Collusion in Markets with Syndication*

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Abstract

Many markets, including the market for IPOs, are syndicated; a bidder who wins a contract often invites competitors to join a syndicate to fulfill the contract. We model syndicated markets as a repeated extensive form game and show that standard intuitions from industrial organization may fail: collusion may become easier as market concentration falls, and market entry may facilitate collusion. In particular, firms can sustain collusion by refusing to join the syndicate of any firm that undercuts the collusive price. Our results can thus rationalize the apparently contradictory facts that IPO underwriting exhibits seemingly collusive pricing despite low market concentration.

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1 Introduction

The fees that investment banks collect for initial public offerings (IPOs) strongly suggest collusive behavior, with investment banks apparently coordinating on fees of 7% of issuance proceeds for moderately sized IPOs (Chen and Ritter, 2000). At the same time, the number of investment banks running moderately sized IPOs is quite large, and there appears to be a nontrivial amount of entry and exit in the market (Hansen, 2001); this presents a puzzle, as standard industrial organization intuitions would therefore suggest that pricing should be competitive.

One possible explanation lies in the structure of the IPO underwriting market. The market for running IPOs is syndicated; once the bid to run an IPO is accepted, the winning investment bank must then organize a syndicate to complete the IPO. In this paper, we show that syndication can explain how collusion may be maintained in the presence of many small firms. We show that the presence of syndication can reverse the standard intuition regarding the effect of market concentration: below a certain level of concentration, the scope for collusion in a syndicated market *increases* as concentration declines. Because syndication follows the pricing stage, colluding firms can punish a firm that undercuts the collusive price by refusing to participate in that firm's syndicate. This type of in-period punishment is not available in non-syndicated markets. Moreover, these in-period punishments become more powerful as a market becomes less concentrated; when the market is comprised of many small firms, joint production lowers production costs dramatically.

Figure 1 shows a snapshot of IPO spreads[1] in 1999.[2] In the late 1970s, spreads for IPOs tended to be quite high, exceeding 7%. In the early 1980s, spreads for IPOs with proceeds in excess of $20 million began to fall below 7%. However, over the course of the late 1980s, and particularly through the 1990s, spreads for IPOs with proceeds between $20 and $100 million became increasingly clustered at 7%. This clustering continues in the 2000s. Notably, the IPO market largely ceased to operate following the 2007-2008 financial crisis, and very few IPOs took place; nevertheless, those that did still paid the 7% spread. Yet, as first documented by Hansen (2001), the market for IPOs since the 1990s appears "competitive," in the sense that many firms were active in the market; indeed, the largest four firms together only make up between 40% and 50% of the market in this period, as depicted in Figure 2.

We model a market with syndication as a repeated extensive form game: In each period, firms compete on price for the opportunity to complete a single project and, upon being

[1]The spread on an IPO is the difference between the price that the underwriters pay for the issuer's stock and the price that investors pay, expressed as a percentage of the price investors pay.

[2]See the version of the paper posted at `http://bit.ly/2uOfEWv` for an animation showing how spreads evolved over time.

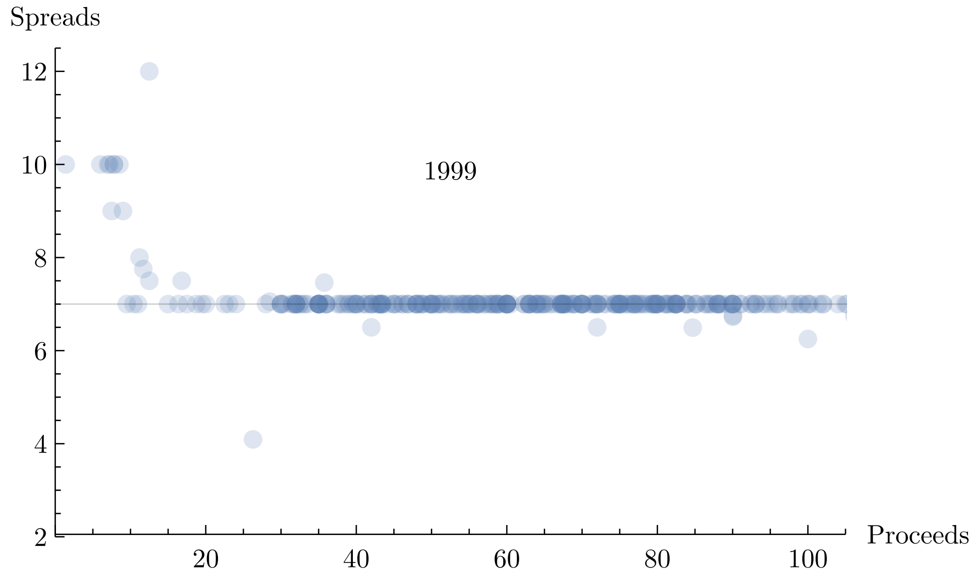

Figure 1: This figure shows the relationship between IPO proceeds and spreads for IPOs with proceeds less than \$100 million in 1999. Proceeds (in millions of dollars) are plotted on the x-axis and spreads (i.e., the percent of the proceeds that go to the underwriter) are plotted on the y-axis. A description of the data used for this figure can be found in Appendix A.

selected, the chosen firm may invite additional firms to join in the production process. Recruiting additional firms is valuable because production costs are convex in the amount of production done by a single firm. Each invited firm then decides whether to join the syndicate. The project is then completed by the syndicate members, payoffs are realized, and play proceeds to the next period.

We show that, in markets with syndication, less concentrated markets may have prices that are farther from the marginal cost of production. In particular, the highest price that can be sustained under equilibrium play is a U-shaped function of market concentration: When markets are very concentrated, collusion can be sustained as in many standard industrial organization models: after a firm undercuts on price, all firms revert to a "competitive" equilibrium in which firms earn no profits in every subsequent period.[3] However, when many small firms are present, collusion can be sustained by in-period punishments: after a firm undercuts on price, other firms can punish the undercutting firm in the same period by refusing to join its syndicate. Of course, such behavior by other firms must itself be incentive compatible. Thus, firms that reject offers of syndication from a firm that undercut on price must be rewarded in future periods. Moreover, firms that turn down more attractive syndication offers receive greater rewards in subsequent periods.

In repeated normal form games, punishments can be enforced using the *simple penal*

[3]See, for instance, Tirole (1988).

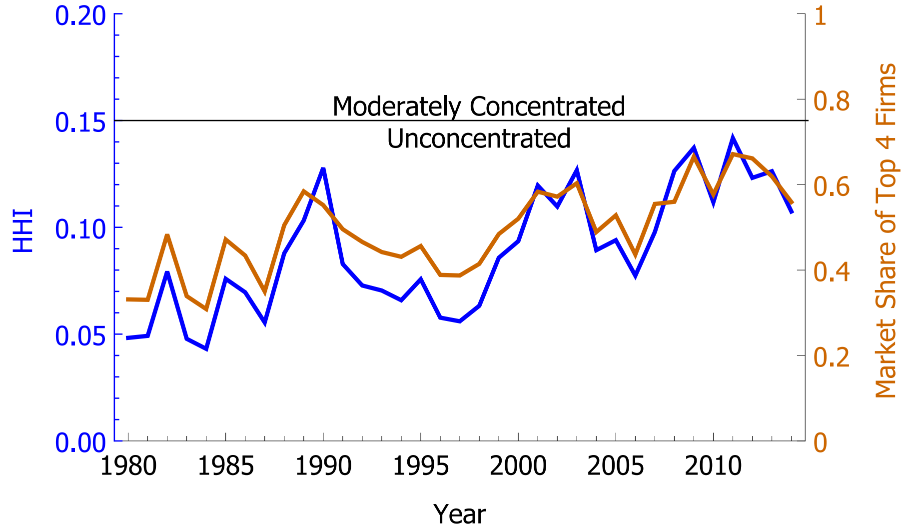

Figure 2: The Herfindahl-Hirschman index (HHI) of, and the market share of the largest four firms in, the market for IPOs. The U.S. Department of Justice defines an industry with an HHI of less than .15 to be an "unconcentrated market" (Department of Justice and Federal Trade Commission, 2010, p. 19). A description of the data used for this figure can be found in Appendix A.

codes of Abreu (1986), under which only one punishment strategy is needed for each player, regardless of the timing or nature of the deviation. However, as noted by Mailath et al. (2016), in the analysis of repeated extensive form games it is necessary to consider more complex responses to deviations.[4,5] In particular, in our setting, it is key that firms punish a price undercutter *in-period* by refusing the undercutter's offers of syndication; to do this, we must construct strategies that simultaneously punish a firm that undercuts on price and reward firms which refuse to join a price undercutter's syndicate.[6]

[4]It is not sufficient to consider the repeated version of the reduced normal form game, as the equilibria of that game will not necessarily correspond to subgame-perfect equilibria of the original repeated extensive form game.

[5]Nocke and White (2007) were the first to use the theory of repeated extensive form games to study collusion, showing that vertical mergers can facilitate collusion under certain circumstances. Byford and Gans (2014) consider collusion via market segmentation by considering a repeated extensive form game with market segment entry decisions followed by production decisions; they, however, restrict attention to a class of equilibria in which agents' decisions regarding production can not depend on past play, eliminating the extensive-form considerations which are central to our work here. See also the work of Atakan and Ekmekci (2011), who consider how reputation may be built in a repeated extensive form game with initial uncertainty about one player's type.

[6]To our knowledge, we are the first to model syndication, i.e., subcontracting, in a repeated extensive

Our baseline model considers the case of symmetric firms; we extend our results to markets with heterogeneous firms. As in the case with symmetric firms, firms can collude even when the market is very fragmented; indeed, heterogeneity itself can increase firms' ability to collude. Moreover, the entry of small firms enhances the scope for collusion in markets with syndication, again counter to standard results in the theory of industrial organization.

Whether spreads on IPOs are set in a competitive or collusive manner has been debated in the finance literature since Chen and Ritter (2000) first documented the clustering of IPO spreads at 7%. Abrahamson et al. (2011) documented that the spreads for IPOs are significantly higher in the United States than in Europe, and cited this as evidence that pricing in the U.S. underwriting market is collusive. Kang and Lowery (2014) presented and estimated a formal model of why collusion would lead to clustering on spreads, combining insights on collusive behavior from Rotemberg and Saloner (1986) and Athey et al. (2004).[7] By contrast, Hansen (2001) claims that the clustering of IPO spreads is likely to be the result of efficient contracting, documenting the apparent relative ease of entry and lack of concentration in the market.[8] Our work helps reconcile the apparently conflicting evidence: we show that collusion in IPO markets is possible despite—and in fact may be facilitated by—low levels of market concentration.

There also is a related debate over whether the pricing of the IPO securities themselves is collusive. IPO shares generally gain about 15% on their first day of public trading, suggesting that issuers are "leaving money on the table" (Loughran and Ritter, 2004). Some authors argue that underpricing is a means for underwriters to extract rents from issuers—likely a feature of an uncompetitive market (Biais et al., 2002; Cliff and Denis, 2004; Loughran and Ritter, 2004; Liu and Ritter, 2011; Kang and Lowery, 2014). On the other hand, other authors argue that issuers may desire underpricing, and thus underpricing can occur even when underwriters compete aggressively (Rock, 1986; Allen and Faulhaber, 1989; Benveniste and Spindt, 1989; Chemmanur, 1993; Brennan and Franks, 1997; Stoughton and Zechner, 1998; Lowry and Shu, 2002; Smart and Zutter, 2003). While our work does not address the issue of underpricing directly, it does show that underwriters could collude in the market for IPOs, even though—or even because—the market is highly fragmented.

The remainder of the paper is organized as follows: Section 2 introduces our model of a market with syndicated production. Section 3 characterizes the highest price sustainable via

form game. There is, however, a large literature on horizontal subcontracting in the context of one-shot interactions, starting with the work of Kamien et al. (1989); see also the work by, among others, Spiegel (1993) and Shy and Stenbacka (2003).

[7] Kang and Lowery's work also helps to explain why, under collusion, spreads may not change with IPO size or changes over time in the cost of performing an IPO.

[8] Torstila (2003) documents the clustering of spreads in countries other than the United States at lower levels, arguing that this provides evidence that clustering does not imply collusive behavior.

collusion in such markets. Section 4 considers how the highest price sustainable via collusion depends on market conditions. Section 5 extends the model to allow for contracting over production shares. Section 6 explores the effect of firm heterogeneity and market entry on the highest sustainable price. Section 7 concludes.

2 Model

We introduce a model of price competition in markets with syndication. There is a finite set of long-lived identical *firms* F and an infinite sequence of short-lived identical *buyers* $\{b_t\}_{t\in\mathbb{N}}$; we let $\varphi \equiv \frac{1}{|F|}$ be the *market concentration*. Time is discrete and infinite; firms discount the future at the rate $\delta \in (0,1)$.

Each firm f is endowed with a production technology with a *cost function* $c(s,m)$, where s is the quantity of production done by firm f and m is the mass of the productive capacity controlled by firm f. We assume that the cost function is strictly increasing and strictly convex in the production done by the firm and strictly decreasing and strictly convex in the productive capacity of the firm. We also assume that a firm which does not engage in production incurs no costs, i.e., $c(0,m) = 0$ for all m, and that production becomes arbitrarily costly as the productive capacity of the firm goes to 0, i.e., $\lim_{m\to 0} c(s,m) \geq \infty$ for all $s > 0$. Finally, we assume that the cost function is homogeneous of degree one.[9]

We let the *total productive capacity* in the economy be given by $k > 0$; in this section, we assume that the total productive capacity is evenly divided among the firms, so that the cost of producing s for any one firm is $c(s,\varphi k)$.

In each period t, the firms and the buyer b_t play the following extensive-form stage game:

Step 1: Each firm $f \in F$ simultaneously makes a *price offer* $p_t^f \in [0,\infty)$. All offers to the buyer are immediately and publicly observed.

Step 2: The buyer accepts at most one offer; the buyer's action is immediately and publicly observed. If no offer is accepted, the stage game ends.

Step 3: If the offer from some firm is accepted, then that firm becomes the *syndicate leader*, ℓ. Firm ℓ then simultaneously offers each *non-leader firm* $g \in F \setminus \{\ell\}$ a *fee* w_t^g. These

[9] This last assumption is stronger than is generally necessary for our analysis but it greatly simplifies our presentation here. It is enough for our results that, as we proportionately increase the production required and the productive capacity, the cost function increases at a slower rate, i.e., $\frac{\partial^2 c(s,sm)}{\partial s^2} \leq 0$ for all $s,m > 0$; in the homogeneous case, this expression holds with equality. Economically, this implies that larger firms are weakly more efficient, in the sense that one firm with productive capacity sm can complete a production share s at a (weakly) lower cost than multiple firms with combined productive capacity sm.

offers are immediately and publicly observed.[10]

Step 4: Each firm $g \in F \setminus \{\ell\}$ either accepts or rejects the fee w_t^g from ℓ. We call the set of firms that accept ℓ's offer, along with the firm ℓ, the *syndicate* G_t. At the end of the period, all agents observe the syndicate.[11]

The buyer b_t has a fixed *value* of $v > c(1,1)$ for the finished product.[12] Thus, the payoff to the buyer b_t is $v - p_t^f$ if he accepts the price offer from firm f and 0 if he does not accept any offer.

If the buyer b_t does not accept any offer, then each firm $f \in F$ obtains a payoff of 0. If firm ℓ becomes the syndicate leader, i.e., the buyer b_t accepts the offer of firm ℓ, then production is performed efficiently *ex post* by the members of ℓ's syndicate, and so each member of the syndicate performs an equal share of production.[13] Thus, the stage game payoffs for the firms after a successful offer to the buyer from firm ℓ are as follows:

1. The payoff for ℓ is $p_t^\ell - c\Big(\frac{1}{|G_t|}, \varphi k\Big) - \sum_{g \in G_t \setminus \{\ell\}} w_t^g$, i.e., the price paid by the buyer less the cost of ℓ's production less the fees paid to other firms.

2. The payoff for $g \in G_t \setminus \{\ell\}$ is $w_t^g - c\Big(\frac{1}{|G_t|}, \varphi k\Big)$, i.e., the fee paid to g less the cost of g's production.

3. The payoff for $h \in F \setminus G_t$ is 0.

3 Optimal Collusion

We now characterize the highest price sustainable via collusion in markets with syndication. A price p is *sustainable* if there exists a subgame perfect Nash equilibrium in which, along the equilibrium path, the buyer accepts a price offer of p in every period.

When the market is very concentrated, i.e., there are a small number of firms, any price (less than or equal to v) can be sustained by "grim trigger" strategies in which deviations from the collusive price are punished in subsequent periods by play in which every firm obtains 0 profits. This type of equilibrium is standard in the analysis of markets with Bertrand competition; in such markets, however, once there are enough firms in the market, no price above the cost of production can be sustained.

[10]In Section 5, we consider the case where an offer specifies not only a fee but also the share of production done by the firm.

[11]Consequently, all agents know which syndication offers were accepted.

[12]We assume that $v > c(1,1)$ to avoid the trivial case where no trade is efficient.

[13]In Section 5, we consider a "complete contracting" version of the model in which a syndicate offer specifies a firm's production share as well as its fee.

In markets with syndication, as in Bertrand competition markets, grim trigger strategies lose their bite as the number of firms in the market grows. However, unlike in the standard Bertrand competition model, markets with syndication admit a second method of maintaining collusion: if a firm becomes a *price deviator*—i.e., if a firm bids lower than the price mandated by the collusive equilibrium—other firms can punish that firm "in period" by refusing offers of syndication. This raises the cost of production for that firm, as it must now complete the project on its own instead of engaging in (more efficient) syndicated production. To incentivize firms to not join the price deviator's syndicate, we need to promise them rewards in future periods; reverting to "perfect competition" after a price deviation does not accomplish this goal, as all firms would earn 0 profits in all future periods. For this reason, reverting to "perfect competition" in periods after a price deviation is not the best continuation plan to sustain collusion. Instead, an optimal continuation plan should simultaneously reward firms for refusing offers of syndication while punishing the price deviator. In particular, the higher the price deviator's syndication offer to a firm g, the higher the continuation payoff needed to induce g to reject the offer of syndication; "the reward should fit the temptation" (Mailath et al., 2016). It is also important to punish a firm if it accepts an offer of syndication from the deviating firm: to do this, we *do* revert to perfect competition if any firm accepts a price deviator's offer of syndication. This punishes *both* the initial deviator and any firm which joins the syndicate as harshly as possible; these strategies make recruiting a syndicate sufficiently costly that lone production is a more attractive option than recruiting a syndicate.

Unlike grim trigger strategies, syndicate punishment strategies become *more* powerful as the market becomes less concentrated, as the cost of completing the project alone becomes increasingly expensive. Consequently, the preceding observations imply that, in general, the highest sustainable price is *not* monotone in market concentration: At high levels of market concentration, firms can collude at the monopoly price, as in the standard Bertrand competition model. When market concentration is sufficiently low, syndicate punishments again enable firms to collude at the monopoly price. However, at intermediate levels of market concentration, there are no subgame-perfect Nash equilibrium strategies which sustain the monopoly price.

We now formally derive the highest sustainable price.

Theorem 1. *For $\delta \geq \frac{1}{2}$, the highest sustainable price, $p^\star$, is given by*[14]

$$p^\star = \begin{cases} v & \varphi \in [1-\delta, 1] \\ \min\left\{\frac{(1-\delta)c(1,\varphi k) - \varphi c(1,k)}{1-\delta-\varphi}, v\right\} & \varphi \in (0, 1-\delta). \end{cases}$$

[14] Our result also obtains for some discount factors less than $\frac{1}{2}$, but assuming that $\delta \geq \frac{1}{2}$ greatly simplifies our presentation here.

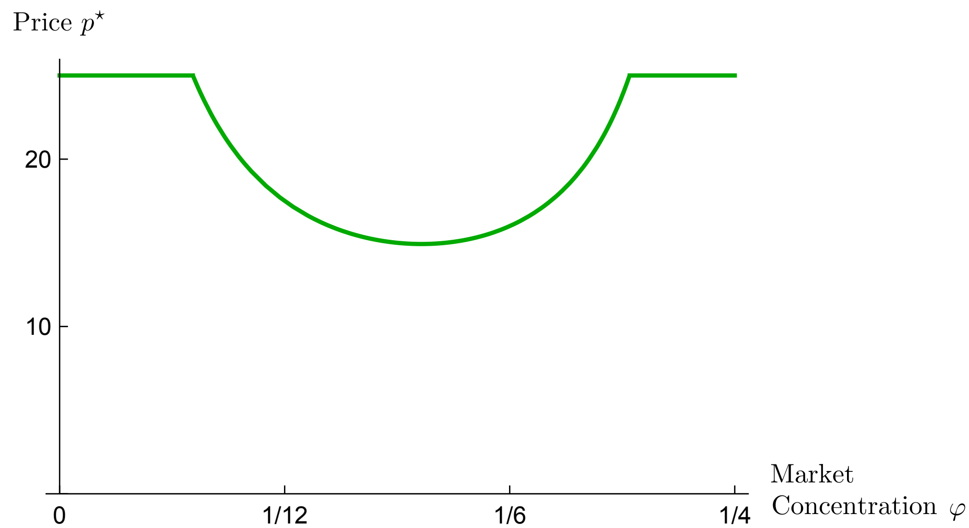

Figure 3: The highest sustainable price $p^\star$ as a function of market concentration φ. Here, $c(s, m) = \frac{s^2}{m}$, $k = 1$, $\delta = \frac{3}{4}$, and the maximum price that the buyer is willing to pay is $v = 25$. For sufficiently concentrated industries, the monopoly price can be sustained through grim trigger strategies. The highest sustainable price is lower for intermediate industry concentration levels, but as market concentration goes to 0 the highest sustainable price reaches the buyer's value v. The cost of efficient production (i.e., when the syndicate includes all firms) is 1 for all market concentrations φ.

Moreover, $p^\star$ is quasiconvex in φ and $\lim_{\varphi \to 0} p^\star = v$.

Figure 3 plots the highest sustainable price $p^\star$ as a function of φ. We call an equilibrium in which, along the equilibrium path, the buyer accepts an offer of the highest sustainable price $p^\star$ and firms engage in efficient joint production an *optimal collusion equilibrium*. In an optimal collusion equilibrium, the combined per-period profits for all firms are given by $p^\star - c(1, 1)$. An optimal collusion equilibrium maximizes industry profits; the buyer accepts the highest sustainable price, and efficient joint production ensures that costs are as low as possible.

In the rest of this section, we show that the $p^\star$ defined in Theorem 1 can be sustained as a subgame-perfect Nash equilibrium of the game defined in Section 2 and, moreover, $p^\star$ is the highest sustainable price. For ease of exposition, we set $k = 1$ throughout the rest of this section.

3.1 Bertrand Reversion Nash Equilibrium

We first describe the *Bertrand reversion Nash equilibrium* of the stage game, i.e., the subgame-perfect equilibrium in which all firms make zero profits and the buyer obtains the good at the lowest possible cost of production. In this equilibrium, each firm f offers a price

$p_t^f = c(1,1)$, which is exactly the cost of producing the good under full participation in the syndicate. The buyer then chooses each firm as syndicate leader with equal probability. The syndicate leader then offers each non-leader firm g a fee $w_t^g = c(\varphi, \varphi)$ equal to g's cost of production (assuming all syndication offers are accepted). Each firm $g \in F \setminus \{f\}$ accepts this offer. Under this behavior, each firm in the syndicate other than f then incurs production costs of $c(\varphi, \varphi)$ and thus breaks even. Moreover, the syndicate leader also breaks even as he obtains $c(1,1) = |F|c(\varphi, \varphi)$ from the buyer, he incurs production costs of $c(\varphi, \varphi)$, and he pays $(|F|-1)c(\varphi, \varphi)$ in total to the syndicate, leaving him with exactly 0 in profit.[15]

If any firm makes an offer other than $c(1,1)$ to the buyer, the buyer chooses the lowest offer.[16] Firms' responses to syndication offers do not depend on the set of offers made to the buyer. If the syndicate leader offers a fee of $c(\varphi, \varphi)$ to each other firm, then each other firm accepts this offer. If the syndicate leader offers a fee other than $c(\varphi, \varphi)$ to any firm, then within-period continuation play can follow any profile of actions for the other firms $g \neq f$ that constitutes a Nash equilibrium of the within-period continuation game.[17] Note, however, that regardless of the equilibrium play after a fee other than $c(\varphi, \varphi)$ has been offered to some firm, the syndicate leader f's profits are no greater than $p^f - c(\varphi, \varphi) - (|F|-1)c(\varphi, \varphi) \leq c(1,1) - |F|c(\varphi, \varphi) = 0$. This follows as no offer greater than $c(1,1)$ will be accepted by the buyer, and no firm will accept a syndication offer of less than $c(\varphi, \varphi)$, which is its minimal cost of production as a member of a syndicate. Thus, the syndicate leader will not wish to deviate from the strategy prescribed above. Given his play, other firms will not wish to deviate from their prescribed strategies either.

Our first result shows that the Bertrand reversion Nash equilibrium strategies just described in fact constitute a subgame-perfect Nash equilibrium of the stage game in which each firm obtains its lowest individually rational payoff.

Proposition 1. *There exists a subgame-perfect Nash equilibrium of the stage game, i.e., the Bertrand reversion Nash equilibrium, in which each firm obtains a payoff of* 0*, its lowest individually rational payoff.*

In the analysis of repeated normal form games, reverting to the stage game equilibrium described in Proposition 1 would be sufficient to punish any off-equilibrium behavior. That is, the Bertrand reversion Nash equilibrium can be used to implement the simple penal codes of Abreu (1986). However, as noted by Mailath et al. (2016), simple penal codes are insufficient to characterize the set of equilibrium payoffs in repeated extensive form games. Nevertheless,

[15] Recall that $c(\cdot, \cdot)$ is homogeneous of degree one.

[16] If there are multiple lowest offers, the buyer chooses each with equal probability.

[17] Note that there may be multiple such Nash equilibria, as whether a syndication offer is profitable for an agent may depend on whether other agents accept their syndication offers.

as we will show, the Bertrand reversion equilibrium is a key component in constructing the equilibrium that supports the highest sustainable price.

3.2 Maintaining Collusion with Grim Trigger Strategies When the Market Is Concentrated

We first show that the monopoly price v is sustainable when firms are patient and the number of firms is sufficiently small. Moreover, under these conditions, collusion can be sustained via "grim trigger" strategies: after a deviation in either step of the stage game, play in all subsequent periods reverts to the Bertrand reversion Nash equilibrium described in Section 3.1.

Proposition 2. *If the discount factor is sufficiently high, i.e., $\delta \geq 1-\varphi$, then there exists a subgame-perfect Nash equilibrium in which every firm offers the monopoly price, i.e., $p_t^f = v$ for any $v \geq c(1,1)$, for all $f \in F$ and for all t.*

To prove Proposition 2, we construct an equilibrium in which, in every period, each firm bids the monopoly price v; the short-lived buyer then accepts one such offer (choosing each offer with equal probability). If the offer from firm ℓ is accepted, ℓ offers a fee $w_t^g = c(\varphi,\varphi)$ to each other firm $g \in F \setminus \{\ell\}$; each other firm g then accepts and joins the syndicate.

If a firm offers a lower price in the first step, i.e., becomes a *price deviator*, the buyer chooses this lower offer. Then, the price deviator makes a syndication offer to every other firm; every other firm accepts the offer of syndication if the price deviator offers $c(\varphi,\varphi)$ to each firm.[18] However, in every subsequent period following such a deviation, play reverts to the Bertrand reversion Nash equilibrium described in Section 3.1. Finally, if any firm chooses to not accept an offer of syndication with fee $c(\varphi,\varphi)$, play also reverts to the Bertrand reversion Nash equilibrium. Thus, in each period, the syndicate leader has profits of

$$v - c(\varphi,\varphi) - (|F|-1)c(\varphi,\varphi) = v - |F|\varphi c(1,1) = v - c(1,1)$$

and each other member of the syndicate has profits of

$$c(\varphi,\varphi) - c(\varphi,\varphi) = 0.$$

Given this proposed equilibrium structure, in which the syndicate leader offers every other firm a syndication fee of $c(\varphi,\varphi)$, it is clear that it is a best response for each firm to accept

[18] If the syndicate leader offers a fee other than $c(\varphi,\varphi)$ to any firm, then within-period continuation play can follow any profile of actions for the other firms $g \neq \ell$ that constitutes a Nash equilibrium of the within-period continuation game.

its offer of syndication, as accepting leads to (weakly) higher profits than rejecting. It is also clear that the buyer in each period is acting optimally. Thus, to ascertain whether this is an equilibrium, we need only check whether each firm is willing to offer the monopoly price in the first step, or would rather offer an infinitesimally lower price to the buyer and have its offer accepted with certainty. The expected discounted value of the current payoff and all future payoffs from following the equilibrium strategies is

$$\sum_{t=0}^{\infty} \delta^t \varphi(v - c(1,1)) = \frac{\varphi}{1-\delta}(v - c(1,1)).$$

This expression is greater than $v - c(1,1)$ so long as $\varphi > 1 - \delta$. Meanwhile, the expected discounted value of all future payoffs from offering an infinitesimally lower price is

$$v - c(1,1).$$

Proposition 2 is the analogue in our setting to the familiar result that, in models of Bertrand competition, collusion at any price can be maintained by grim trigger strategies when the industry is sufficiently concentrated. However, in the standard model of Bertrand competition, collusion cannot be maintained at any price when $\delta < 1 - \varphi$; in the next section, we show that this is *not* true in our setting.

3.3 Maintaining Collusion with Syndicate Punishments

In this section, we first provide an intuitive description of an equilibrium which sustains the price $p^\star$ defined in Theorem 1. We then give a formal construction of the strategy profile, and show that the strategy profile constitutes a subgame-perfect Nash equilibrium. Finally, we show that no subgame-perfect Nash equilibrium can sustain a price higher than $p^\star$.

The key idea is to construct strategies that exploit syndicate boycotting to enforce higher prices. Play begins in the *cooperation phase*, in which each firm offers the same price $p^\star$ and a firm, upon having its offer accepted, engages in efficient syndication. Play continues in the cooperation phase so long as no one deviates. If some firm deviates in the first step—i.e., offers a lower price to the buyer in order to guarantee that it wins the bid—we call such a firm a *price deviator*. Because of the efficiency gains from syndicated production, the price deviator will wish to induce the non-leading firms to join its syndicate, and thus will be willing to offer each firm a fee above its cost of production as an inducement. By the same token, if the non-leading firms refuse to join the price deviator's coalition, they can raise the deviator's cost of production, punishing the price deviator in-period. Thus, the optimal collusion plan will promise future-period rewards to non-leading firms that reject above-cost syndication offers from the price deviator. For this reason, Bertrand reversion after a price deviation is not necessarily the best continuation plan to sustain collusion. Moreover, it is also important to punish a firm if it joins a price deviator's syndicate; to do this, we *do* use Bertrand reversion, as it punishes both the initial deviator and any firm that joins the syndicate as harshly as possible. Thus, whenever any firm deviates by accepting a price deviator's syndication offer—or rejecting a non-price deviator's equilibrium syndication offer—play enters the *Bertrand reversion phase*, in which firms play the Bertrand reversion Nash equilibrium each period.

After a period in which a firm f is a price deviator, but no firm joins its syndicate, we enter a *collusive punishment phase* which both punishes the price deviator and rewards those who refused to join its syndicate. In the collusive punishment phase, each firm offers the same price q to the buyer. The higher that q is, the higher total industry profits will be, which permits larger rewards to firms that reject a price deviator's syndication offer. At the same time, behavior during such a collusive punishment phase must itself be subgame-perfect. If the price q is too high, the collusive punishment phase will not be subgame-perfect, as the price deviator or another firm will wish to price-deviate in this phase.

Moreover, the continuation payoff to a firm other than the price deviator during a collusive punishment phase may depend on the offer that was made to that firm by the price deviator. In particular, "the reward should fit the temptation" (Mailath et al., 2016)—the larger the fee offered to the firm by the price deviator, the greater the continuation payoff offered to that firm to induce it to reject the offer of syndication.

Thus, to characterize the highest sustainable price, we specify a subgame-perfect Nash equilibrium that exploits the possibility of in-period punishments. This equilibrium is composed of three types of phases: In the *cooperation phase*, each firm offers p to the short-lived buyer, who then chooses each firm with equal probability; afterwords, an efficient syndicate is formed. If any firm f price-deviates, but no other firm joins its syndicate, then we enter a *collusive punishment phase with continuation values* ψ, in which the continuation values are determined by the syndication offers. In a collusive punishment phase with continuation values ψ, each firm offers a specific price q to the short-lived buyer, who then chooses each firm with equal probability; we call q the *collusive punishment price*. The winning bidder then efficiently syndicates production; in so doing, it offers each non-leading firm g a fee equal to its assigned continuation value, ψ^g, plus g's production cost, $c(\varphi, \varphi)$. Finally, if any firm deviates from equilibrium play with respect to accepting or rejecting offers of syndication, play enters the *Bertrand reversion phase*, in which firms play the Bertrand reversion Nash equilibrium each period.

By making future play conditional on offers of syndication, firms are incentivized to punish price deviators in-period, by refusing to join their syndicates. This then reduces the incentive for agents to deviate on price, since each firm is aware that, if it deviates on price, it will have to engage in lone production. Since lone production becomes costlier as the market becomes more fragmented, reducing market concentration may make it easier to sustain collusion at a given price.

We now give a formal construction of the strategy profile that sustains $p^\star$. The equilibrium is constructed as follows:

- There are three phases of equilibrium play:

  1. In the *cooperation phase*,
     - every firm submits the same bid $p = p^\star$,
     - the short-lived buyer accepts one such offer of $p^\star$, choosing each offer with equal probability,
     - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi)$ to every non-leading firm $g \in F \smallsetminus \{\ell\}$ to join the syndicate, and
     - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
  2. In the *collusive punishment phase with continuation values ψ*,
     - every firm submits the same bid $q = \min\{c(1, \varphi), v\}$,
     - the short-lived buyer accepts one such offer of q, choosing each offer with equal probability,
     - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi) + \psi^g$ to every non-leading firm $g \in F \smallsetminus \{\ell\}$ to join the syndicate, and
     - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
  3. In the *Bertrand reversion phase*, agents play the Bertrand reversion Nash equilibrium.

- Under equilibrium play, play continues in the same phase. In the cooperation phase or a collusive punishment phase, some firm f may price-deviate in the first step, in which case the buyer accepts this offer, or deviate with respect to the prescribed set of syndication offers. If so, future play depends on the sum over the non-leading firms of the (positive) difference between the syndication fee offered to each firm and the cost to that firm of doing φ of the project, $\sum_{g \in F \smallsetminus \{f\}} (w^g - c(\varphi, \varphi))^+$.[19] Based on this sum, we

[19] Here, $(x)^+ \equiv \max\{0, x\}$.

categorize the set of offers made by a deviating firm f into three cases: *uniformly low offers*, *insufficient offers*, and *sufficient offers*. Future play in each case is as follows:

Uniformly Low Offers: $\sum_{g\in F\setminus\{f\}}(w^g - c(\varphi,\varphi))^+ = 0$**.** In this case, rejecting the syndication offer is a best response for each non-leading firm, as the fee offered is weakly less than each non-leading firm's cost of production. Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{g\in F\setminus\{f\}}(w^g - c(\varphi,\varphi))^+ \leq \frac{\delta}{1-\delta}(q - c(1,1))$**.** In this case, absent dynamic rewards and punishments, some non-leading firms may be tempted to accept their syndication offers. All non-leading firms *do* reject their syndication offers and play proceeds going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{(w^h - c(\varphi,\varphi))^+}{\sum_{g\in F\setminus\{f\}}(w^g - c(\varphi,\varphi))^+}(q - c(1,1)) & h \neq f \\ 0 & h = f. \end{cases}$$

Sufficient Offers: $\sum_{g\in F\setminus\{f\}}(w^g - c(\varphi,\varphi))^+ > \frac{\delta}{1-\delta}(q - c(1,1))$**.** In this case, play enters the Bertrand reversion phase in the next period. In period, each non-leading firm h accepts its syndication offer if and only if $w^h \geq \bar{w}$, where $\bar{w} = c\Big(\sum_{g\in F\setminus\{f\}} \mathbb{1}_{\{w^g\geq\bar{w}\}}, \varphi\Big)$; i.e., each non-leading firm accepts or rejects its syndication offer so as to maximize its in-period payoff given the actions of other firms. Here, each firm accepts its syndication offer if and only if that offer is profitable within-period, as play enters the Bertrand reversion phase in the next period regardless of the firm's behavior.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

Figure 4 provides an automaton representation of the subgame-perfect Nash equilibrium described here.

It is immediate that the conjectured equilibrium delivers a price of $p^\star$ in each period. We now verify that the prescribed strategies constitute a subgame-perfect Nash equilibrium.

Responding to Syndication Offers

We first show that the prescribed actions regarding accepting or rejecting syndication offers are best responses. It is immediate that, after equilibrium play in either the cooperation phase or a collusive punishment phase, it is a best response for each non-leading firm to

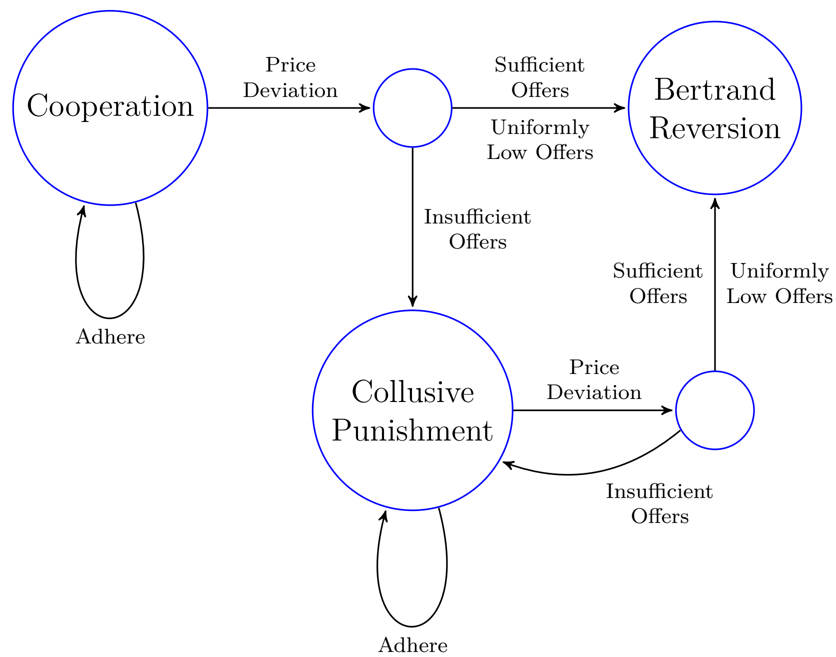

Figure 4: Automaton representation of the class of equilibria we consider. Labeled nodes are phases; unlabeled nodes are intermediate phases, which represent the branching of transitions based on behavior in the second step of the game.

accept its syndication offer.[20] It is also immediate that, in the case of uniformly low offers, it is a best response for each non-leading firm to reject its syndication offer.[21] Finally, it is immediate that, in the case of sufficient offers, each non-leading firm plays a best response; each non-leading firm only accepts its syndication offer if accepting provides a non-negative payoff in this period, and play continues to the Bertrand reversion phase regardless of the firm's actions.

To show that, in the case of insufficient offers, it is a best response for each non-leading firm to reject the offer of syndication, we calculate the total payoff for h from accepting the offer as

$$w^h - c\left(\frac{1}{2}, \varphi\right) < w^h - c(\varphi, \varphi),$$

as play reverts to the Bertrand reversion phase if h accepts the offer (even if other firms reject their syndication offers).[22] Meanwhile, the total payoff for h in the continuation game

[20]This follows as each syndication offer provides the firm with non-negative surplus and, if the firm rejects the syndication offer, play continues to the Bertrand reversion phase, in which the firm's future payoffs are 0.

[21]This follows as each syndication offer provides the firm with non-positive surplus and play continues to the Bertrand reversion phase regardless of the firm's actions.

[22]Note that, since the equilibrium calls for each firm to reject its offer of syndication, h expects that, if it accepts its offer of syndication, it will be the only firm to join the syndicate and thus will have production costs of $c\left(\frac{1}{2}, \varphi\right)$.

from rejecting the offer is

$$\begin{aligned}\frac{\delta}{1-\delta}\psi^h &= \frac{\delta}{1-\delta}\left(\frac{(w^h - c(\varphi,\varphi))^+}{\sum_{g\in F\smallsetminus\{f\}}(w^g - c(\varphi,\varphi))^+}(q - c(1,1))\right)\\ &\geq w^h - c(\varphi,\varphi),\end{aligned}$$

where the inequality follows from the fact that $\sum_{g\in F\smallsetminus\{f\}}(w^g - c(\varphi,\varphi))^+ \leq \frac{\delta}{1-\delta}(q - c(1,1))$, as we are in the insufficient offers case. Thus, it is a best response for every non-leading firm to rejects its syndication offer in the insufficient offers case.

Responding to Price Offers

It is immediate that each short-lived buyer b_t is acting optimally as b_t always chooses one of the lowest price offers less than or equal to its reservation price v.

Deviating on Price or Syndication Offers in the Collusive Punishment Phase

We begin by verifying that, during a collusive punishment phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most q from the buyer. Thus, firm f's profit in-period is at most $q - c(1,\varphi) \leq c(1,\varphi) - c(1,\varphi) = 0$ as $q = \min\{v, c(1,\varphi)\}$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most 0. On the other hand, firm f enjoys a continuation value $\psi^f \geq 0$ by not deviating; consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers.

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during a collusive punishment phase. Recall that sufficient offers require that the price deviator provide the non-leading firms with *dynamic compensation* totaling at least $\frac{\delta}{1-\delta}(q - c(1,1))$ above their costs of production. Thus, the in-period payoff to the deviating firm f is at most

$$\underbrace{q}_{\text{Price}} - \underbrace{c(1,1)}_{\substack{\text{Cost of production}\\ \text{when all firms participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,1))}_{\substack{\text{Dynamic compensation}\\ \text{to other firms}}} = \left(1 - \frac{\delta}{1-\delta}\right)(q - c(1,1)) \leq 0,$$

where the last inequality follows as $\delta \geq \frac{1}{2}$. In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to 0. By contrast, if firm f continues with equilibrium play, it receives a non-negative payoff. Thus, not deviating is a best response for firm f.

Deviating on Price or Syndication Offers in the Cooperation Phase

Finally, we verify that, during the cooperation phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most $p^\star$ from the buyer. Thus, firm f's profit in-period is at most $p^\star - c(1,\varphi)$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most $p^\star - c(1,\varphi)$. On the other hand, firm f enjoys profits each period of $\varphi(p^\star - c(1,1))$ by not deviating. Consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers so long as

$$\frac{1}{1-\delta}\varphi(p^\star - c(1,1)) \geq p^\star - c(1,\varphi),$$

which holds as $p^\star \leq \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$ by construction.

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during the cooperation phase. Recall that sufficient offers require that the price deviator provide the non-leading firms with dynamic compensation totaling at least $\frac{\delta}{1-\delta}(q - c(1,1))$ above their costs of production. Thus, the in-period payoff to the deviating firm f is at most

$$\underbrace{p^\star}_{\text{Price}} - \underbrace{c(1,1)}_{\substack{\text{Cost of production}\\ \text{when all firms participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,1))}_{\substack{\text{Dynamic compensation}\\ \text{to other firms}}}. \tag{1}$$

In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to that given by (4). By contrast, if firm f continues with equilibrium play, firm f enjoys profits each period of $\varphi(p^\star - c(1,1))$. Consequently, it is not profitable for f to deviate and make sufficient offers so long as

$$\frac{1}{1-\delta}\varphi(p^\star - c(1,1)) \geq p^\star - c(1,1) - \frac{\delta}{1-\delta}(q - c(1,1)),$$

which reduces to

$$p^\star \leq \frac{(1-\delta)c(1,1) + \delta(q - c(1,1)) - \varphi c(1,1)}{1-\delta-\varphi}.$$

There are now two cases to consider, depending on $q = \min\{c(1,\varphi), v\}$: In the first case, $q = c(1,\varphi)$. Thus, as $p^\star = \min\{\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}, v\} \leq \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$, it is not profitable for f to deviate by making sufficient offers so long as

$$\begin{aligned} \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi} &\leq \frac{(1-\delta)c(1,1)+\delta(c(1,\varphi)-c(1,1))-\varphi c(1,1)}{1-\delta-\varphi} \\ (1-\delta)c(1,\varphi) &\leq (1-\delta)c(1,1)+\delta(c(1,\varphi)-c(1,1)) \\ (2\delta-1)c(1,1) &\leq (2\delta-1)c(1,\varphi), \end{aligned}$$

which holds since $\delta \geq \frac{1}{2}$ and $c(1,1) < c(1,\varphi)$.

In the second case, $q = v$, which implies that $p^\star = v$.[23] Thus, it is not profitable for f to deviate by making sufficient offers so long as

$$\begin{aligned} v &\leq \frac{(1-\delta)c(1,1)+\delta(v-c(1,1))-\varphi c(1,1)}{1-\delta-\varphi} \\ (1-\delta-\varphi)v &\leq \delta v + (1-2\delta-\varphi)c(1,1) \\ (2\delta+\varphi-1)c(1,1) &\leq (2\delta+\varphi-1)v. \end{aligned}$$

This holds since $\delta \geq \frac{1}{2}$, $\varphi > 0$, and $v \geq c(1,1)$.

Thus, for $\delta \geq \frac{1}{2}$, $p^\star$ can be sustained.

Maximality of $p^\star$

It now remains to show that no price higher than $p^\star$ can be sustained. There are two cases to consider, depending on whether $p^\star = v$ or $p^\star = \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$: In the former case, no price greater than $p^\star = v$ can be sustained as no buyer will accept an offer higher than v.

In the latter case, suppose there existed an equilibrium in which the buyer accepted an offer of $p > p^\star$ each period. We show that at least one firm is not playing a best response: The total industry profits generated each period are at most $p - c(1,1)$, and so the total expected industry profits are at most $\frac{1}{1-\delta}(p - c(1,1))$. Thus, there must exist at least one firm f with total expected profits of at most $\frac{1}{1-\delta}\varphi(p-c(1,1))$. If firm f deviated by offering a price of $p - \epsilon$ and engaging in lone production, f's in-period profits approach $p - c(1,\varphi)$ as $\epsilon \to 0$. No matter the behavior of other firms in subsequent play, f can guarantee itself non-negative profits in each subsequent period.[24] Therefore, firm f has profits of deviating of at least

[23] Note that when $1-\delta-\varphi > 0$, we may calculate that $p^\star \geq q$, as $\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi} - c(1,\varphi) = \frac{\varphi(c(1,\varphi)-c(1,1))}{1-\delta-\varphi} > 0$, and so $\min\{\frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}, v\} - \min\{c(1,\varphi), v\} \geq 0$.

[24] For example, f could offer a price of $c(1,\varphi)$ and, if chosen by the buyer, offer a syndication fee of 0 to all other firms and, if not chosen, reject all syndication offers.

$p - c(1,\varphi) > \frac{1}{1-\delta}\varphi(p - c(1,1))$, its profits from not deviating, as $p > p^\star = \frac{(1-\delta)c(1,\varphi)-\varphi c(1,1)}{1-\delta-\varphi}$.

Behavior of $p^\star$

We now show that $p^\star$ is quasiconvex. In the region where $p^\star$ is less than v, we have that the second derivative of $p^\star$ with respect to φ is given by

$$\frac{\partial^2 p^\star}{\partial\varphi^2} = \frac{(1-\delta)\frac{\partial^2 c(1,\varphi)}{\partial\varphi^2}}{1-\delta-\varphi} + \frac{2}{1-\delta-\varphi}\underbrace{\frac{(1-\delta)\Big(c(1,\varphi) - c(1,1) + (1-\delta-\varphi)\frac{\partial c(1,\varphi)}{\partial\varphi}\Big)}{(1-\delta-\varphi)^2}}_{\frac{\partial p^\star}{\partial\varphi}},$$

which is positive at any critical point of $p^\star$: The first term is positive as the cost function is convex in its second argument and the second term must be 0 at any critical point. Thus, $p^\star$ is quasiconvex over the region where $p^\star < v$. It is then immediate that $p^\star$ is quasiconvex over its entire domain as it is the minimum of a quasiconvex function and a constant.

Finally, given that, for $\varphi < 1-\delta$,

$$p^\star = \min\left\{\frac{(1-\delta)c(1,\varphi) - \varphi c(1,1)}{1-\delta-\varphi}, v\right\}$$

it is immediate that $\lim_{\varphi\to 0} p^\star = v$ as $\lim_{\varphi\to 0} c(1,\varphi) = \infty$ by assumption.

4 Prices, Profits, and Capacity

We now consider the question of how the highest sustainable price and industry profits in an optimal collusion equilibrium vary as a function of the productive capacity k. In standard industrial organization models, industry profits are increasing in the productive efficiency of firms. However, in our setting, this is not necessarily the case: for a large class of cost functions, industry profits in an optimal collusion equilibrium are strictly *decreasing* in the productive capacity k.

Proposition 3. *If $c(s,\varphi) - c(s,1)$ is convex in s for all $\varphi \in (0, 1-\delta)$,*[25] *then the highest sustainable price $p^\star$ and industry profits in an optimal collusion equilibrium are decreasing in productive capacity k.*

Increasing the productive capacity affects the highest sustainable price, $p^\star$, through two channels: First, it lowers the cost of efficient joint production, making collusion more profitable. Second, it also lowers the cost of lone production, making price deviation and lone production

[25] For instance, all cost functions of the form $c(s,m) = s\left(\frac{s}{m}\right)^\alpha$, where $\alpha > 0$, satisfy this condition.

more profitable. Since the sustainability of collusion depends on the relative profitability of these two options—recall from our derivation of $p^\star$ in Section 3.3 that $p^\star$ is chosen so that price-deviating and then engaging in lone production is unprofitable—increasing capacity could potentially make collusion easier or harder to sustain. When the difference between the cost of lone production ($c(s, \varphi)$) and the cost of efficient joint production ($c(s, 1)$) is increasing and convex in the quantity produced s, the second effect dominates. This makes collusion harder to sustain and thus the highest sustainable price falls with productive capacity.

Intuitively, one might expect that increasing the productive capacity k would enhance industry profits in an optimal collusion equilibrium, as it lowers the cost of production $c(1, k)$. However, as described above, when the difference between the cost of lone production ($c(s, \varphi)$) and the cost of efficient joint production ($c(s, 1)$) is increasing and convex in the quantity produced s, the highest sustainable price $p^\star$ falls as productive capacity increases. Moreover, as productive capacity increases, the highest sustainable price (and thus industry revenues) drops faster than the cost of efficient production. Thus, industry profits decline as productive capacity increases.

5 Contracting over Production Shares

We now consider a model in which each syndication offer to a non-leading firm g specifies not only the fee that g will receive but also the share of production that g will complete. Under this form of contracting, in Step 3 of the extensive form stage game, the syndicate leader ℓ offers each other firm g a contract (s_t^g, w_t^g). If g accepts this syndication offer, it will receive a *fee* of w_t^g from ℓ (as before) and will produce a *production share* s_t^g. The stage game payoffs in this case (where, as before, the set of firms who accept the offer of syndication is denoted by G_t) are given by

1. The payoff for ℓ is $p_t^\ell - c\Big(1 - \sum_{g \in G_t \setminus \{\ell\}} s_t^g, \varphi k\Big) - \sum_{g \in G_t \setminus \{\ell\}} w_t^g$, i.e., the price paid by the buyer less both the cost of ℓ's production and the fees paid to other firms.

2. The payoff for $g \in G_t \setminus \{\ell\}$ is $w_t^g - c(s_t^g, \varphi k)$, i.e., the fee paid to g less the cost of g's production.

3. The payoff for $h \in F \setminus G_t$ is 0.

Surprisingly, the highest sustainable price in this game is the same as in the case described in Theorem 1, in which firms are unable to contract over production shares.

Theorem 2. *If syndication offers specify both production shares and fees, then for $\delta \geq \frac{1}{2}$, the highest sustainable price is given by $p^\star$, as defined in Theorem 1; moreover, $p^\star$ is quasiconvex in φ and $\lim_{\varphi \to 0} p^\star = v$.*

We give a full proof of Theorem 2 in Appendix B.3. To prove that $p^\star$ is sustainable when syndication offers specify production shares, we construct an equilibrium that sustains $p^\star$; this equilibrium is very similar to the one constructed in Section 3. In particular, the equilibrium has the same set of phases and the circumstances under which play transitions from one phase to another are comparable.

The sustainability of collusion depends on the relative profitability for each firm of colluding versus price-deviating and then engaging in lone production. Recall from our derivation of $p^\star$ in Section 3.3 that $p^\star$ is chosen so that price-deviating and then engaging in lone production is unprofitable. Because price-deviating and then engaging in lone production does not involve multi-firm syndicates, changing the contracting structure between syndicate leaders and non-leading firms does not affect $p^\star$ directly.

Changing the contracting structure does make recruiting syndicate members after a price deviation easier. Thus, one might worry that collusion might not be sustainable because a different type of deviation would become attractive: price-deviating and then building a syndicate. However, so long as $\delta \geq \frac{1}{2}$, it is still more costly for a price deviator to make sufficient offers (and thus recruit a syndicate) than to engage in lone production; see Appendix B.3 for details.

6 Heterogeneous Firms

We now extend the model of Section 5 to consider the case in which firms' productive capacities differ.[26] Thus, for each $f \in F$, let κ^f be the *productive capacity controlled by firm* f. It will be helpful to define $\kappa^{\max}$ as the largest share of productive capacity controlled by a single firm, i.e., $\kappa^{\max} \equiv \max_{f \in F}\{\kappa^f\}$. Moreover, the total productive capacity is given by $k = \sum_{f \in F} \kappa^f$.

6.1 Equilibrium Characterization

We now characterize the highest sustainable price as a function of the firms' productive capacities, which we denote $\hat{p}^\star(\kappa; \delta)$. To prove that $\hat{p}^\star(\kappa; \delta)$ is sustainable, we construct an

[26] Here, modeling syndication contracts as specifying both a fee and a production share is natural, since efficient production requires firms with different productive capacities to perform differing production shares.

equilibrium that sustains $\hat{p}^\star(\kappa;\delta)$; this equilibrium is very similar to the one constructed in Section 5.

In our constructed equilibrium, if a firm is small enough, it is allocated no surplus in the cooperation phase. This is because, if a firm is small enough, the highest sustainable price will be less than that firm's cost of production. Accordingly, it will not be profitable for that firm to price-deviate and then engage in lone production. Therefore, no surplus is needed to disincentivize this firm from price-deviating and then engaging in lone production. This frees up additional surplus that can be allocated to larger firms that *will* be tempted to price-deviate and then engage in lone production. We call firms that obtain positive surplus in an equilibrium supporting the highest sustainable price $\hat{p}^\star(\kappa;\delta)$ *collusion beneficiaries* and denote the set of collusion beneficiaries as $\hat{F}$.

To prevent a collusion beneficiary f from undercutting on price and engaging in lone production, f's profits from colluding must be large enough that f prefers to adhere to the equilibrium. Consider an equilibrium that sustains the price p and let r^f denote the fraction of surplus allocated to f. In an equilibrium, f must not be tempted to engage in lone production, so the following constraint must hold:

$$\frac{1}{1-\delta} r^f (p - c(1,k)) \geq p - c\Big(1,\kappa^f\Big). \tag{2}$$

Maximizing price subject to constraint (2) for each collusion beneficiary, along with the constraints that $r^f \geq 0$ for all firms and that $\sum_{f\in F} r^f = 1$, yields the highest sustainable price $\hat{p}^\star(\kappa;\delta)$, as expressed in Theorem 3.

Theorem 3. *If syndication offers specify both production shares and fees, firms may have heterogeneous production capacities, and* $c(1,\kappa^{\max}) \leq v$,[27] *then the highest sustainable price* $\hat{p}^\star(\kappa;\delta)$ *is given by the* $\hat{p}^\star(\kappa;\delta)$*-maximizing solution to*

$$\begin{aligned}
\hat{p}^\star(\kappa;\delta) &= \begin{cases} v & \varphi \in [1-\delta, 1] \\ \min\left\{\frac{(1-\delta)\hat{\varphi}(\kappa;\delta)\sum_{f\in\hat{F}} c\left(1,\kappa^f\right) - \hat{\varphi}(\kappa;\delta)c(1,k)}{1-\delta-\hat{\varphi}(\kappa;\delta)}, v\right\} & \varphi \in (0, 1-\delta), \end{cases} \\
\hat{F}(\kappa;\delta) &= \left\{f \in F : \hat{p}^\star(\kappa;\delta) \geq c\Big(1,\kappa^f\Big)\right\}, \\
\hat{\varphi}(\kappa;\delta) &= \frac{1}{|\hat{F}(\kappa;\delta)|},
\end{aligned}$$

so long as $\delta \geq \hat{\delta}(\kappa;\delta) \equiv \frac{\hat{p}^\star(\kappa;\delta) - c(1,k)}{\hat{p}^\star(\kappa;\delta) - c(1,k) + \min\{\hat{p}^\star(\kappa;\delta), c(1,\kappa^{\max})\} - c(1,k)} \in [\frac{1}{2}, 1)$.

We give a full proof of Theorem 3 in Appendix B.4. To prove that $\hat{p}^\star(\kappa;\delta)$ is sustainable

[27] When $c(1,\kappa^{\max}) > v$, the highest sustainable price is simply v; this corresponds to the case where price-deviating and engaging in lone production is not profitable in-period for any firm.

when firms are heterogeneous, we construct an equilibrium that sustains $\hat{p}^{\star}(\kappa;\delta)$; this equilibrium is very similar to the one constructed in Section 5. In particular, the equilibrium has the same set of phases and the circumstances under which play transitions from one phase to another are comparable.

In the cooperation phase of our constructed equilibrium, each firm submits a bid of $\hat{p}^{\star}(\kappa;\delta)$. However, the amount of surplus received by each firm now depends on that firm's productive capacity. Larger firms, i.e., firms with a larger productive capacity, receive a greater share of surplus, as the cost of lone production is lower for a larger firm. After a price deviation, if every non-leading firm rejects the price deviator's offer of syndication play enters a collusive punishment phase. The price in this collusive punishment phase is given by $\min\{c(1,\kappa^{\max}), v\}$. This ensures that no firm has an incentive to deviate and engage in lone production (as the cost of lone production will be no less than the price). Finally, there is also a Bertrand reversion phase, in which the price is the cost of efficient joint production $c(1,k)$. Play enters this stage whenever any firm deviates with respect to accepting or rejecting offers of syndication.

To understand how the highest sustainable price $\hat{p}^{\star}(\kappa;\delta)$ varies, note that the highest sustainable price depends on the average cost for lone production *among the collusion beneficiaries*, $\hat{\varphi}(\kappa;\delta)\sum_{f\in\hat{F}} c\Big(1,\kappa^{f}\Big)$. To see why this is the case, suppose the productive capacity of a collusion beneficiary f decreases, increasing f's cost of lone production; then constraint (2) slackens, and firm f could be allocated a smaller amount of surplus and still not be tempted to price-deviate and engage in lone production. Hence, we can reallocate some of firm f's profits to other firms, thereby making collusion relatively more attractive for these firms and thus raising the highest sustainable price. Since this is true for every collusion beneficiary, constraint (2), which depends on each collusion beneficiary's cost of lone production, must hold with equality for each collusion beneficiary. Summing constraint (2) across all the collusion beneficiaries, we can then derive the expression for the highest sustainable price given in Theorem 3.

The restriction on the discount factor $\hat{\delta}(\kappa;\delta)$ ensures that undercutting on price and recruiting a syndicate is not profitable—i.e., that the binding constraint on the highest sustainable price remains the profits available from price-deviating followed by lone production. The restriction on $\hat{\delta}(\kappa;\delta)$ is analogous to the $\frac{1}{2}$ threshold for δ when firms are symmetric.[28]

6.2 Effects of Heterogeneity

Using Theorem 3, we can now characterize the effects of a small degree of heterogeneity.

[28]The expression for $\hat{\delta}$ does not immediately reduce to $\frac{1}{2}$ in the case of symmetric firms, as the expression is derived allowing for the possibility that there is at least one firm obtaining no surplus.

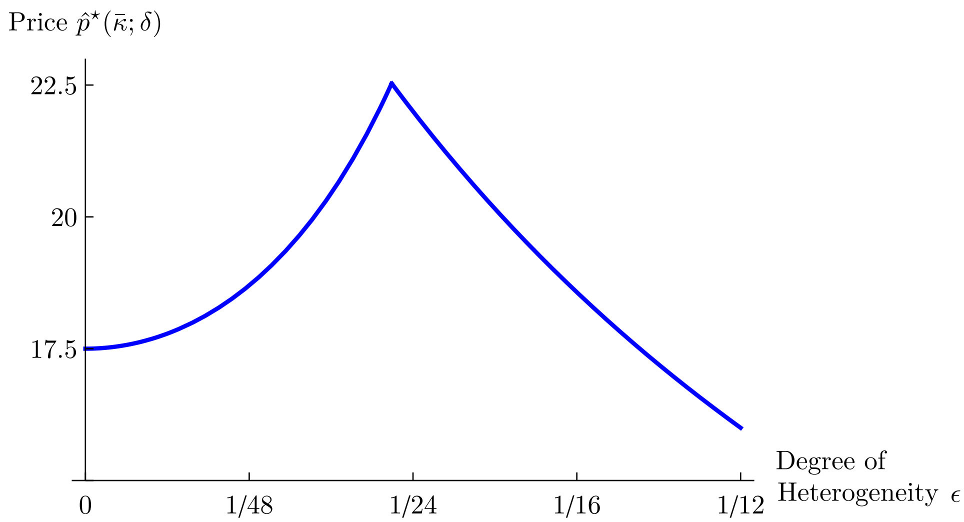

Figure 5: The highest sustainable price $\hat{p}^{\star}(\bar{\kappa};\delta)$ as a function of the degree of heterogeneity ϵ. Here, $c(s,m) = \frac{s^2}{m}$, $\delta = \frac{3}{4}$, and there are 12 firms; half of the firms have productive capacity $\frac{1}{12} + \epsilon$, and half of the firms have productive capacity $\frac{1}{12} - \epsilon$.

Proposition 4. *If syndication offers specify both production shares and fees, κ is given by $\kappa^f = (\varphi k)_{f \in F}$ for some k, $\delta > \hat{\delta}(\kappa;\delta)$, and $\hat{p}^{\star}(\kappa;\delta) < v$, then there exists an $\epsilon > 0$ such that, for every distribution of productive capacities $\bar{\kappa} \neq \kappa$ such that $|\bar{\kappa}^f - \kappa^f| < \epsilon$ for all $f \in F$, we have that*

$$\hat{p}^{\star}(\bar{\kappa};\delta) > \hat{p}^{\star}(\kappa;\delta).$$

To provide intuition for Proposition 4, consider the example illustrated in Figure 5. When the 12 firms are nearly homogenous, each firm is a collusion beneficiary, so that $\hat{F}(\bar{\kappa};\delta) = F$. Accordingly, by Theorem 3, the highest sustainable price is linearly increasing in the average cost of lone production across all firms, $\varphi \sum_{f \in F} c\Big(1, \bar{\kappa}^f\Big)$. Moreover, since the cost of lone production by a firm is convex in that firm's productive capacity, this sum is increasing in the degree of heterogeneity—the larger firms' production cost savings are smaller than the increased costs for the smaller firms, raising the average cost of lone production. However, when some firms are very small, their costs of lone production rise above the highest sustainable price. Hence, these firms are no longer a threat to price-deviate and engage in lone production, so they are allocated no surplus; they are no longer collusion beneficiaries and F no longer equals $\hat{F}(\bar{\kappa};\delta)$. The six larger firms now comprise the set of collusion beneficiaries $\hat{F}(\bar{\kappa};\delta)$. Thus, in Figure 5, the relevant average becomes the average cost of lone production across the six large firms. This average is decreasing in the degree of heterogeneity in this example, as additional heterogeneity increases each large firm's productive capacity. Thus, as the degree of heterogeneity increases above a certain point ($\epsilon \approx \frac{1}{24}$), the highest sustainable price is decreasing in the degree of heterogeneity.

6.3 Market Entry

We now consider the effect of entry by a small firm on the highest sustainable price. When a firm enters the market, there are three possible effects: First, it may become easier for a price deviator to form a syndicate, making collusion more difficult. However, when the discount factor is high enough, a price-deviator will find forming a syndicate more costly than engaging in lone production, so this effect does not affect the highest sustainable price. Second, the new entrant may itself price-deviate and engage in lone production; this may make collusion more difficult. But, for a small enough entrant, the cost of lone production is higher than the highest sustainable price when the entrant is not present,[29] and so the entrant will not price-deviate and engage in lone production. Third, the additional productive capacity of the entrant reduces the cost of joint production, which makes collusion at the current price more profitable. This last effect always has bite, and so entry by a small enough entrant raises the highest sustainable price.

Proposition 5. *If syndication offers specify both production shares and fees,* $\delta > \hat{\hat{\delta}}(\kappa;\delta)$, $\hat{p}^{\star}(\kappa;\delta) < v$, *and* $\lim_{m\to 0} c(s,m) = \infty$ *for all* $s > 0$, *then there exists an* $\epsilon > 0$ *such that entry by a firm* f *with productive capacity* $\kappa^f < \epsilon$ *will increase the highest sustainable price, i.e.,*

$$\hat{p}^{\star}((\kappa,\kappa^f);\delta) > \hat{p}^{\star}(\kappa;\delta).$$

Figure 6 depicts the highest sustainable price for a simple economy as a function of the size of the entrant. When no entrant is present, the highest sustainable price is 15; however, for small entrants, the highest sustainable price is (slightly) higher than 15. This happens because an entrant of sufficiently small capacity does not have the productive capacity to profitably undercut the collusive price and engage in lone production. Moreover, the entrant's capacity makes collusion more profitable for the incumbent firms, as it decreases the cost of joint production. This makes collusion relatively more attractive to the incumbent firms, compared to price-deviating and engaging in lone production. Thus, entry by a sufficiently small firm will facilitate collusion as opposed to hampering it.

However, for a sufficiently large entrant, collusion will become more difficult. An entrant with enough productive capacity can profitably undercut the collusive price by price-deviating and engaging in lone production; this occurs when κ^f becomes approximately $\frac{1}{16}$ in Figure 6. Thus, when the entrant has sufficient production capacity, some industry profits must be allocated to the entrant in order to make colluding a more rewarding option for the entrant than price-deviating and engaging in lone production. Allocating some profits to the entrant

[29] Similarly, if entrants are unable to bid but instead can only participate in the syndicate, the highest sustainable price will increase after entry.

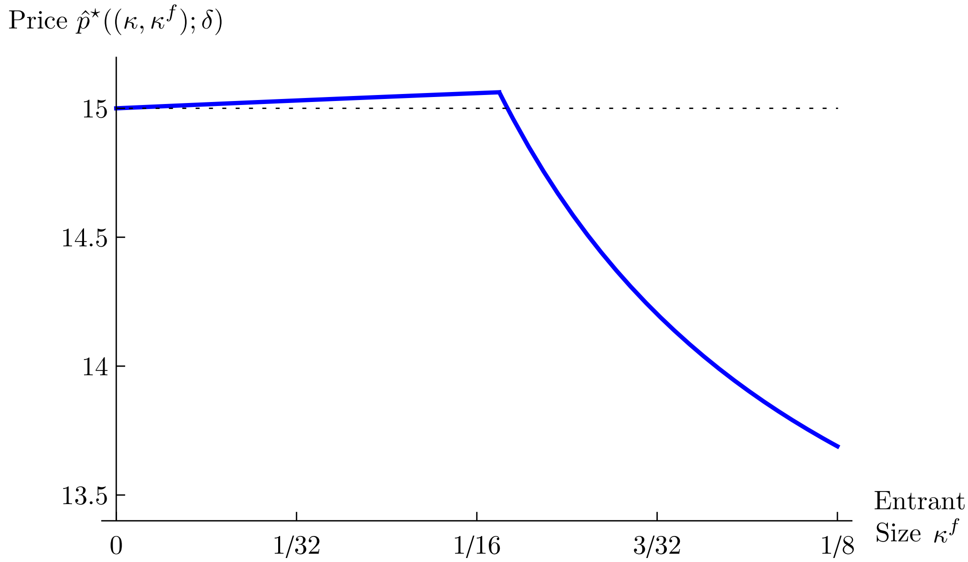

Figure 6: The highest sustainable price $\hat{p}^{\star}((\kappa, \kappa^f); \delta)$ as a function of entrant size κ^f. Here, $c(s, m) = \frac{s^2}{m}$, $\delta = \frac{3}{4}$, and there are 8 incumbent firms each with productive capacity $\frac{1}{8}$. The dashed line denotes the highest sustainable price without entry.

leaves fewer industry profits for the other firms, making collusion relatively less attractive to them. This makes collusion more difficult, reducing the highest sustainable price.

7 Conclusion

Our results show that, in markets with syndication, classical industrial organization intuitions are not always valid: Decreasing market concentration can *raise* prices, as it strengthens firms' ability to punish a deviator in-period by refusing offers of syndication.[30] Moreover, entry can also *raise* prices; a small entrant cannot credibly threaten to disrupt the collusive equilibrium, but does make collusion more profitable (and thus more attractive) to incumbent firms. Thus, our analysis suggests that some standard antitrust remedies—such as breaking up firms or facilitating entry—are of questionable use in thwarting collusion in markets with syndication.

Our analysis also adds to the ongoing scholarly debate on whether the IPO underwriting market is collusive and, if so, how collusion persists despite low market concentration in the industry. Our results offer potential insight into other features of the financial industry as well: For example, regulatory barriers routinely restrict participation in certain types of investments to investors that meet net worth or financial sophistication requirements. One

[30] Although here we work in a complete information environment, in ongoing work, we show that our conclusions are largely robust to relaxing our assumption that syndication offers are public.

might predict that the industry would oppose such restrictions, on the grounds that higher capacity (i.e., more investors) reduces the total cost of production. However, our work shows that increased capacity may reduce industry profits by making collusion more difficult. Our analysis thus suggests that the financial sector may actively support such restrictions, as they can facilitate collusion.

Finally, our work also highlights the importance of considering the full extensive form of firm interactions in industrial organization settings. Many industries are characterized by repeated, complex interactions that are best modeled as repeated extensive form games, such as IPO underwriting, debt origination, municipal auctions followed by horizontal subcontracting between bidders, and real estate transactions with agent selection. Further exploring repeated extensive form games is thus crucial to understanding subtle but important strategic interactions in these, and many other, markets.

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Appendix – For Online Publication

A Data

The data on IPOs used in Figures 1 and 2 comes from the Securities Data Company (SDC) database. Data are from 1976–2013. We make the usual exclusions, dropping real estate investment trusts (REITs), American depositary receipts (ADRs), and unit offerings, as in the work of Chen and Ritter (2000) and Kang and Lowery (2014). For Figure 1, we additionally drop any observation that is missing data for the "price at close of offer/1st trade" for all IPOs from December 1985 forward. Prior to December 1985, such data are not recorded, while after this date, IPOs with this field missing appear to frequently be duplicate entries or non-standard deals; this filter eliminates 1930 out of 11982 IPOs.

B Proofs

B.1 Proof of Proposition 2

We construct a subgame-perfect Nash equilibrium where every firm offers the monopoly price as follows:

- There are two phases of equilibrium play:

    1. In the *cooperation phase*:
        - Every firm submits the same bid $p = v$,
        - The buyer accepts the lowest price offer so long as one such offer is less than or equal to v. If there are multiple such offers, the buyer accepts each such offer with equal probability. If there are no such offers, the buyer rejects all the offers.
        - Every firm, if it becomes the syndicate leader, offers every other firm $c(\varphi, \varphi)$ to join the syndicate, and
        - Every other firm accepts this offer.
    2. In the *Bertrand reversion phase*, agents play the Bertrand reversion Nash equilibrium.

- Under equilibrium play, play continues in the same phase. If, in the cooperation phase, any firm f deviates in the first step or deviates with respect to the prescribed set of offers, then play proceeds to the Bertrand reversion phase. Moreover, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

It is immediate that along prescribed path of play every firm offers v for all t.

It is also immediate that play in the Bertrand reversion phase is subgame-perfect, as play is a subgame-perfect Nash equilibrium of the stage game (Proposition 1).

In the cooperation phase, an argument analogous to that used to prove Proposition 1 shows that offering $c(\varphi, \varphi)$ to each other firm minimizes the syndicate leader's production costs; moreover, only by offering $c(\varphi,\varphi)$ to each other firm can the syndicate leader possibly obtain positive profits in the future. Thus, offering $c(\varphi,\varphi)$ to each other firm is the optimal action by the syndicate leader during the cooperation phase.

It is immediate that the buyer is acting optimally given the price offers.

Finally, we consider whether any firm will wish to be a price deviator. The expected profits from the equilibrium strategy are given by

$$\frac{1}{1-\delta}\varphi(v-c(1,1)).$$

Again using an argument analogous to that used to prove Proposition 1, we have that offering $c(\varphi,\varphi)$ to each other firm minimizes the syndicate leader's production costs; thus, a price deviator's production costs are given by $c(1,1)$. Moreover, as we revert to Bertrand competition after a price deviation, profits in all future periods will be 0. Thus, the profits from deviating on price are bounded by

$$v-c(1,1).$$

Thus, so long as $\delta \geq 1-\varphi$, the strategies described here constitute a subgame-perfect Nash equilibrium.

B.2 Proof of Proposition 3

We first show that industry profits in the optimal collusion equilibrium are decreasing in k. It is easy to verify that price is now given by:

$$p^\star = \frac{(1-\delta)c(1,k\varphi)-\varphi c(1,k)}{1-\delta-\varphi}.$$

Industry profits per period are thus

$$\Pi \equiv \frac{(1-\delta)c(1,k\varphi)-\varphi c(1,k)}{1-\delta-\varphi} - c(1,k) = \frac{1-\delta}{1-\delta-\varphi}k\left(c\left(\frac{1}{k},\varphi\right)-c\left(\frac{1}{k},1\right)\right).$$

where the equality follows from the fact that the cost function is homogeneous of degree 1. Differentiating profits with respect to k, and then multiplying by $\frac{1-\delta-\varphi}{1-\delta}$ gives

$$\frac{1-\delta-\varphi}{1-\delta}\frac{\partial \Pi}{\partial k} = \left(c\left(\frac{1}{k},\varphi\right)-c\left(\frac{1}{k},1\right)\right) - \frac{1}{k}\left(c_s\left(\frac{1}{k},\varphi\right)-c_s\left(\frac{1}{k},1\right)\right)$$

Letting $g(x) = c(x, \varphi) - c(x, 1)$ and $x = \frac{1}{k}$, we have that

$$\begin{aligned}\frac{1-\delta-\varphi}{1-\delta}\frac{\partial \Pi}{\partial k} &= g(x) - xg'(x) \\ &= g(x) - g(0) - (x-0)g'(x) \\ &< 0,\end{aligned}$$

where the second equality follows from the from the fact that $c(0, y) = 0$ for all $y \geq 0$, and the inequality follows from the convexity assumption of the theorem.

Since both the cost of efficient joint production and industry profits in the optimal collusion equilibrium are decreasing in k, the highest sustainable price must be decreasing in k.

B.3 Proof of Theorem 2

To show that $p^\star$ is the highest sustainable price, we construct an equilibrium of the following form:[31]

- There are three phases of equilibrium play:
  1. In the *cooperation phase*,
     - every firm submits the same bid $p = p^\star$,
     - the short-lived buyer accepts one such offer of $p^\star$, choosing each offer with equal probability,
     - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi k)$ to every non-leading firm $g \in F \setminus \{\ell\}$ for agreeing to perform φ of production, and
     - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
  2. In the *collusive punishment phase with continuation values* ψ,
     - every firm submits the same bid $q = \min\{c(1, \varphi k), v\}$,
     - the short-lived buyer accepts one such offer of q, choosing each offer with equal probability,
     - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi, \varphi k) + \psi^g$ to every non-leading firm $g \in F \setminus \{\ell\}$ for agreeing to perform φ of production, and

[31] It is immediate that, when $\varphi \in [1-\delta, 1]$, we can sustain collusion exactly as in the proof of Theorem 1.

- every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.

3. In the *Bertrand reversion phase*, agents play the Bertrand reversion Nash equilibrium.[32]

- Under equilibrium play, play continues in the same phase. In the cooperation phase or a collusive punishment phase, some firm f may price-deviate in the first step, in which case the buyer accepts this offer, or deviate with respect to the prescribed set of syndication offers. If so, future play depends on the sum over the non-leading firms of the (positive) difference between the syndication fee w^g offered to each firm g and the cost to that firm of doing s^g of the project, $\sum_{g\in F\setminus\{f\}}(w^g - c(s^g, \varphi k))^+$. Based on this sum, we categorize the set of offers made by a deviating firm f into three cases: *uniformly low offers*, *insufficient offers*, and *sufficient offers*. Future play in each case is as follows:

Uniformly Low Offers: $\sum_{g\in F\setminus\{f\}}(w^g - c(s^g, \varphi k))^+ = 0$**.** In this case, rejecting the syndication offer is a best response for each non-leading firm, as the fee offered is weakly less than each non-leading firm's cost of production. Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{g\in F\setminus\{f\}}(w^g - c(s^g, \varphi))^+ \leq \frac{\delta}{1-\delta}(c(1, \varphi k) - c(1, k))$**.** In this case, absent dynamic rewards and punishments, some non-leading firms may be tempted to accept their syndication offers. All non-leading firms *do* reject their syndication offers and play proceeds going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{\left(w^h - c\left(s^h, \varphi k\right)\right)^+}{\sum_{g\in F\setminus\{f\}}(w^g - c(s^g, \varphi k))^+}(c(1, \varphi k) - c(1, k)) & h \neq f \\ 0 & h = f. \end{cases}$$

Sufficient Offers: $\sum_{g\in F\setminus\{f\}}(w^g - c(s^g, \varphi k))^+ > \frac{\delta}{1-\delta}(c(1, \varphi k) - c(1, k))$**.** In this case, play enters the Bertrand reversion phase in the next period; in period, each firm h accepts if and only if $w^h \geq c(s^g, \varphi k)$.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

The proof that this strategy profile is a subgame-perfect Nash equilibrium and that it

[32] Here, in the Bertrand reversion Nash equilibrium, the syndicate leader offers every other firm $c(\varphi, \varphi k)$ for agreeing to perform φ of the production.

attain the highest sustainable price of any subgame-perfect Nash equilibrium then follows as in the discussion following Theorem 1.

B.4 Proof of Theorem 3

To find $\hat{p}^{\star}(\kappa;\delta)$, we solve the problem

$$\max_{p,r}\{p\} \tag{3}$$

subject to the constraints

$$\begin{aligned} \frac{1}{1-\delta} r^f(p - c(1,k)) &\geq p - c\Big(1,\kappa^f\Big) && \text{for all } f \in F \\ r^f &\geq 0 && \text{for all } f \in F \\ \sum_{f\in F} r^f &= 1. \end{aligned}$$

We transform this problem by letting $\pi^f = r^f(p - c(1,k))$ be the continuation value for f from adhering to the equilibrium strategy in the cooperation phase, and so obtain the problem

$$\max_{\pi}\left\{\sum_{f\in F}\pi^f\right\}$$

subject to the constraints

$$\begin{aligned} \frac{1}{1-\delta}\pi^f &\geq \sum_{g\in F}\pi^g + c(1,k) - c\Big(1,\kappa^f\Big) && \text{for all } f \in F \\ \pi^f &\geq 0 && \text{for all } f \in F. \end{aligned}$$

The first constraint is the *no lone deviation constraint*. This is a convex optimization problem, and moreover it is immediate that it satisfies Slater's condition. Thus, by Theorem 7.16 of Sundaram (1996), there exists a vector of continuation payoffs π and Lagrangian multipliers λ and μ that satisfy the Kuhn-Tucker conditions, i.e., for all $f \in F$,

$$\begin{aligned} 1 + \frac{\delta}{1-\delta}\lambda^f - \sum_{g\in F}\lambda^g + \mu^f &= 0 \\ \lambda^f \geq 0 \text{ and } \lambda^f\left(\frac{1}{1-\delta}\pi^f - \sum_{g\in F}\pi^g - c(1,k) + c\Big(1,\kappa^f\Big)\right) &= 0 \\ \mu^f \geq 0 \text{ and } \mu^f\pi^f &= 0. \end{aligned}$$

Let the set of firms for which $\lambda^f \neq 0$ be denoted $\hat{F}(\kappa;\delta)$; thus, for each $f \in \hat{F}(\kappa;\delta)$, we have that

$$\frac{1}{1-\delta}\pi^f - \sum_{g\in F}\pi^g - c(1,k) + c\Big(1,\kappa^f\Big) = 0.$$

Summing over firms in $\hat{F}(\kappa;\delta)$, we obtain

$$\begin{aligned}
\frac{1}{1-\delta}\sum_{f\in\hat{F}(\kappa;\delta)}\pi^f &= \sum_{f\in\hat{F}(\kappa;\delta)}\left(\sum_{g\in F}\pi^g + c(1,k) - c\Big(1,\kappa^f\Big)\right)\\
\frac{1}{1-\delta}(p-c(1,k)) &= \sum_{f\in\hat{F}(\kappa;\delta)}\Big(p - c(1,k) + c(1,k) - c\Big(1,\kappa^f\Big)\Big)\\
p - c(1,k) &= (1-\delta)|\hat{F}|p - \sum_{f\in\hat{F}(\kappa;\delta)} c\Big(1,\kappa^f\Big)\\
p &= \frac{(1-\delta)\hat{\varphi}(\kappa;\delta)\sum_{f\in\hat{F}(\kappa;\delta)} c\Big(1,\kappa^f\Big) - \hat{\varphi}(\kappa;\delta)c(1,k)}{1-\delta-\hat{\varphi}(\kappa;\delta)}
\end{aligned}$$

where $\hat{\varphi}(\kappa;\delta) = \frac{1}{|\hat{F}(\kappa;\delta)|}$. Note that if $\lambda^f \neq 0$, then we can rewrite $\frac{1}{1-\delta}\pi^f - \sum_{g\in F}\pi^g - c(1,k) + c\Big(1,\kappa^f\Big) = 0$ as $\frac{1}{1-\delta}\pi^f = p - c\Big(1,\kappa^f\Big)$; thus, $\hat{F}(\kappa;\delta) = \Big\{f \in F : p \geq c\Big(1,\kappa^f\Big)\Big\}$.

To show that $\hat{p}^\star(\kappa;\delta)$ is the highest sustainable price, we construct an equilibrium as follows:[33]

- There are three phases of equilibrium play:
  1. In the *cooperation phase*,
     - every firm submits the same bid $p = \hat{p}^\star(\kappa;\delta)$,
     - the short-lived buyer accepts one such offer of $\hat{p}^\star(\kappa;\delta)$, choosing each offer with equal probability,
     - every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi^g,\kappa^g) + \pi^g$ to each non-leading firm ℓ for agreeing to perform φ^g of production, where $\varphi^g \equiv \frac{\kappa^g}{k}$, and
     - every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.
  2. In the *collusive punishment phase with continuation values* ψ,
     - every firm submits the same bid $q = \min\{c(1,\kappa^{\max}), \hat{p}^\star(\kappa;\delta)\}$,
     - the short-lived buyer accepts one such offer of q, choosing each offer with equal probability,

[33] It is immediate that, when $\varphi \in [1-\delta, 1]$, we can sustain collusion exactly as in the proof of Theorem 1.

- every firm, if it becomes the syndicate leader ℓ, offers a fee $c(\varphi^g, \kappa^g) + \psi^g$ to every non-leading firm $g \in F \smallsetminus \{\ell\}$ to join the syndicate, and
- every non-leading firm accepts the offer by the syndicate leader ℓ to join the syndicate.

3. In the *Bertrand reversion phase*, agents play the Bertrand reversion Nash equilibrium.[34]

- Under equilibrium play, play continues in the same phase. In the cooperation phase or a collusive punishment phase, some firm f may price-deviate in the first step, in which case the buyer accepts this offer, or deviate with respect to the prescribed set of syndication offers. If so, future play depends on the sum over the non-leading firms of the (positive) difference between the syndication fee w^g offered to each firm g and the cost to that firm of doing s^g of the project, $\sum_{g \in F \smallsetminus \{f\}} (w^g - c(s^g, \kappa^g))^+$.[35] Based on this sum, we categorize the set of offers made by a deviating firm f into three cases: *uniformly low offers*, *insufficient offers*, and *sufficient offers*. Future play in each case is as follows:

Uniformly Low Offers: $\sum_{g \in F \smallsetminus \{f\}} (w^g - c(s^g, \kappa^g))^+ = 0$. In this case, rejecting the syndication offer is a best response for each non-leading firm, as the fee offered is weakly less than each non-leading firm's cost of production. Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{g \in F \smallsetminus \{f\}} (w^g - c(s^g, \kappa^g))^+ \leq \frac{\delta}{1-\delta}(q - c(1, k))$. In this case, absent dynamic rewards and punishments, some non-leading firms may be tempted to accept their syndication offers. All non-leading firms *do* reject their syndication offers and play proceeds going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{\left(w^h - c\left(s^h, \kappa^h\right)\right)^+}{\sum_{g \in F \smallsetminus \{f\}} (w^g - c(s^g, \kappa^g))^+} (q - c(1, k)) & h \neq f \\ 0 & h = f. \end{cases}$$

Sufficient Offers: $\sum_{g \in F \smallsetminus \{f\}} (w^g - c(s^g, \kappa^g))^+ > \frac{\delta}{1-\delta}(q - c(1, k))$. In this case, play enters the Bertrand reversion phase in the next period; in period, each firm h accepts if and only if $w^h \geq c\left(s^h, \kappa^h\right)$.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

[34] Here, in the Bertrand reversion Nash equilibrium, the syndicate leader offers every other firm $c(\varphi^g, \kappa^g)$ for agreeing to perform φ^g of the production.

[35] Here, $(x)^+ \equiv \max\{0, x\}$.

It is immediate that the conjectured equilibrium delivers a price of $\hat{p}^\star(\kappa;\delta)$ in each period. We now verify that the prescribed strategies constitute a subgame-perfect Nash equilibrium.

Responding to Syndication Offers

We first show that the prescribed actions regarding accepting or rejecting syndication offers are best responses. It is immediate that, after equilibrium play in either the cooperation phase or a collusive punishment phase, it is a best response for each non-leading firm to accept its syndication offer.[36] It is also immediate that, in the case of uniformly low offers, it is a best response for each non-leading firm to reject its syndication offer.[37] Finally, it is immediate that, in the case of sufficient offers, each non-leading firm plays a best response; each non-leading firm only accepts its syndication offer if accepting provides a non-negative payoff in this period, and play continues to the Bertrand reversion phase regardless of the firm's actions.

To show that, in the case of insufficient offers, it is a best response for each non-leading firm to reject the offer of syndication, we calculate the total payoff for h from accepting the offer as

$$w^h - c\Big(s^h, \kappa^h\Big),$$

as play reverts to the Bertrand reversion phase if h accepts the offer (even if other firms reject their syndication offers). Meanwhile, the total payoff for h in the continuation game from rejecting the offer is

$$\begin{aligned}\frac{\delta}{1-\delta}\psi^h &= \frac{\delta}{1-\delta}\left(\frac{\Big(w^h - c\Big(s^h, \kappa^h\Big)\Big)^+}{\sum_{g\in F\smallsetminus\{f\}}(w^g - c(\varphi,\varphi))^+}(q - c(1,k))\right)\\ &\geq w^h - c\Big(s^h, \kappa^h\Big),\end{aligned}$$

where the inequality follows from the fact that $\sum_{g\in F\smallsetminus\{f\}}(w^g - c(s^g,\kappa^g))^+ \leq \frac{\delta}{1-\delta}(q - c(1,k))$, as we are in the insufficient offers case. Thus, it is a best response for every non-leading firm to rejects its syndication offer in the insufficient offers case.

Responding to Price Offers

It is immediate that each short-lived buyer b_t is acting optimally as b_t always chooses one of the lowest price offers less than or equal to its reservation price v.

[36] This follows as each syndication offer provides the firm with non-negative surplus and, if the firm rejects the syndication offer, play continues to the Bertrand reversion phase, in which the firm's future payoffs are 0.

[37] This follows as each syndication offer provides the firm with non-positive surplus and play continues to the Bertrand reversion phase regardless of the firm's actions.

Deviating on Price or Syndication Offers in the Collusive Punishment Phase

We begin by verifying that, during a collusive punishment phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most q from the buyer. Thus, firm f's profit in-period is at most $q - c\Big(1, \kappa^f\Big) \leq c(1, \kappa^{\max}) - c\Big(1, \kappa^f\Big) \leq 0$ as $q = \min\{v, c(1, \kappa^{\max})\}$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most 0. On the other hand, firm f enjoys a continuation value $\psi^f \geq 0$ by not deviating; consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers.

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during a collusive punishment phase. Recall that sufficient offers require that the price deviator provide the non-leading firms with *dynamic compensation* totaling at least $\frac{\delta}{1-\delta}(q - c(1,k))$ above their costs of production. Thus, the in-period payoff to the deviating firm f is at most

$$\underbrace{q}_{\text{Price}} - \underbrace{c(1,k)}_{\substack{\text{Cost of production} \\ \text{when all firms participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,k))}_{\substack{\text{Dynamic compensation} \\ \text{to other firms}}} = \left(1 - \frac{\delta}{1-\delta}\right)(q - c(1,k)) \leq 0,$$

where the last inequality follows as $\delta \geq \frac{1}{2}$. In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to 0. By contrast, if firm f continues with equilibrium play, it receives a non-negative payoff. Thus, not deviating is a best response for firm f.

Deviating on Price or Syndication Offers in the Cooperation Phase

Finally, we verify that, during the cooperation phase, no firm has an incentive to price-deviate or, if selected as the syndicate leader, not make the prescribed syndication offers. First, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes uniformly low or insufficient offers. No other firm will join f's syndicate, and f will receive a payment of at most $\hat{p}^{\star}(\kappa;\delta)$ from the buyer. Thus, firm f's profit in-period is at most $\hat{p}^{\star}(\kappa;\delta) - c\Big(1, \kappa^f\Big)$. Moreover, firm f's profits in every future period will be 0. Therefore, firm f's total profits from making uniformly low or insufficient offers are at most $\hat{p}^{\star}(\kappa;\delta) - c\Big(1, \kappa^f\Big)$. On the other hand, firm f enjoys profits each period of $r^f(\hat{p}^{\star}(\kappa;\delta) - c(1,k))$ by not deviating. Consequently, it is not profitable for f to deviate and make uniformly low or insufficient offers so long as

$$\frac{1}{1-\delta} r^f (\hat{p}^\star(\kappa;\delta) - c(1,k)) \geq p^\star - c\Big(1, \kappa^f\Big);$$

but this constraint is satisfied by the construction of $\hat{p}^\star(\kappa;\delta)$—see (3).

Second, consider the payoff to a deviating firm f that is selected as syndicate leader and then makes sufficient offers during the cooperation phase. Recall that sufficient offers require that the price deviator provide the non-leading firms with dynamic compensation totaling at least $\frac{\delta}{1-\delta}(q - c(1,k))$ above their costs of production. Thus, the in-period payoff to the deviating firm f is at most

$$\underbrace{p^\star}_{\text{Price}} - \underbrace{c(1,k)}_{\substack{\text{Cost of production}\\ \text{when all firms participate}}} - \underbrace{\frac{\delta}{1-\delta}(q - c(1,k))}_{\substack{\text{Dynamic compensation}\\ \text{to other firms}}}. \tag{4}$$

In future periods, play reverts to the Bertrand reversion Nash equilibrium, and so firm f's future payoffs will be 0. Thus, f's total payoff from deviating is less than or equal to that given by (4). By contrast, if firm f continues with equilibrium play, firm f enjoys profits each period of $r^f(\hat{p}^\star(\kappa;\delta) - c(1,1))$. Consequently, it is not profitable for f to deviate and make sufficient offers so long as

$$\frac{1}{1-\delta}\varphi(\hat{p}^\star(\kappa;\delta) - c(1,1)) \geq p^\star - c(1,1) - \frac{\delta}{1-\delta}(q - c(1,1)).$$

Note that, for a small enough firm f, we could have $r^f = 0$. Thus, we must have δ large enough to that

$$0 \geq \hat{p}^\star(\kappa;\delta) - c(1,k) - \frac{\delta}{1-\delta}(q - c(1,k)).$$

Thus, solving for δ, we have

$$\delta \geq \frac{\hat{p}^\star(\kappa;\delta) - c(1,k)}{(\hat{p}^\star(\kappa;\delta) - c(1,k)) + (q - c(1,k))},$$

which will be satisfied since $q = \min\{c(1, \kappa^{\max}, \hat{p}^\star)\}$.

Thus, for $\delta \geq \hat{\delta}(\kappa;\delta)$, $\hat{p}^\star(\kappa;\delta)$ can be sustained.

Maximality of $\hat{p}^\star(\kappa;\delta)$

It now remains to show that no price higher than $\hat{p}^\star(\kappa;\delta)$ can be sustained. There are two cases to consider, depending on whether $\hat{p}^\star(\kappa;\delta) = v$ or $\hat{p}^\star(\kappa;\delta) < v$: In the former case, no price greater than $\hat{p}^{\star}(\kappa;\delta) = v$ can be sustained as no buyer will accept an offer higher than v.

It is also immediate that we can not construct an equilibrium with a price higher than $\hat{p}^{\star}(\kappa;\delta) = \frac{(1-\delta)\hat{\varphi}(\kappa;\delta)\sum_{f\in\hat{F}(\kappa;\delta)} c\left(1,\kappa^f\right) - \hat{\varphi}(\kappa;\delta)c(1,k)}{1-\delta-\hat{\varphi}(\kappa;\delta)}$, since, by construction, under any such price some firm will have an incentive to slightly underprice and engage in lone production.

B.5 Proof of Proposition 4

First, note that $\hat{F}(\kappa;\delta) = F$ for all κ when ϵ is sufficiently small. Moreover, $\delta(\kappa;\delta)$ is continuous in κ, and so, for ϵ sufficiently small, we have that $\delta > \delta(\kappa;\delta)$ since $\delta > \delta((\varphi k)_{f\in F};\delta)$. If $\hat{p}^{\star}(\kappa;\delta) = v$, we are done, since $\hat{p}^{\star}((\varphi k)_{f\in F};\delta) < v$ by assumption. Thus, when $\hat{p}^{\star}(\kappa;\delta) < v$, we can write

$$\hat{p}^{\star}(\kappa;\delta) - \hat{p}^{\star}((\varphi k)_{f\in F};\delta) = (1-\delta)\hat{\varphi}(\kappa;\delta)\frac{\sum_{f\in\hat{F}} c\left(1,\kappa^f\right) - \sum_{f\in\hat{F}} c(1,\varphi k)}{1-\delta-\hat{\varphi}(\kappa;\delta)} > 0$$

where the inequality follows from the strict convexity of $c(s,m)$ with respect to m.

B.6 Proof of Proposition 5

Let ϵ be small enough so that $c(1,\epsilon) > v$. Note that such an ϵ must exist, as $c(1,\epsilon) \to \infty$ as $\epsilon \to 0$. Solving for the highest sustainable price when f is present, i.e., solving the problem given in (3), we obtain

$$\hat{p}^{\star}((\kappa,\kappa^f);\delta) = \min\left\{\frac{(1-\delta)\hat{\varphi}(\kappa;\delta)\sum_{f\in\hat{F}} c\left(1,\kappa^f\right) - \hat{\varphi}(\kappa;\delta)c\left(1,k+\kappa^f\right)}{1-\delta-\hat{\varphi}(\kappa;\delta)}, v\right\}.$$

Note that ϵ has been chosen to ensure that $f \notin \hat{F}$. Thus,

$$\hat{p}^{\star}((\kappa,\kappa^f);\delta) - \hat{p}^{\star}(\kappa;\delta) = \min\left\{\hat{\varphi}(\kappa;\delta)\frac{c(1,k) - c\left(1,k+\kappa^f\right)}{1-\delta-\hat{\varphi}(\kappa;\delta)}, v - \hat{p}^{\star}(\kappa;\delta)\right\} > 0.$$

Finally, note that $\hat{\delta}((\kappa,\kappa^f);\delta) \geq \hat{\delta}(\kappa;\delta)$ as $\hat{p}^{\star}((\kappa,\kappa^f);\delta) > \hat{p}^{\star}(\kappa;\delta)$.