stronger PPC lies in inducing a higher probability of default conditional on a policy failure on the equilibrium path or, if in equilibrium Country chooses a sure-to-succeed policy, off the equilibrium path. The higher prospect of default in turn increases Country's total payments upon a policy failure and thereby curbs Country's incentives to choose too risky a policy.

If Country were certain to default upon a policy failure, it would pay 1 if the policy either succeeds or fails (to Creditors in the former case and in default costs in the latter). Country would consequently fully internalize the risk of failure and choose the socially optimal policy. But unless the socially optimal policy is a sure-to-succeed one, making Country fully internalize the risk of failure is costly because Country incurs wasteful default costs all too often and Creditors obtain nothing if the policy fails.

To set up the welfare maximization problem associated with an optimal PPC, let CR(w) and DC(w) denote, respectively, Creditors' equilibrium recovery and Country's equilibrium

rium default costs given a policy failure as a function of w. Country's equilibrium total payments given a policy failure are then $TP(w) \equiv CR(w) + DC(w)$.

Because Country pays Creditors 1 if the policy succeeds (with probability p) and its total payments given a policy failure are TP(w) (with probability 1-p), Country's interim payoff is R(p) - [p + (1-p)TP(w)], where R(p) is the policy payout. Differentiating with respect to p, equating to zero, and rearranging, Country's privately optimal (interior) policy in period 1 satisfies

$$R'(p) = 1 - TP(w). \tag{7}$$

Country thus equates the policy's marginal payout (left-hand side) with the marginal cost of a safer policy, i.e., the difference between Country's payments when the policy succeeds and fails (right-hand side). Let p(w) denote Country's privately optimal policy as a function of w; i.e., the value of p that satisfies (7) as an equality.

We can now express social (and Country) welfare as a function of w as

$$R(p(w)) - (1 - p(w))DC(w).$$
 (8)

The first term is the policy's expected payout given that Country chooses p(w). The second term is Country's expected default costs as a function of w and p(w). Let w^* denote the value of w that maximizes (8) and $p^* = p(w^*)$ Country's privately optimal policy given w^* . The next proposition follows immediately.

Proposition 5 An optimal pari passu clause with $w = w^*$ under which Country chooses a policy $p^*(w^*)$ minimizes the sum of (i) Country's moral hazard costs of $R(\widehat{p}) - R(p^*(w^*))$, where \widehat{p} is the first-best policy, and (ii) Country's expected default costs of $(1-p^*(w^*)DC(w^*))$.

An optimal PPC trades off (i) a lower payout difference between the first-best policy and Country's policy (moral hazard costs) and (ii) a lower probability that Country's policy fails and precipitates default against (iii) a higher probability that the clause would be interpreted broadly given a policy failure and (iv) a higher probability of Country's default given a broad interpretation of the clause (under an equal recovery equilibrium). Moral hazard costs decrease as Country's total payments given a policy failure increase (up to 1). Consequently, any w that maximizes Country's total payments upon a policy failure for a given magnitude of default costs is potentially optimal.

Figure 2A shows Country's total payments given a policy failure and the associated default costs as a function of w for $s = \frac{1}{2}$ and $a_{\min} = \frac{1}{2}$. For $w \in (0, \frac{1}{2}]$, a pari passu clause produces a full-participation equilibrium where Country's total payments are $\frac{1}{2}$ and default costs are 0. For $w \in (\frac{1}{2}, w']$, where $w' \approx 0.809$, a pari passu clause produces an equal recovery (unconstrained) equilibrium where Country's total payments are w and default costs increase with w. For $w \in (w', 1]$, a pari passu clause produces a constrained equilibrium, where Country's total payments increase with w but Country's expected default costs remain fixed. In this range of PPC strengths, therefore, the only

potentially-optimal w is $1.^{16}$

A higher level of information asymmetry may render a $pari\ passu$ clause more effective in modulating moral hazard. A $pari\ passu$ clause is more effective as a tool to generate higher levels of total payments (and hence lower moral hazard costs) at the cost of higher default costs when it generates equal recovery equilibria. But for lower levels of information asymmetry, the range of values of w that generate such equilibria shrinks, and ultimately disappears, As a result, for low levels of information asymmetry, it becomes either impossible or very costly (in terms of associated default costs) to design the clause to generate high levels of total payments.

To show the effect of asymmetric information on the potential benefit of a PPC, we compare in Figures 2B and 2C. Country's default costs and total payments given a policy failure for s=1/2 and s=2/5 (again assuming $a_{\min}=\frac{1}{2}$). Figure 2B shows Country's default costs and total payments as a function of w; Figure 2C shows a parametric plot of Country's default costs and total payments as w goes from \underline{s} up to the value of w for which the (equal recovery) equilibrium participation rate is 1/2. As these graphs show, for low default costs Country's total payments are higher under s=2/5, whereas for high default costs Country's total payments are higher under s=1/2. Thus, a less transparent Country might be better able to constrain moral hazard through a PPC than a more transparent one.

6 Conclusion

This paper presented a sovereign debt model that explicates the strategic consequences of pari passu clauses and their persistent ambiguous phrasing. We showed that an ambiguous pari passu clause along with asymmetric information on Country's asset value give rise to restructuring equilibria in which some creditors hold out, default is probabilistic, and both the probability of default and Country's total expected payments upon a policy failure increase with the probability that the pari passu clause is interpreted broadly. By varying the probability that the pari passu clause will be interpreted broadly, therefore, parties to a sovereign debt contract can generate the optimal (second-best) trade-off between reducing moral hazard and imposing dead-weight default costs. A reduction in asymmetric information has two opposite effects. First, the degree of moral hazard in the absence of a pari passu clause is lower if information asymmetry is lower. Second, the pari passu clause becomes less effective in constraining moral hazard. As a result, it is indeterminate whether Countries benefit from a lower level of asymmetric information during restructuring negotiations.

 $^{^{16}}$ Country's expected default costs remain constant because under any constrained equilibrium the participation rate is fixed and CC's optimal demand decreases as w increases. As a result, Country's probability of no-default increases with w.

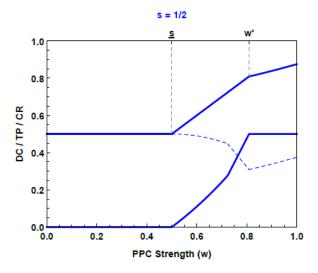
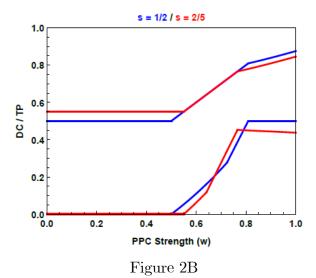


Figure 2A



s = 1/2 / s = 2/5

0.8

0.7

0.6

0.4

0.0

0.1

0.2

0.3

0.4

0.5

Default Costs

Figure 2C

Appendix

This Appendix proves Propositions 2 and 3. We begin by presenting some ancillary concepts and notations.

Recall that CC's demand is an "interior demand" if Country's probability of default conditional on a broad interpretation of the PPC $(P_{nd}(a,d,s))$ is strictly between 0 and 1 given CC's participation rate (α) and the level of information asymmetry between Country and Creditors (s).

We define the set of (potentially optimal) interior demands as

$$D_i = \{ d \in (0, \underline{s}] : P_{nd}(a, d, s) \in (0, 1) \}. \tag{A1}$$

That is, D_i consists of all strictly positive restructuring demands (weakly) less than \underline{s} for which Country's probability of default is strictly between 0 and 1.

The following lemma presents the infimum and supremum of a non-empty D_i .

Lemma A1
$$\inf(D_i) = \max\{0, \underline{s} - (1 - \alpha)\} \equiv \underline{d} \text{ and } \sup(D_i) = \min\{\overline{s} - (1 - \alpha), \underline{s}\} \equiv \overline{d} \text{ for } D_i \neq \emptyset.$$

The infimum of a non-empty D_i is the maximum of the largest no-default demand of $\underline{s} - (1 - \alpha)$ and 0. The supremum of a non-empty D_i is the maximum of the lowest default demand of $\overline{s} - (1 - \alpha)$ and \underline{s} .

The next lemma presents lower and upper bounds of α for which D_i includes \underline{s} or is empty.

Lemma A2 inf
$$(\alpha \mid \underline{s} \in D_i) = 1 - s \equiv \overline{\alpha}$$
 and inf $(\alpha \mid D_i \neq \emptyset) = 1 - \overline{s} \equiv \underline{\alpha}$.

The infima in Lemma A2 are derived from the fact that $P_{nd}(a,d,s)>0$ iff $d+1-\alpha<\overline{s}$; that is, Country's probability of default conditional on a broad interpretation of the PPC is strictly positive iff the sum of Creditors' demands is strictly less than the upper bound of the support of Country's payout distribution. In particular, if $\alpha>\overline{\alpha}$, HC's demand is strictly less than s $(1-\alpha< s)$ whereas if $\alpha\leq\underline{\alpha}$ HC's demand is greater than or equal to \overline{s} $(1-\alpha\geq\overline{s})$. The sum of Creditors' demands is therefore strictly less than \overline{s} for any $d\leq\underline{s}$ if $\alpha>\overline{\alpha}$ and is greater than or equal to \overline{s} for any d>0 if $\alpha\leq\underline{\alpha}$.¹⁷

Now, for $\alpha > \underline{\alpha}$ and $d \in D_i$ (implying that $D_i \neq \emptyset$ and $P_{nd}(a,d,s) \in (0,1)$), the strictly-positive difference $\overline{s} - [(1-\alpha) + d]$ represents the measure of policy payout realizations that exceeds the sum of Creditors' demands. Writing $\alpha - \underline{\alpha}$ for $\overline{s} - (1-\alpha)$ (by substituting $\underline{\alpha}$ for $1-\overline{s}$) and dividing by s, Country's probability of no default conditional on a broad interpretation of the PPC as a function of d, α , and s is

¹⁷Note that $\underline{s} \in (\underline{\alpha}, \overline{\alpha})$.

$$P_{nd}(a,d,s) = \frac{(\alpha - \underline{\alpha}) - d}{s}.$$
 (A2)

We now turn to the proofs of Propositions 2 and 3.

Proposition 2 (Full-Participation Equilibrium) There exists a payoff dominant full-participation equilibrium iff $w \leq \underline{s}$ (Area A in 1).

Proof. The "if" part follows from the text. To show the "only" part, suppose that $w > \underline{s}$ and that all Creditors participate and make a demand of \underline{s} , which Country accepts. Creditors accordingly obtain a recovery rate of \underline{s} . If a sufficiently small mass ϵ of Creditors deviate to holding out, the participation rate will decrease to $1 - \epsilon$. By (A2), the probability that Country does not default given a broad interpretation of the PPC is

$$P_{nd}(1 - \epsilon, \underline{s}, s) = \frac{(1 - \epsilon - \underline{\alpha}) - \underline{s}}{s}$$

$$= 1 - \frac{\epsilon}{s}, \tag{A3}$$

where the second line follows by substituting $1 - \overline{s}$ for $\underline{\alpha}$ and s for $\overline{s} - \underline{s}$. The deviating creditors' recovery rate, $wP_{nd}(1 - \epsilon, \underline{s}, s)$, is therefore $w(1 - \frac{\epsilon}{s})$, which for any $w > \underline{s}$ is greater than the putative equilibrium recovery rate of \underline{s} for a sufficiently small ϵ .

The next proposition characterizes and presents the conditions for the existence of a partial-participation equilibrium where CC make a no-default demand (f.n. 13).

Proposition 2A (partial-participation, no-default equilibrium) For $w \in [\widehat{w}(s), \underline{s})$, where $\widehat{w}(s) \equiv \max\{\frac{1}{2}, \frac{2(-s+\sqrt{s+3s^2})}{1+2s}\}$, there exists a partial-participation equilibrium in which CC make a no-default demand, Creditors' equilibrium recovery is w, and the equilibrium participation rate and restructuring demand are $(\alpha_{nd}^*, d_{nd}^*) = (\frac{1+2s}{4(1-w)}, \frac{w(1+2s)}{4(1-w)})$.

Proof. Recall that a no-default demand is $\underline{s} - (1 - \alpha)$. Because HC's recovery rate under a no-default demand is w (the probability that the PPC would be interpreted broadly), CC's recovery rate must be w as well. Solving for α that satisfies the equality $\frac{\underline{s} - (1 - \alpha)}{\alpha} = w$ gives the equilibrium participation rate. The equilibrium restructuring demand is obtained by substituting the equilibrium participation rate for α in $\underline{s} - (1 - \alpha)$.

Now, CC maximize their recovery by making a no-default demand of $\widehat{d}_{nd} = \frac{w(1+2s)}{4(1-w)}$ given a participation rate of $\widehat{\alpha}_{nd} = \frac{1+2s}{4(1-w)}$ iff:

$$1 \le -wd \cdot \frac{\partial P_{nd}(a, d, s)}{\partial d} \text{ for } d = \widehat{d}_{nd}$$
(A4)

subject to :

$$\hat{d} \geq (1 - w) \min\{\hat{\alpha}_{nd}, \underline{s}\}.$$
 (A5)

A no-default demand maximizes CC's recovery iff the marginal benefit of increasing that demand (left-hand side of (A4)) is less than or equal to the corresponding marginal cost

(right-hand side of (A4)), provided that CC's equilibrium recovery is higher than their recovery under a default demand (A5). The marginal benefit of increasing a no-default demand is 1 because $P_{nd}(a,d,s)=1$ for $d=\widehat{d}_{nd}$ and therefore $1-w+wP_{nd}(a,d,s)$ is equal to 1. Plugging $-\frac{1}{s}$ into $\frac{\partial P_{nd}(a,d,s)}{\partial d}$ in the right-hand side, the inequality (A4) reduces to $1 \leq \frac{w}{s} \times \widehat{d}_{nd}$. Plugging $\frac{w(1+2s)}{4(1-w)}$ for \widehat{d}_{nd} gives $w \geq \frac{2(-s+\sqrt{s+3s^2})}{1+2s}$.

To satisfy the payoff constraint in (A5), CC's recovery under a no-default equilibrium demand must be greater than their recovery under a default demand of $\min\{\widehat{\alpha}_{nd}, \underline{s}\}$. Because CC's recovery under a default demand is capped at \underline{s} it suffices to consider the case where the equilibrium participation rate is lower than or equal to \underline{s} , in which case the payoff constraint reduces to $\widehat{d}_{nd} \geq (1-w)\widehat{\alpha}_{nd}$, which further simplifies to $w \geq \frac{1}{2}$ because $\widehat{d}_{nd} = w\widehat{\alpha}_{nd}$.

To prove Proposition 3, we shall prove the following, more specific, proposition.

Proposition 3A (Equal recovery equilibria) Let $\underline{w}(s) \equiv \frac{s}{5/4 - \sqrt{\underline{s}^2 + (\overline{\alpha}/2)^2}}$ for $s > \widetilde{s}$ and $\overline{w}(s) \equiv \frac{s}{1-\underline{s}}$ for $s > \widehat{s}$, where $\widetilde{s} \equiv \frac{7}{2} - \sqrt{10}$ (≈ 0.33) and $\widehat{s} \equiv \sqrt{3} - \frac{3}{2} (\approx 0.23)$ (see dashed and blue curves in Figure 1, respectively).

For $w \in (\underline{s}, \overline{w}(s)]$ there exists a unique equal recovery equilibrium where the participation rate (a^*) and CC's restructuring demand (d^*) as a function of the PPC's strength (w) and the information asymmetry between Country and Creditors (s) are:

$$\begin{cases} a^*(w,s) = \underline{s} + \frac{\underline{\alpha}}{2} + \sqrt{\underline{s}Ws + \left(\frac{\underline{\alpha}}{2}\right)^2} & \text{if } w \in (\underline{s}, \ \underline{w}(s)] \\ d^*(w,s) = \underline{s} & \end{cases}$$

and

$$\begin{cases} a^*(w,s) = 2Ws + \sqrt{(Ws - \underline{\alpha})^2 + (2Ws)^2} \\ d^*(w,s) = \frac{1}{2}((a^* - \underline{\alpha}) + Ws) \end{cases} \quad \text{if } w \in (\underline{w}(s), \ \overline{w}(s)],$$

where $W \equiv \frac{1-w}{w}$. The equilibrium participation rate strictly decreases with w and is bounded below by $4\overline{\alpha}$, the equilibrium restructuring demand weakly decreases with w and is bounded below by $2\overline{\alpha}$, and Country's equilibrium probability of no-default strictly decreases with w and is bounded below by $\frac{\overline{\alpha}}{s}$.

Proof. We begin by presenting a lemma that expresses the equilibrium participation rate under an equal recovery equilibrium as a function of d, s, and w. Because under an equal recovery equilibrium CC's demand must be interior, we restrict attention to $d \in D_i$.

Lemma A3 In an equal-recovery equilibrium, the participation rate as a function of $d \in D_i$, s, and w is

$$\widehat{\alpha}(d, w, s) = d + \underline{\alpha}/2 + \sqrt{dWs + \left(\frac{\underline{\alpha}}{2}\right)^2}.$$
 (A6)

Proof. Recall that from (6) CC and HC obtain equal recovery rates if $\frac{1}{\alpha} \times d[(1-w) + wP_{nd}(a,d,s)] = wP_{nd}(a,d,s)$ for $d \in D_i$. Solving for $P_{nd}(a,d,s)$ that satisfies this equality

gives

$$P_{nd}(a,d,s) = W \times \frac{d}{\alpha - d}.$$
 (A7)

Plugging $\frac{(\alpha-\underline{\alpha})-d}{s}$ into $P_{nd}(a,d,s)$ (from (A2)) and solving for α gives (A6).

The rest of the proof proceeds in two parts. We first find the equilibrium participation rate as a function of w and s given that CC make an interior demand of \underline{s} . We then find the equilibrium participation rate and CC's demand as a function of w and s where CC make an interior demand strictly less than \underline{s} . At the end of each part we present the equilibrium outcome as a function of w for the special case where $s = \frac{1}{2}$.

(i) Consider an equal-recovery equilibrium where CC make a demand of \underline{s} . Such a demand maximizes CC's recovery iff

$$1 - w + w P_{nd}(a, d, s) > -wd \cdot \frac{\partial P_{nd}(a, d, s)}{\partial d} \text{ for } d \in (\underline{d}, \underline{s})$$
(A8)

subject to :

$$1 > \alpha \geq \overline{a}.$$
 (A9)

A restructuring demand of \underline{s} is optimal iff CC's marginal benefit of increasing any lower demand (left-hand side of (A8)) is greater than the corresponding marginal cost (right-hand side of (A8)), where $\underline{s} \in D_i = (0, \underline{s}]$ is an interior demand (A9).

Because CC's marginal benefit of making a higher interior demand decreases with d and the corresponding marginal cost increases with d, an interior demand of \underline{s} is optimal iff the limit of the marginal benefit of making a higher interior demand as $d \to \underline{s}$ is greater than or equal to the limit of the corresponding marginal cost. Plugging $\frac{(\alpha-\underline{\alpha})-d}{s}$ into $P_{nd}(a,d,s)$ (from (A7)), $\frac{-d}{s}$ into $\frac{\partial P_{nd}(a,d,s)}{\partial d}$, and \underline{s} into d (because both limits involve continuous functions), the inequality in (A8) holds iff

$$a \ge \min\{\overline{\alpha} + \overline{s} - \frac{s}{w}, 1\} \equiv \alpha^m,$$
 (A10)

where we omit the argument of α^m (s and w). Moreover, $\alpha^m \geq \overline{\alpha}$ is satisfied for any $w > \underline{s}, \frac{18}{s}$ whereas $\alpha^m < 1$ holds provided that $w < \frac{s}{\underline{s}}$.

Substituting \underline{s} for d in $\widehat{\alpha}(d, w, s)$ (in (A6)), plugging the resulting expression into a in (A10), and solving for w that satisfies (A10) we get

$$w \le \frac{s}{5/4 - \sqrt{\underline{s}^2 + (\overline{\alpha}/2)^2}} \equiv \underline{w}(s),$$
 (A11)

where $\underline{w}(s) > \underline{s}$ iff $s > \frac{7-2\sqrt{10}}{2} \equiv \widetilde{s}$ (see Figure 1). It is straightforward but tedious to show that if $w \leq \underline{w}(s)$ then $w < \frac{s}{\underline{s}}$ so that $\alpha^m < 1$ for $w \leq \underline{w}(s)$. The equilibrium participation rate is obtained by substituting \underline{s} for d in (A6). It is straightforward as well

¹⁸This follows because $\overline{s} \cdot \underline{s} \geq s$ for all $s \in (0, \frac{1}{2}]$, which in turn implies that if $w > \underline{s}$ then $w \geq \frac{\underline{s}}{\overline{s}}$ and therefore that $\alpha^m > \overline{\alpha}$.

to verify that the equilibrium participation rate decreases continuously with w.¹⁹

The following example illustrates an equal-recovery equilibrium where $d^* = \underline{s}$.

Example A1 Suppose that $s = \frac{1}{2}$. For $w \in (\frac{1}{2}, \underline{w}(\frac{1}{2})]$ the equilibrium participation rate and restructuring demand are $(\alpha^*, d^*) = (\frac{1}{2} + \sqrt{\frac{W}{2}}, \frac{1}{2})$, where $\underline{w}(\frac{1}{2}) \equiv \frac{1}{2}(1 + \frac{1}{\sqrt{5}})$ (≈ 0.72).

(ii) Next, consider an equal-recovery equilibrium where CC make a demand $d < \underline{s}$. For such a demand to maximize CC's recovery we must have

$$1 - w + w P_{nd}(a, d, s) = -w d \frac{\partial P_{nd}(a, d, s)}{\partial d} \text{ for } d \in (\underline{d}, \overline{d})$$
 (A12)

subject to :

$$(1-w)d + w \cdot d \cdot P_{nd}(a,d,s) \ge (1-w)\min\{\alpha,\underline{s}\} \text{ for } (\alpha,d) = (\alpha^*,d^*).$$
 (A13)

A demand $d \in (\underline{d}, \overline{d})$ maximizes CC's recovery iff the marginal benefit of increasing such a demand (left-hand side of (A12)) is equal to the corresponding marginal cost (right-hand side of (A12)),²⁰ provided that CC's expected equilibrium payoff is higher than their expected payoff from a default demand of either α or \underline{s} (A13).

Solving for d which satisfies (A12) yields

$$\widehat{d}(\alpha, w, s) = \frac{1}{2} \left((\alpha - \underline{\alpha}) + Ws \right). \tag{A14}$$

Substituting $\hat{d}(\alpha, w, s)$ for d in $\hat{\alpha}(d, w, s)$ in (A6) gives the unique equilibrium participation rate as a function of w and s. The equilibrium restructuring demand is obtained by plugging the candidate equilibrium participation rate into α in $\hat{d}(\alpha, w, s)$.

To satisfy the payoff constraint in (A13), CC's recovery rate under an equal-recovery equilibrium must be greater than their recovery rate under a default demand of $\min\{\alpha, \underline{s}\}$ (by dividing both sides of (A13) by α). CC's recovery rate under an equal-recovery equilibrium is $wP_{nd}(a,d,s)$, whereas their recovery rate under a default demand is either 1-w (if $\alpha < \underline{s}$) or $(1-w)\frac{\underline{s}}{\alpha} < 1-w$ (if $\alpha > \underline{s}$). Because CC's recovery rate increases with α^* under an equal recovery equilibrium but decreases with α under a default demand for $\alpha > \underline{s}$ (because $\frac{\underline{s}}{\alpha}$ decreases with α), it suffices to consider the case where $\alpha^* \leq \underline{s}$.

For any $\alpha^* \leq \underline{s}$, CC's recovery rate is higher under an equilibrium interior demand than under a default demand of α iff $wP_{nd}(a,d,s) \geq 1 - w$ or $P_{nd}(a,d,s) \geq W$. Plugging $W \times \frac{d^*}{\alpha^* - d^*}$ for $P_{nd}(a,d,s)$ (from A7) gives

$$d^* \ge \frac{\alpha^*}{2}.\tag{A15}$$

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²⁰The second-order condition for a maximum is satisfied because the derivative of $1-w+wP_{nd}(a,d,s)+wd\frac{\partial P_{nd}(a,d,s)}{\partial d}$ w.r.t. to d is $-\frac{1}{s}<0$.

Thus, to satisfy the payoff constraint, the equilibrium restructuring demand must be greater than half the equilibrium participation rate. Substituting α^* for α in $\widehat{d}(\alpha, w, s)$ (in (A14)), the inequality in (A15) holds iff $(\alpha^* - \underline{\alpha}) + Ws \geq \alpha^*$ or $Ws > \underline{\alpha}$. Plugging $\frac{1-w}{w}$ into W and solving for w gives

$$w \le \frac{s}{1-s} \equiv \overline{w}(s),\tag{A16}$$

where $\overline{w}(s) > \underline{s}$ for $\widehat{s} > An$ equal-recovery equilibrium thus exists iff $w \leq \overline{w}(s)$.

The following example illustrates an equal-recovery equilibrium where $d^* < \underline{s}$.

Example A2 Suppose that $s = \frac{1}{2}$. For $w \in (\underline{w}(\frac{1}{2}), w(1)]$ the equilibrium participation rate and restructuring demand are $(a^*, d^*) = (CW, \frac{W}{2}(C + \frac{1}{2}))$, where $\underline{w}(\frac{1}{2}) \equiv \frac{1}{2}(1 + \frac{1}{\sqrt{5}})$, $\underline{w}(1) = 1$, and $C \equiv 1 + \sqrt{5}/2$.

Finally, the lower bound of α^* is obtained by substituting Ws for $\underline{\alpha}$ and $\overline{w}(s)$ for w in $\alpha^*(w,s)$.²¹ Because $d^* \geq \frac{\alpha^*}{2}$, the lower bound of d^* is one-half of the lower bound of α^* . The lower bound of the equilibrium probability of no-default is obtained by substituting the lower bounds of α^* and d^* into (A2), which gives $\frac{(4\underline{a}-\underline{a})-2\underline{a}}{s}=\frac{\underline{a}}{s}$.

Remark Given a full-participation equilibrium for $w \leq \underline{s}$ (Area A in Figure 1), both α^* and d^* decrease continuously with w for $s \in [\widetilde{s}, 1/2]$, but drop discontinuously at $w = \underline{s}$ for $s \in (\widehat{s}, \widetilde{s})$. By contrast, Country's equilibrium probability of no-default drop continuously with w for any $s \in (\widehat{s}, 1/2]$.

First, consider the case where $s \in (\tilde{s}, 1/2]$. In the limit as $\alpha^* \to \alpha_m^+$, the equilibrium restructuring demand converges to $\lim_{\alpha^* \to \alpha_m +} \widehat{d}(\alpha^*, s, w) = \frac{1}{2}((a_m - \underline{\alpha}) + Ws)$ (by (A14)). Plugging $\overline{\alpha} + \overline{s} - \frac{s}{w}$ for a_m and simplifying yields \underline{s} (after substituting \underline{s} for $\overline{\alpha} - \underline{\alpha}$). The equilibrium restructuring demand, $d^*(w, s)$, thus decreases continuously with w. Because the equilibrium participation rate decreases continuously with d (by A6), the equilibrium participation rate, $\alpha^*(w, s)$, decreases continuously with w as well.

Next, suppose that $s \in (\hat{s}, \hat{s})$, where $\hat{s} \equiv \sqrt{3} - \frac{3}{2}$ (≈ 0.23). Because the equilibrium participation rate for any $w > \underline{s}$ is less than α_m , CC's equilibrium restructuring demand must be strictly less than \underline{s} . Both the equilibrium restructuring demand and participation rate thus drop discontinuously at $w = \underline{s}$.

The discontinuous drop in CC's restructuring demand results from the fact that for high participation rates CC's optimal demand is a no-default demand under which HC's recovery rate is higher than CC's recovery rate. HC's higher recovery rate produces a pressure to hold out, which causes the equilibrium participation rate and restructuring demand to drop until CC's optimal demand becomes an interior demand.

Finally, the equilibrium probability of no-default decreases *continuously* with w even when a^* and d^* drop discontinuously with w. To see this, observe that the lowest

²¹Substituting $\frac{s}{1-s}$ for w in 4Ws gives $4(1-\underline{s}-s)=1-2s$.

participation rate for which CC's optimal demand is a no-default demand is $\alpha_{nd} \equiv \underline{\alpha} + s(1 + \frac{1}{w})$. Now, in the limit as $\alpha^* \to \alpha_{nd}^+$, the equilibrium restructuring demand goes to $\lim_{\alpha^* \to \alpha_{nd}^+} \widehat{d}(\alpha^*, s, w) = \frac{1}{2}((a_{nd} - \underline{\alpha}) + Ws)$ (by (A14)). Plugging $\underline{\alpha} + s(1 + \frac{1}{w})$ for a_{nd} and simplifying yields $\frac{s}{w}$ (after substituting $\frac{1-w}{w}$ for W). As $\alpha^* \to \alpha_{nd}^+$, therefore, the equilibrium probability of no-default converges to $\lim_{\alpha^* \to \alpha_{nd}^+} \lim_{d^* \to \frac{s}{w}} P_{nd}(a^*, d^*, s) = \frac{(\alpha_{nd} - \underline{a}) - \frac{s}{w}}{s} = \frac{(\underline{\alpha} + s(1 + \frac{1}{w}) - \underline{a}) - \frac{s}{w}}{s} = 1$.

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