Disclosure Rules in Contract Law

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Abstract

How does the prospect of sale affect the seller's incentive to investigate — to acquire socially valuable information about the asset? How do the disclosure rules of contract law influence the investigation decision? Shavell (1994) showed that, if sellers and buyers are symmetrically informed, at the pre-investigation stage, then a mandatory disclosure rule leads to a first-best outcome, and a voluntary disclosure rule leads to a suboptimal outcome. But in many real-world cases owners of assets have better information about their assets, even before they investigate. In such asymmetric information settings, we show, mandatory disclosure no longer attains a first-best outcome. And, under certain conditions, voluntary disclosure is the more efficient rule. We further enrich the analysis by introducing a third rule: the mandatory post-disclosure rule, which requires disclosure of material information, but only after the contract is concluded. We show that this rule can be more efficient than both voluntary disclosure and mandatory (pre-contract) disclosure.

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1 Introduction

1.1 Motivation

Consider the following example:

> *Example: Underground Water.* Having lived in a house for the past ten years, Seller suspects that there is underground water beneath the house, which might damage its foundations. (Namely, Seller knows that the probability that underground water exists beneath her house is larger than the average probability of underground water in the relevant area.) Seller could hire a surveyor and find out whether underground water in fact exists.
>
> (a) *Improvement.* If underground water is not found, Seller could improve the property. Specifically, Seller could construct an office building on the property. An office building can only be constructed on property with solid foundations — foundations that are not threatened by underground water.
>
> (b) *Remediation.* If underground water is found, Seller could invest in remediation — to fix any water damage and to avoid further damage.
>
> In the future, Seller may want to sell the house to Buyer. If and when Buyer becomes the owner of the house, Buyer can also make improvement or remediation investments.[1]

In such an example, it may be efficient for Seller to hire the surveyor. The information that the surveyor would provide — about the existence of underground water — is socially valuable (Hirshleifer 1971). It can be used to enhance the value of the property. Specifically, we distinguish between two types of value-enhancing investments — improvement investments and remediation investments. Improvement investments refer to investment opportunities that arise for high-value assets — in our example, when there is no underground water. (We call this the Improvement case.) Remediation investments refer to investment opportunities that arise for low-value assets — when underground water is found. (We call this the Remediation case.)

When the value of information — reflecting the value-enhancing investments that can only be made by an informed owner — exceeds the cost of acquiring the information, e.g., the cost of hiring a surveyor, it

[1] The example is based on cases that considered the duty of a seller (or subdivider) of real estate to disclose to a purchaser that the property has a history of underground water. See, e.g., Barnhouse v. City of Pinole, 133 Cal. App. 3d 171, 189 (1982); Buist v. C. Dudley De Velbiss Corp., 182 Cal. App. 2d 325 (1960). *See generally* Janet Fairchild, *Fraud Predicated on Vendor's Misrepresentation or Concealment of Danger or Possibility of Flooding or other Unfavorable Water Conditions*, 90 A.L.R.3d 568 (Originally published in 1979). The example, and the analysis in this paper, assumes that information acquisition is costly. In the example, this cost is the price that Seller would pay the surveyor. In the terminology of Kronman (1978), we are focusing on deliberately acquired information.

is socially desirable for the information to be acquired. Assume for the moment that Seller is in the best position to acquire the information and that Buyer is in the best position to invest. Will Seller acquire the information? Will she disclose the information to Buyer? (Assuming that the disclosure itself is costless and credible, e.g., delivery of the surveyor's report. Compare: Grossman 1981 and Milgrom 1981.)

The answers to these questions depend on the legal regime. In the United States, courts impose a duty to disclose material information prior to a sale, but the scope of this duty is subject to ongoing debate.[2] It is, therefore, important to study the efficiency properties of different disclosure rules. Should we adopt a mandatory disclosure (MD) rule, where Seller must disclose the surveyor's report (if such a report is obtained), or should we rather prefer a voluntary disclosure (VD) rule, where Seller can choose whether or not to disclose the surveyor's report?

The question of information acquisition prior to a sale is relevant for a broad range of economic applications, corresponding to the broad range of assets that are bought and sold — from land, to a used car, to a business, and more. The question, and the analysis in this paper, also applies, with appropriate adjustments, to the sale of services, e.g., when a property owner hires a contractor to build or improve a house and the owner can acquire information that affects the contractor's cost of performance.

In an important paper, Shavell (1994) compared the efficiency of MD and VD under the assumption that, prior to any investigation, Seller and Buyer are symmetrically informed, namely, they both know the average likelihood that underground water exists beneath the house. Indeed, this assumption of (pre-investigation) informational symmetry is common in the literature on the acquisition of information prior to sale (see Section 1.5 below). But, in many cases, information is asymmetric. Sellers of assets often know more about their assets than potential buyers. In our example, even before hiring the surveyor Seller's prior about the likelihood that underground water threatens the foundations was higher than the statistical average. We introduce information asymmetry into the conventional model and show how it alters standard results about the acquisition and disclosure of information.

[2] See Restatement (Second) Contracts § 161 cmt d. (1981) ("A seller of real or personal property is, for example, ordinarily expected to disclose a known latent defect of quality or title that is of such a character as would probably prevent the buyer from buying at the contract price"). See also Obde v. Schlemeyer 353 P. 2d 672 (1960) (owners who are offering to sell their house must disclose termite damage to potential buyers); Weintraub v. Krobatsch, 317 A.2d 68 (N.J. 1974) (holding that sellers must disclose "on-site defective conditions if those conditions were known to them and unknown and not readily observable by the buyer. Such conditions, for example, would include radon contamination and a polluted water supply"); Cooter and Ulen (2011), at pp. 360–361; Posner (2003), at p. 111.

1.2 Mandatory Disclosure vs. Voluntary Disclosure

Shavell (1994) showed that, with symmetric information, a mandatory disclosure regime provides Seller with optimal incentives to acquire information — to hire the surveyor. Namely, if Seller must disclose the surveyor's findings to Buyer, Seller will hire a surveyor when it is efficient to do so.[3] Asymmetric information qualifies this result. A duty to disclose no longer provides optimal incentives for sellers to collect information. In the Improvement case, it is socially optimal to investigate, when the expected value of the asset is above a certain threshold (namely, when the likelihood of underground water is below a certain threshold). MD leads to excessive investigation: Sellers of assets with an expected value below the optimal threshold will investigate, in order to avoid pooling with sellers of even worse assets. In the Remediation case, it is socially optimal to investigate, when the expected value of the asset is *below* a certain threshold (namely, when the likelihood of underground water is *above* a certain threshold). MD creates distorted incentives: Sellers with higher-value assets investigate in order to avoid pooling with sellers of lower-value assets, and sellers with lower-value assets do not investigate in order to pool with sellers of higher-value assets. MD induces investigation by sellers of high-value assets who should not be investigating; and it fails to induce investigation by sellers of low-value assets who should be investigating.

In the conventional, symmetric-information model, a mandatory disclosure (MD) rule provides first-best incentives. It is clearly better than the alternative rule, the voluntary disclosure (VD) rule, which induces excessive investigation (and only selective disclosure of information — when the surveyor reports good news).[4] With asymmetric information, the analysis is more subtle. MD no longer provides first-best incentives. VD still provides stronger incentives to investigate, but it is no longer clear whether this extra investigation decreases or increases social welfare.

In our basic model, MD emerges as the dominant rule. In the Improvement case, MD leads to excessive investigation and VD only exacerbates this problem. In the Remediation case, MD induces both excessive investigation and insufficient investigation — sellers who should not be investigating will investigate and sellers who should be investigating will not investigate. VD again leads to more investigation, which could have mitigated the insufficient investigation problem. But this extra investigation creates no social value. Indeed, in a VD regime (in the Remediation case) any investigation is socially wasteful, since only bad news can lead to value-enhancing investments and sellers will not voluntarily disclose bad news. Therefore, in

[3] This result relies on the assumption, in Shavell (1994), that Seller has all the bargaining power—an assumption that we retain (see Section 2.1 below). This assumption is relaxed in Schweizer (2017).

[4] VD does not induce complete revelation of information (as in, e.g., Grossman and Hart 1980, Grossman 1981, and Milgrom 1981), because information is costly to acquire. Compare: Shavell (1994).

both the Improvement case and the Remediation case, MD is more efficient than VD.

Moving beyond the basic model, we identify several important advantages of VD. Starting with the Improvement case, we show that the extra investigation induced by VD can be socially beneficial, when heterogeneous investigation costs are added to the basic model. (The basic model includes heterogeneity only with respect to the expected value of the asset.) Whereas in the basic model the only problem, in the Improvement case, was one of excessive investigation, we now also have a problem of inadequate investigation. Specifically, sellers with low-value assets will not investigate enough, as they seek to pool with sellers of high-value assets who fail to investigate because they face especially high investigation costs. VD reduces the inadequate investigation problem. With this new advantage, VD can be more efficient than MD. (Note that investigation is socially valuable, even in a VD regime: In the Improvement case, good news leads to value-enhancing investments and sellers will voluntarily disclose good news.)

Next consider the Remediation case. In the basic model, investigation is socially wasteful in a VD regime, because bad news is never disclosed and thus Buyer cannot make value-enhancing remediation investments. This result is driven by the assumption, in the basic model, that investment is only possible post-investigation (and post-disclosure). In some cases, however, remediation investment is possible even if Buyer does not know for sure whether the asset is of low or high value. For example, Buyer can decide to treat the entire property for termites, even if she does not know for sure whether the property is infested or which part of the property is infested. In a VD regime, a Buyer who faces a silent (non-disclosing) Seller may infer a sufficiently high probability that the asset is of low value — especially since with VD many sellers remain silent after their investigation revealed bad news — and proceed to invest in remediation. If so, investigation is no longer without value in a VD regime. Recall that, in the Remediation case, MD leads to excessive investigation by sellers of high-value assets and insufficient investigation by sellers of low-value assets. The added investigation under VD mitigates the insufficient investigation problem, but also exacerbates the excessive investigation problem. When VD induces enough extra investigation by sellers of low-value assets, VD can be more efficient than MD.

Finally, the basic model focuses on post-sale investment by Buyer. VD becomes more attractive, when we also allow for pre-sale investment by Seller. When the extra investigation induced by VD results in more pre-sale investment, VD can be more efficient than MD. To summarize, the basic model provides a useful benchmark for our analysis, but it is only a benchmark. Extending the analysis beyond the basic model — introducing heterogeneous investigation costs, investment without investigation and pre-sale investment — we show that VD can be more efficient than MD. This result stands in contrast with the conventional analysis

that unequivocally supports MD. It is the introduction of asymmetric information (even pre-investigation) that qualifies the dominance of MD.

1.3 Mandatory Post-Contract Disclosure

In addition to the two standard rules (MD and VD), we introduce a third rule — the mandatory post-contract disclosure (MPCD) rule. This new rule requires disclosure of material information, but only after the contract is concluded.[5] MPCD is a hybrid rule, providing incentives to investigate that are stronger than those provided by MD but weaker than those provided by VD. Therefore, returning to the basic model: In the Improvement case, MPCD is more efficient than VD, but less efficient than MD. (In the heterogeneous investigation costs extension, MPCD can be more efficient than MD and thus the most efficient rule overall.) In the Remediation case, MPCD is more efficient than VD, because it results in disclosure, albeit post-sale, of bad news that is kept secret in a VD regime. MPCD can also be more efficient than MD: MD leads to excessive investigation by sellers of high-value assets and insufficient investigation by sellers of low-value assets. The added investigation that MPCD generates exacerbates the former, but mitigates the latter. When MPCD induces enough extra investigation by sellers of low-value assets, MPCD can be more efficient than MD and thus the most efficient rule overall.

1.4 Reality Checks

Asymmetric information and warranties. We replace the conventional symmetric information assumption with an asymmetric information assumption. We believe that in many cases sellers have better information than potential buyers, even before any investigation. Our analysis applies where the seller's informational advantage is based on a non-observable (and non-verifiable) signal. Otherwise, an unraveling dynamic could restore informational symmetry.

Warranties are a well-known solution to the asymmetric information problem (Grossman 1981). A seller who knows that the probability of underground water in her property is below average could promise the buyer: "I'll pay you $X (or you can rescind the contract), if you ever find underground water." And, again, an unraveling dynamic could restore informational symmetry. In reality, however, the buyer's concern would not be limited to underground water, and the warranty would need to be more general: "I'll pay you $X (or

[5] The MPCD rule would need to come with several "technical" supplements. In particular, we would need a rule that denies enforcement to a contractual clause that purports to rescind the contract if Seller discloses bad news post-contract. Also, there is a concern that a buyer who received bad news post-contract would turn around and sell the property to another buyer. We could address this concern by reverting back to MD after the first sale (or if buyer resells within a specified period of time).

you can rescind the contract), if you ever find *anything wrong* with the asset." But such a broad warranty might interfere with the efficient allocation of risk between the parties. Indeed, a narrower warranty focused on underground water might also interfere with efficient risk allocation, since even a better-informed seller would have only incomplete information about the risk of underground water. Therefore, it seems unlikely that warranties will completely eliminate the asymmetric information problem. Indeed, there are many real-world examples where, for one reason or another, the contract did not include a warranty. (See also Section 6 below.)

Buyer's questions and the viability of VD. In a VD regime, what happens if the buyer asks a silent seller: "Did you investigate and get bad news?" If the law forces the seller to respond truthfully, then such a question transforms VD into MD. The viability of VD, therefore, requires a legal permission to lie in response to this question. If the law (through a VD rule) allows you to keep certain information private, then it cannot allow another person to extract that information by asking a simple question. This general observation applies to any analysis of VD rules, not only to the analysis in this paper (see Porat and Yadlin 2016).[6]

Investigation by Buyer. We focus on the seller's incentives to investigate, and how these incentives are affected by the legal regime. In many real-world cases, the buyer can also investigate (before deciding to purchase the asset). The implications of possible investigation by the buyer are briefly discussed in Section 6 below. It is important to emphasize that investigation by the buyer, when it occurs, is generally a poor substitute for disclosure by the seller (Lefcoe 2004).[7] (For an analysis, in a different setting, that focuses on investigation by the buyer — see Bilancini and Boncinelli 2016.)

1.5 Literature

The literature on acquisition and disclosure of information prior to sale begins with Farrell and Sobel (1983). Shavell (1994) builds on Farrell and Sobel (1983), adding the possibility that information has a social value. Another early contribution is Matthews and Postlewaite (1985) who focus on quality testing in product markets, but assume costless investigation (or "testing") and do not allow for value-enhancing investments. More recently, Polinsky and Shavell (2012) study firms' incentives to acquire information about product

[6] Compare: Faure-Grimaud et al (2009) who analyze a firm's decision to purchase a rating from a ratings agency and argue that the firm's ownership of the rating implies a right to keep it secret, specifically to keep secret the acquired information (the rating) and the fact that information was acquired. Faure-Grimaud et al (2009) note that rating agency contracts guarantee such secrecy.

[7] Since investigation by Buyer is relatively ineffective, the probability that Buyer's investigation will reveal the problem can be low. And if Buyer's investigation happens to reveal the problem, Seller can just wait for a second buyer who fails to investigate or conducts a failed investigation, or a third buyer, etc. The buyer who ends up purchasing the asset is, therefore, likely to be uninformed (as in the Winner's Curse).

risks; Dye (2017) transforms the binary MD v. VD policy choice into a continuous instrument by allowing different levels of damages for failure to disclose; and Schweizer (2017) analyzes the implications of different assumptions about the allocation of bargaining power between the information-acquiring party and the non-acquiring party. All of these models assume that pre-investigation the seller and buyer are symmetrically informed.

A related literature studies incentives to disclose information, under the assumption that initially, and without investigation, the seller is informed; and the buyer is not. This literature does not consider incentives to acquire information. Classic contributions to this literature include Grossman and Hart (1980), Grossman (1981) and Milgrom (1981). Excellent surveys of this important literature can be found in Daughety and Reinganum (2013, Section 3.B.2), Dranove and Jin (2010), and Bolton and Dewatripont (2005, chapter 5).[8]

Our paper seeks to combine these two strands in the literature. Like the first strand, we focus on the incentives to acquire information. And, like the second strand, we assume that initially (i.e., pre-investigation) the parties are asymmetrically informed. In addition, we extend the literature by distinguishing between improvement investments and remediation investments.[9] This distinction is particularly important when analyzing disclosure rules, since the incentive to voluntarily disclose good news, which gives rise to improvement investments, is quite different from the incentive to voluntarily disclose bad news, which gives rise to remediation investments.

Our paper is also related to work by Faure-Grimaud et al (2009) who study a firm's decision to purchase a rating from a ratings agency. The decision to purchase a rating is similar to the investigation decision in our model. Like us, Faure-Grimaud et al assume that the firm has private information (i.e., receives a private signal), before it decides whether to obtain a rating. But their focus is on deriving the optimal contract between the firm and the rating agency. More importantly, Faure-Grimaud et al assume that information does not have a direct social value (specifically, in their model the acquired information does not trigger value-enhancing investments).

The paper proceeds as follows: Section 2 lays out our framework of analysis. Section 3 derives and compares outcomes and welfare levels for the two standard rules, MD and VD. In the basic model, MD emerges as the more efficient rule. Section 4 considers several extensions that challenge MD's dominance: (1) Adding heterogeneity in investigation costs; (2) Allowing for investment without prior investigation; and (3) Allowing for investment by Seller. Section 5 introduces the new, MPCD rule and compares it to the two

[8] Both literatures generally assume that all buyers understand the disclosure (when it is made). But see Fishman and Hagerty (2003) who study a model where some buyers do not understand the disclosure.

[9] For example, Shavell (1994) focuses only on improvement investments (by assuming that the value of investment increases with the value of the asset).

standard rules. Section 6 offers concluding remarks, briefly discussing investigation by Buyer and liability rules (as compared to disclosure rules). Proofs are relegated to the Appendix.

2 Framework of Analysis

2.1 Setup

There are two parties: Seller and Buyer. There are five time periods: T=0, 1, 2, 3, and 4. The structure of the game is as follows:

T=0: Seller owns an asset. The asset has two possible values $\nu \in \{\nu_L, \nu_H\}$, where $\nu_H > \nu_L$.

T=1: Seller receives a non-verifiable signal that reveals, to Seller only, whether the asset is more likely to be a low-value asset or a high-value asset. Specifically, Seller receives a non-verifiable signal $\alpha \in [0,1]$, drawn from the distribution $F(\alpha)$. A type α seller, who received a signal $\alpha \in [0,1]$, knows that $Pr(\nu_H) = \alpha$ and $Pr(\nu_L) = 1 - \alpha$, and that the expected value of the asset is: $\bar{\nu}(\alpha) = \alpha\nu_H + (1-\alpha)\nu_L$. The seller's type is private information. The distribution function, $F(\alpha)$, is common knowledge. We can think of $F(\alpha)$ as the distribution of seller types, from which Seller is drawn. We assume a unit mass of sellers.

T=2: Seller decides whether to investigate. If Seller investigates, then Seller learns for sure whether the asset is a low-value asset or a high-value asset. Specifically, Seller can choose to investigate, namely, to invest k, a publicly known constant (e.g., the market price of hiring a surveyor), and obtain a verifiable signal $s \in \{L, H\}$. An investigation reveals the true value of the asset with certainty. If the actual value of the asset is $\nu = \nu_L$, then a seller who invests k will obtain a verifiable signal $s = L$. If the actual value of the asset is $\nu = \nu_H$, then a seller who invests k will obtain a verifiable signal $s = H$. Formally, $Pr(\nu_L|s = L) = 1$ (and, correspondingly, $Pr(\nu_H|s = L) = 0$) and $Pr(\nu_H|s = H) = 1$ (and, correspondingly, $Pr(\nu_L|s = H) = 0$). Seller's decision whether to investigate is private information (compare: Faure-Grimaud et al 2009).

T=3: Buyer appears and the parties negotiate a sale of the asset from Seller to Buyer. (Seller has no outside option.[10]) Before the negotiations commence, there is a disclosure stage: With MD, a seller who inves-

[10] This assumption (which follows Shavell 1994) prevents an unraveling that may occur, if sellers who are offered a low price

tigated at T=2 discloses the investigation results — the verifiable signal s — to Buyer. (In other words, with MD, the legal sanction for a failure to disclose is sufficiently high, such that an investigating Seller always discloses the investigation results.) With VD, a seller who investigated at T=2 decides whether to disclose the investigation results to Buyer. We assume that disclosure is truthful or verifiable (as in Grossman and Hart 1980, Grossman 1981, Milgrom 1981, and Shavel 1994). Namely, a disclosing seller can only disclose the actual results of the investigation; and a non-investigating seller cannot disclose anything and must remain silent. This assumption captures situations where the investigation produces verifiable results, such as a surveyor's report. After the disclosure stage, negotiations commence. Following Shavell (1994), we assume that Buyer pays what he believes to be the (expected) value of the asset. This assumption gives Seller all the bargaining power (and it corresponds, e.g., to a situation where Seller auctions the asset to two or more buyers). It also rules out the possibility that Seller would use the price to signal her private information.

T=4: Buyer decides whether to invest in the asset. We assume that knowing the true value of the asset increases social welfare, by enabling value-enhancing investments. These value-enhancing investments can take one of two forms:

1. Improvement investments: These investment opportunities arise uniquely for high-value assets. Specifically, if $\nu = \nu_H$, Buyer can increase the value of the asset from ν_H to $\nu_H + \Delta v$, at a cost of $x < \Delta v$. Let $\Delta\tilde{v} \equiv \Delta v - x$.

2. Remediation investments: These investment opportunities arise uniquely for low-value assets. Specifically, if $\nu = \nu_L$, Buyer can increase the value of the asset from ν_L to $\nu_L + \Delta v$, at a cost of $x < \Delta v$. Let $\Delta\tilde{v} \equiv \Delta v - x$. We assume that $\Delta v \leq \nu_H - \nu_L$, namely, remediation cannot increase the value of the asset above ν_H. (That possibility is left to improvement investments.)

We study the implications of these two investment opportunities separately. (In some applications, both types of investment opportunities may be present. It would be straightforward to extrapolate from our analysis to such scenarios.) We initially assume that investment is possible only if Buyer knows for sure, as a result of disclosure by a seller who decided to investigate, whether the asset is a low-value asset or a high-value asset. This assumption seems plausible in many cases. For example, one would need to know the source, and precise location, of the underground water problem, before it can be remediated. In an extension

can simply exit the market. This assumption is relaxed in Schweizer (2017).

(in Section 4), we allow for investment without precise knowledge of the asset's value.

In our framework, a sale (at T=3) occurs immediately after Seller investigates or after Seller decides not to investigate (at T=2), so that Seller does not have the opportunity to invest herself — in improvement or in remediation. We consider only post-sale investments by Buyer in order to focus on the effects of a sale, and of disclosure rules, on the investigation decision. In an extension (in Section 4), we allow for pre-sale investments by Seller.

2.2 The First-Best

We derive the first-best outcome and welfare level, as a benchmark for the analysis of the alternative disclosure rules (in the following sections). Consider the Improvement case first. For a type α seller, the expected value of the asset, without investigation, is: $\bar{\nu}(\alpha) = \alpha\nu_H + (1-\alpha)\nu_L$. The expected value, with investigation, is: $\alpha(\nu_H + \Delta\tilde{\nu}) + (1-\alpha)\nu_L = \bar{\nu}(\alpha) + \alpha\Delta\tilde{\nu}$, and the social value of information is $I^*(\alpha) = \alpha\Delta\tilde{\nu}$. The social value of information is increasing in $\alpha, dI^*(\alpha)/d\alpha > 0$, as a higher α implies a higher probability that investigation would lead to value-enhancing improvement investments.

Next consider the Remediation case: The expected value, without investigation, is $\bar{\nu}(\alpha)$ as in the Improvement case. For a type α seller, the expected value, with investigation, is $\alpha\nu_H + (1-\alpha)(\nu_L + \Delta\tilde{\nu}) = \bar{\nu}(\alpha) + (1-\alpha)\Delta\tilde{\nu}$, and the social value of information is: $I^*(\alpha) = (1-\alpha)\Delta\tilde{\nu}$. The social value of information is decreasing in $\alpha, dI^*(\alpha)/d\alpha < 0$, as a higher α implies a lower probability that investigation would lead to value-enhancing remediation investments.

In the first-best, sellers with $I^*(\alpha) > k$ investigate and sellers with $I^*(\alpha) \leq k$ do not investigate. Let $\hat{\alpha}^*$ denote the first-best investigation threshold, which is implicitly defined by $I^*(\hat{\alpha}^*) = k$. In the Improvement case, we have $\hat{\alpha}^* = k/\Delta\tilde{\nu}$. Sellers with $\alpha > \hat{\alpha}^*$ investigate; and sellers with $\alpha \leq \hat{\alpha}^*$ do not. In the Remediation case, we have: $\hat{\alpha}^* = 1 - k/\Delta\tilde{\nu}$. Sellers with $\alpha < \hat{\alpha}^*$ investigate; and sellers with $\alpha \geq \hat{\alpha}^*$ do not. In the first-best, a seller who investigates discloses the information to the buyer. In the Improvement case, the buyer invests x, if he learns that $\nu = \nu_H$. In the Remediation case, the buyer invests x, if he learns that $\nu = \nu_L$.

In the Improvement case the first-best social welfare level is:

$$W^* = E[\bar{\nu}(\alpha)] + \int_{\hat{\alpha}^*}^{1} (I^*(\alpha) - k) f(\alpha) d\alpha$$

And in the Remediation case, the first-best social welfare level is:

$$W^* = E[\bar{\nu}(\alpha)] + \int_0^{\hat{\alpha}^*} (I^*(\alpha) - k) f(\alpha) d\alpha$$

2.3 The Investigation Decision

A central decision in our model is the seller's decision whether to investigate. The different disclosure regimes produce different incentives to investigate, and these differences play an important role in determining the relative efficiency of the alternative regimes. To determine whether the seller will choose to investigate, we compare the payoff of an investigating seller to the payoff of a seller who chooses not to investigate. Let $\pi_I(\alpha)$ denote the expected payoff of a type α seller who chooses to investigate and let $\pi_{NI}(\alpha)$ denote the expected payoff of a type α seller who chooses not to investigate. The private value of information to the seller is thus given by $I(\alpha) = \pi_I(\alpha) - \pi_{NI}(\alpha)$. A seller with $I(\alpha) > k$ will investigate; a seller with $I(\alpha) \leq k$ will not.

Of course, $\pi_I(\alpha)$, $\pi_{NI}(\alpha)$ and $I(\alpha)$ depend on the disclosure rule (MD or VD) as detailed below. Yet, several general features are worth highlighting at this point. First, a non-investigating seller always remains silent. Let π_S denote the expected payoff of a silent seller and note that this payoff does not depend on the seller's type (since the buyer cannot distinguish between different types of silent sellers, they are all offered the same price). We thus have $\pi_{NI}(\alpha) = \pi_S$. This payoff, π_S, varies between the two rules — and so we have π_S^{MD} and π_S^{VD} — and these variations will prove critical to the analysis.

Next, consider the expected payoff of an investigating seller. With both rules, if the seller investigates and finds ν_H, she will disclose this information, and buyer will pay $\nu_H + \Delta\tilde{\nu}$ in the Improvement case and ν_H in the Remediation case. The difference between the two rules arises when the seller investigates and finds ν_L. With MD, the seller will disclose this information, and the buyer will pay ν_L in the Improvement case and $\nu_L + \Delta\tilde{\nu}$ in the Remediation case. Therefore, in the Improvement case, the expected profit of a type α seller who investigates is: $\pi_I^{MD}(\alpha) = \bar{\nu}(\alpha) + \alpha\Delta\tilde{\nu}$; and in the Remediation case, the expected profit of a type α who investigates is: $\pi_I^{MD}(\alpha) = \bar{\nu}(\alpha) + (1-\alpha)\Delta\tilde{\nu}$.

With VD, a seller who investigates and finds ν_L will remain silent and get π_S. (This is obviously true in the Improvement case, since $\pi_S^{VD} > \nu_L$. In the Remediation case, a seller who investigates and finds ν_L will remain silent, if $\pi_S^{VD} > \nu_L + \Delta\tilde{\nu}$. We assume that this condition holds; otherwise, VD would be equivalent to MD.) Therefore, in the Improvement case, the expected profit of a type α seller who investigates is $\pi_I^{VD}(\alpha) = \alpha(\nu_H + \Delta\tilde{\nu}) + (1-\alpha)\pi_S^{VD}$; and in the Remediation case, the expected profit of a type α seller

who investigates is $\pi_I^{VD}(\alpha) = \alpha\nu_H + (1-\alpha)\pi_S^{VD}$.

We can now calculate the value of information for each rule. With MD, in the Improvement case, we have: $I^{MD}(\alpha) = \pi_I^{MD}(\alpha) - \pi_{NI}^{MD}(\alpha) = \bar{\nu}(\alpha) + \alpha\Delta\tilde{\nu} - \pi_S^{MD}$. And in the Remediation case, we have: $I^{MD}(\alpha) = \pi_I^{MD}(\alpha) - \pi_{NI}^{MD}(\alpha) = \bar{\nu}(\alpha) + (1-\alpha)\Delta\tilde{\nu} - \pi_S^{MD}$. In both the Improvement case and the Remediation case, the benefit from investigation is higher for higher α sellers, which means that higher α sellers are more likely to investigate: $dI^{MD}(\alpha)/d\alpha > 0$.[11] Therefore, we can identify a threshold value, $\hat{\alpha}^{MD}$, implicitly defined by $I^{MD}(\hat{\alpha}^{MD}) = k$, such that sellers with $\alpha > \hat{\alpha}^{MD}$ investigate, and sellers with $\alpha \leq \hat{\alpha}^{MD}$ don't.

With VD, in the Improvement case, we have: $I^{VD}(\alpha) = \pi_I^{VD}(\alpha) - \pi_{NI}^{VD}(\alpha) = \alpha(\nu_H + \Delta\tilde{\nu} - \pi_S^{VD})$. And in the Remediation case, we have: $I^{VD}(\alpha) = \pi_I^{VD}(\alpha) - \pi_{NI}^{VD}(\alpha) = \alpha(\nu_H - \pi_S^{VD})$. In both the Improvement case and the Remediation case, the benefit from investigation is greater for higher α sellers, which means that higher α sellers are more likely to investigate: $dI^{VD}(\alpha)/d\alpha > 0$. Therefore, we can identify a threshold value, $\hat{\alpha}^{VD}$, implicitly defined by $I^{VD}(\hat{\alpha}^{VD}) = k$, such that sellers with $\alpha > \hat{\alpha}^{VD}$ investigate, and sellers with $\alpha \leq \hat{\alpha}^{VD}$ don't.

As noted above, the expected payoff of a silent seller, and how it varies between the two rules, plays a central role in the analysis. When facing a silent seller, the buyer infers the equilibrium "mix" of possible silent sellers. With MD, silent sellers are those sellers with $\alpha \leq \hat{\alpha}^{MD}$ who choose not to investigate. The number of silent sellers is $F(\hat{\alpha}^{MD})$, and the expected value of assets offered by silent sellers is $E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$. Therefore, the expected payoff of a silent seller is $\pi_S^{MD} = E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$.

With VD, there are two groups of silent sellers:

(i) Non-investigating sellers: Sellers with $\alpha \leq \hat{\alpha}^{VD}$. The number of non-investigating silent sellers is $F(\hat{\alpha}^{VD})$, and the expected value of assets offered by non-investigating silent sellers is $E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{VD}]$.

(ii) Investigating sellers who found ν_L: The number of investigating silent sellers is $\int_{\hat{\alpha}^{VD}}^{1}(1-\alpha)f(\alpha)d\alpha$, and the value of assets offered by non-investigating silent sellers is ν_L.

Aggregating across the two groups, the total number of silent sellers is: $S^{VD} \equiv F(\hat{\alpha}^{VD}) + \int_{\hat{\alpha}^{VD}}^{1}(1-\alpha)f(\alpha)d\alpha$. Let $\theta^{VD} = F(\hat{\alpha}^{VD})/S^{VD}$ denote the share of non-investigating sellers among all silent sellers. Let $1-\theta^{VD}$ denote the share of investigating sellers (who found ν_L) among all silent sellers. The expected payoff of a silent seller is $\pi_S^{VD} = \theta^{VD}E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{VD}] + (1-\theta^{VD})\nu_L$.

[11] This result is immediate in the Improvement case. In the Remediation case, the result follows from our assumption that $\Delta\nu \leq \nu_H - \nu_L$.

3 Outcomes and Welfare

We begin by separately analyzing each regime: mandatory disclosure in Section 3.1 and voluntary disclosure in Section 3.2. We then compare the two regimes in Section 3.3.

3.1 Mandatory Disclosure

We first consider the incentives to investigate. MD leads to excessive investigation by high α sellers and, in the Remediation case, also to insufficient investigation by low α sellers. More precisely -

Lemma 1

(a) *In the Improvement case, $\hat{\alpha}^{MD} < \hat{\alpha}^*$; MD induces inefficient investigation, by sellers with $\alpha \in (\hat{\alpha}^{MD}, \hat{\alpha}^*)$.*

(b) *In the Remediation case, $\hat{\alpha}^{MD} > \hat{\alpha}^*$; MD induces inefficient investigation, by sellers with $\alpha > \hat{\alpha}^{MD}$, and inefficiently fails to induce investigation by sellers with $\alpha < \hat{\alpha}^*$.*

In the Improvement case, it is socially optimal to investigate when $\alpha > \hat{\alpha}^*$, whereas in the MD regime sellers with $\alpha > \hat{\alpha}^{MD}$ choose to investigate (see Figure 1(a) below). By comparing the private value of information under MD, $I^{MD}(\alpha) = \alpha\Delta\tilde{\nu} + \bar{\nu}(\alpha) - \pi_S^{MD}$, to the social value of information, $I^*(\alpha) = \alpha\Delta\tilde{\nu}$, we find that $\hat{\alpha}^{MD} < \hat{\alpha}^*$, which implies excessive investigation. Improvement investments create social value in good realizations (when the asset is of high value). The social benefit derives from the investment value — the value from improving the asset, $\alpha\Delta\tilde{\nu}$. Private incentives to investigate are driven by the higher price that Seller can get, when the investigation reveals a good realization. This higher price reflects both the investment value, $\alpha\Delta\tilde{\nu}$, and the benefit from avoiding pooling with silent sellers, $\bar{\nu}(\alpha) - \pi_S^{MD}$. Because of this additional benefit, the private benefit from investigation always exceeds the social benefit and $\hat{\alpha}^{MD} < \hat{\alpha}^*$. Sellers with $\alpha \in (\hat{\alpha}^{MD}, \hat{\alpha}^*)$ will inefficiently decide to investigate, because they want to avoid pooling with the low-α, non-investigating sellers.

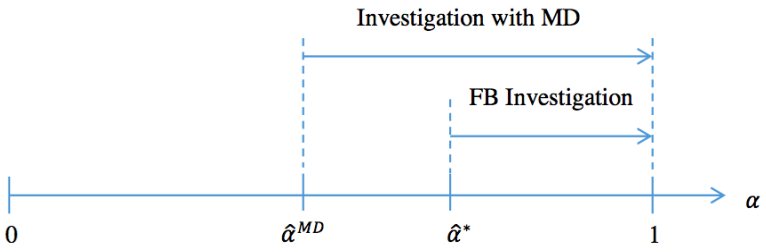

Figure 1(a): Investigation under MD — the Improvement Case

In the Remediation case, it is socially optimal to investigate when $\alpha < \hat{\alpha}^*$, whereas in the MD regime sellers with $\alpha > \hat{\alpha}^{MD}$ choose to investigate (see Figure 1(b) below). Sellers with $\alpha > \hat{\alpha}^{MD}$ decide to investigate, because they expect good news that will allow them to charge a higher price. Such investigation is inefficient, because the investigation cost exceeds the social benefit, which accrues only in the low-probability case where these high-α sellers get bad news that triggers remediation investment. In addition, sellers with $\alpha < \hat{\alpha}^*$ will inefficiently decide not to investigate, because they know that investigation will likely reveal bad news, which they would then have to disclose to Buyer. By not investigating, and remaining silent, these low-α sellers can pool with non-investigating sellers that have a higher α.

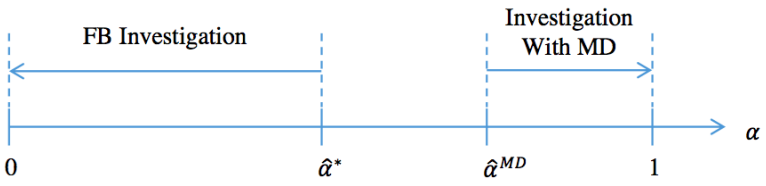

Figure 1(b): Investigation under MD — the Remediation Case

In both the Improvement case and the Remediation case, the social welfare level is

$$W^{MD} = E[\bar{\nu}(\alpha)] + \int_{\hat{\alpha}^{MD}}^{1} (I^*(\alpha) - k) f(\alpha) d\alpha$$

We can compare the welfare level with mandatory disclosure to the first-best welfare level. The comparison is summarized in the following proposition.

Proposition 1: In both the Improvement case and the Remediation case, Mandatory Disclosure does not achieve the first-best social welfare level: $W^{MD} < W^*$.

This inefficiency result contrasts with the efficiency result in the symmetric information case (Shavell 1994).

3.2 Voluntary Disclosure

We first consider the incentives to investigate. VD leads to excessive investigation. More precisely -

Lemma 2

(a) *In the Improvement case,* $\hat{\alpha}^{VD} < \hat{\alpha}^*$*; VD induces inefficient investigation, by sellers with* $\alpha \in (\hat{\alpha}^{VD}, \hat{\alpha}^*)$.

(b) *In the Remediation case, investigation does not result in value-enhancing investment and thus any investigation is socially wasteful. Specifically, the investigation by sellers with* $\alpha > \hat{\alpha}^{VD}$ *is socially wasteful.*

In the Improvement case, it is socially optimal to investigate when $\alpha > \hat{\alpha}^*$, whereas in the VD regime sellers with $\alpha > \hat{\alpha}^{VD}$ choose to investigate. We compare the private value of information under VD, $I^{VD}(\alpha) = \alpha(\nu_H + \Delta\tilde{\nu} - \pi_S^{VD})$, to the social value of information, $I^*(\alpha) = \alpha\Delta\tilde{\nu}$, and find that $\hat{\alpha}^{VD} < \hat{\alpha}^*$. Sellers with $\alpha \in (\hat{\alpha}^{VD}, \hat{\alpha}^*)$ will inefficiently decide to investigate, because they want to avoid pooling with the low-α, non-investigating sellers. In the Remediation case, it is socially optimal to investigate when $\alpha < \hat{\alpha}^*$, whereas in the VD regime sellers with $\alpha > \hat{\alpha}^{VD}$ choose to investigate. Indeed, in the Remediation case any investigation by Seller is socially wasteful, since an investigating seller will not disclose bad news — the only news that can trigger value-enhancing investments. (This result is qualified in Section 4.)

In the Improvement case, the social welfare level is:

$$W^{VD} = E[\bar{\nu}(\alpha)] + \int_{\hat{\alpha}^{VD}}^{1} (I^*(\alpha) - k) f(\alpha) d\alpha$$

In the Remediation case, the social welfare level is $W^{VD} = E[\bar{\nu}(\alpha)] - (1 - F(\hat{\alpha}^{VD}))k$. Comparing the welfare level with voluntary disclosure to the first-best welfare level, we obtain the following result.

Proposition 2: In both the Improvement case and the Remediation case, Voluntary Disclosure does not achieve the first-best social welfare level: $W^{VD} < W^*$**.**

3.3 Comparison: Mandatory Disclosure vs. Voluntary Disclosure

We obtain the intuitive result that the private value of information with voluntary disclosure is larger than the private value of information with mandatory disclosure:

Lemma 3: In both the Improvement case and the Remediation case, $\hat{\alpha}^{VD} < \hat{\alpha}^{MD}$.

Multiple equilibria. The games defined by both MD and VD, in both the Improvement case and the Remediation case, can have multiple equilibria. For example, consider the MD rule and the Improvement case. There can be a "high-$\hat{\alpha}^{MD}$ equilibrium," where parties believe that only a few sellers, with very high α, investigate; and thus the expected profit of a non-investigating seller π_S^{MD}, is high. A high π_S^{MD}, in turn, implies a low $I^{MD}(\alpha)$ (recall that $I^{MD}(\alpha) = \bar{\nu}(\alpha) + \alpha\Delta\tilde{\nu} - \pi_S^{MD}$), confirming parties' beliefs. There can also be a "low-$\hat{\alpha}^{MD}$ equilibrium," where parties believe that many sellers investigate and only sellers with very low α do not investigate; and thus the expected profit of a non-investigating seller, π_S^{MD}, is low. A low π_S^{MD}, in turn, implies a high $I^{MD}(\alpha)$, confirming parties' beliefs.

In theory, it is possible to select a "high-$\hat{\alpha}^{VD}$ equilibrium" in the VD game, and a "low-$\hat{\alpha}^{MD}$ equilibrium" in the MD game, such that $\hat{\alpha}^{VD} > \hat{\alpha}^{MD}$. We rule out this possibility by focusing on comparisons between corresponding equilibria, e.g. between a high-$\hat{\alpha}^{VD}$ equilibrium and a high-$\hat{\alpha}^{MD}$ equilibrium. This equilibrium selection can be justified through the following dynamic reasoning: Assume an MD equilibrium (either a high-$\hat{\alpha}^{MD}$ equilibrium or a low-$\hat{\alpha}^{MD}$ equilibrium), with an equilibrium number of sellers who decided to investigate, and where parties hold equilibrium expectations about this number. Now consider new legislation that replaces the MD rule with a VD rule. The next seller who makes a decision whether or not to investigate will have a negligible effect on the overall share of investigating sellers and thus on parties' beliefs about this share. Taking these beliefs as given, the new VD rule will necessarily increase the seller's incentive to investigate. Similar reasoning applies to the second seller (after the rule change) who decides whether or not to investigate, and to the third, and so on. Over time, the share of sellers who decide to investigate, and expectations about this share, change, until a new VD equilibrium is reached.

We next compare welfare levels. In the Improvement case, we get excessive investigation with MD, and even more excessive investigation with VD. Therefore, MD is more efficient than VD. In the Remediation case, we again get more costly investigation with VD. Moreover, while investigation generates social benefit under MD, it is a complete waste under VD. Therefore, MD is more efficient than VD also in the Remediation case. These results are summarized in the following proposition.

Proposition 3: In both the Remediation case and the Improvement case, MD is more efficient than VD.

Prior work has shown that MD is more efficient than VD, when Seller and Buyer are symmetrically informed pre-investigation (Shavell 1994). In the symmetric information case, MD induces the first-best welfare level and is, therefore, clearly more efficient than VD. In the asymmetric information case, MD no longer achieves the first-best (see Proposition 1 above). Still, in the basic model, MD is more efficient than VD. Below we consider several extensions to the basic model, where VD can be more efficient than MD.

4 Extensions: The Advantages of Voluntary Disclosure[12]

4.1 Heterogeneous Investigation Costs

The basic model focused on heterogeneity in asset type (i.e., on the α dimension). Another possible source of heterogeneity is the cost of investigation, k. In our motivating example, the investigation cost was the market rate of a surveyor, which is relatively constant across all sellers, at least if all sellers have access to the same, competitive market for surveyors. In other scenarios, investigation costs can vary across sellers. Even in the surveyor example, when the market for surveyors is less competitive and price dispersion is greater, different sellers, with different search costs, can be more or less able to find a surveyor who charges a low price. Therefore, instead of a constant investigation cost, k, common to all sellers, we now assume that Seller's cost of investigation is drawn from the distribution $G(k)$. (In this extension, as in the basic model, we assume that Seller's investigation decision is not observable to Buyer. We also assume, in this extension, that Seller's investigation cost is not observable to Buyer.)

Adding this second source of heterogeneity creates a potential advantage for VD in the Improvement case. For a type-α seller, the private value of information is $I^{MD}(\alpha)$ in a MD regime and $I^{VD}(\alpha)$ in a VD regime, as in the basic model; and we still get $I^{VD}(\alpha) > I^{MD}(\alpha)$ for all α. The difference is that, now, a higher $I^{MD}(\alpha)$ or $I^{VD}(\alpha)$ does not immediately translate into more investigation. In the basic model (with the constant k), if a seller with a lower $I^{MD}(\alpha)$ or $I^{VD}(\alpha)$ chooses to investigate, then necessarily a seller with a higher $I^{MD}(\alpha)$ or $I^{VD}(\alpha)$ chooses to investigate. Now, when investigation costs are drawn from a distribution $G(k)$, a seller with a lower $I^{MD}(\alpha)$ or $I^{VD}(\alpha)$ may draw a low k and choose to investigate,

[12] See Bar-Gill and Porat (2017) for a more comprehensive analysis of these extensions.

whereas a seller with a higher $I^{MD}(\alpha)$ or $I^{VD}(\alpha)$ may draw a high k and choose not to investigate.

This setup creates a new problem for MD: Whereas in the basic model (in the Improvement case) the only problem was one of excessive investigation, we now also have a problem of inadequate investigation. Specifically, sellers with low-value assets, i.e., low-α sellers, will not investigate enough. The reason is that these sellers will want to pool with the subset of high-α sellers who fail to investigate because they face especially high investigation costs. VD reduces this inadequate investigation problem, by increasing the private value of information: $I^{VD}(\alpha) > I^{MD}(\alpha)$. Because of this new advantage, VD can be more efficient than MD.

4.2 Investment without Investigation

So far we have assumed that value-enhancing investment — in both the Improvement case and the Remediation case — can only occur post-investigation. In some cases, however, it may be possible to invest without prior-investigation. For example, if the probability of a termite infestation is large enough, an owner could treat the entire property for termites without a prior-investigation that establishes for sure the existence, or absence, of termites and pinpoints the source of the problem.

The possibility of investment without investigation can change the first-best outcome. In particular, when the investment yield, $\Delta\nu/x$, is sufficiently high and investigation costs are sufficiently large, it would be socially desirable to invest without prior investigation. In this case, the social value of information derives from the avoidance of unnecessary investment. In the Improvement case, we get $I^*(\alpha) = (1-\alpha)x$. And in the Remediation case, we get $I^*(\alpha) = \alpha x$.

The possibility of investment without investigation can also change the payoff of a silent seller, since buyers facing a silent seller may decide to invest (without investigating). In particular, when the investment yield, $\Delta\nu/x$, is sufficiently large, a buyer who faces a silent seller can increase his payoff by investing. This also means that the buyer will offer a higher price to the silent seller. And when the payoff of a silent seller increases, the private value of information decreases. While both the social value of information and the private value of information may change when it is possible to invest without investigating, the results derived in Section 3 continue to hold in the Improvement case.

Not so in the Remediation case. Consider a scenario where the investment yield is not sufficiently high to affect the social value of information or to justify investment without investigation under MD, but high enough to justify investment without investigation under VD. Recall that, under VD, the group of silent sellers could contain many investigating sellers who received bad news, and so it is more likely that

the buyer would choose to invest. Such investment can increase welfare under VD. (In the basic model, investigation was socially useless with VD, because the bad news was never disclosed to Buyer. Now, investigation can lead to "actionable" silence and so its social value increases.) Social welfare increases from $W^{VD} = E[\bar{\nu}(\alpha)] - (1 - F(\hat{\alpha}^{VD}))k$ in the basic model to:

$$W^{VD} = E[\bar{\nu}(\alpha)] + \int_{\hat{\alpha}^{VD}}^{1} (I^*(\alpha) - k) f(\alpha) d\alpha$$

And this higher welfare level can exceed the welfare level in an MD regime: When $\hat{\alpha}^{VD} > \hat{\alpha}^*$, VD is less efficient than MD, as it simply induces more inefficient investigation. But when $\hat{\alpha}^{VD} < \hat{\alpha}^*$, VD can be more efficient than MD, as it induces some efficient investigation by sellers with $\alpha \in (\hat{\alpha}^{VD}, \hat{\alpha}^*)$.

4.3 Investment by the Seller (and Pre-Sale Value)

To focus on the effects of a sale, and of disclosure rules, the basic model assumed that sale follows immediately after Seller investigates or chooses not to investigate. This assumption allowed us to abstract from the possibility of investment by Seller and focus on investment by Buyer. A more general model would consider scenarios where Seller holds on to the asset for a period of time before selling it and allow for investment by Seller pre-sale as well as by Buyer post-sale.

Starting with the social optimum, if the investment — by either improving or remediating the asset – confers value on both Seller and Buyer, then an earlier investment by Seller is more efficient than a later investment by Buyer. The social value of information, from the basic model, applies in this extension, with the adjustment that Seller, rather than Buyer, invests based on the acquired information. (In the extension, an investment by Seller generates the full social value; in the basic model, an investment by Buyer generated the full social value.)

Now consider MD. An investigating Seller will make optimal investment decisions, since she internalizes the full benefit from her investment — she directly enjoys the benefit in the pre-sale period and she gets compensated, through a higher price, for the benefit that Buyer enjoys in the post-sale period. Indeed, the investment decisions of an investigating Seller are identical to the investment decisions of Buyer in the basic model, after Seller disclosed the results of her investigation (as required under the MD rule); and Buyer will never invest in this extension. As in the basic model, MD continues to induce excessive investigation. This distortion becomes smaller as the pre-sale period gets longer.

In the VD regime, we must distinguish between the Improvement case and the Remediation case. In the

Improvement case, the investment decisions of an investigating Seller are identical to the investment decisions of Buyer in the basic model after Seller voluntarily disclosed the results of her investigation. Therefore, the analysis of the investigation decision is qualitatively similar to the analysis in the basic model: VD continues to induce excessive investigation, albeit with a reduction in the magnitude of this distortion as the pre-sale period gets longer. Most importantly, the comparison between VD and MD remains as in the basic model.

In the Remediation case, the analysis, under VD, does change, when we allow for investment by Seller (and for pre-sale value). The investment decisions of an investigating Seller depend on the observability of the investment. If the investment is observable to a potential buyer, then a decision to invest also implies a decision to disclose; and VD becomes MD. If the investment is not observable to a potential buyer, then VD induces more investigation than MD, as in the basic model. But, unlike in the basic model, this extra investigation is socially valuable, as it increases the pre-sale value of the asset. This effect increases the relative efficiency of VD, as compared to MD.[13]

4.4 Taking Stock

The extensions discussed in Section 4 reveal several advantages of VD relative to MD. Some of these advantages are relevant in the Improvement case (Section 4.1) and others in the Remediation case (Sections 4.2 and 4.3). We must, therefore, revise proposition 3:

Proposition 3a: In both the Improvement case and the Remediation case, either MD or VD can be the most efficient rule.

5 Mandatory Post-Contract Disclosure

We have thus far focused on the two standard rules: mandatory disclosure (MD) and voluntary disclosure (VD). We now introduce a third rule: mandatory post-contract disclosure (MPCD). MPCD allows the seller to (voluntarily) choose whether to disclose the results of an investigation pre-contract but, if the seller chose not to disclose pre-contract, the rule requires that she disclose post-contract. We show that this new rule can outperform the two standard rules.

We return to the basic model. Our framework of analysis (Section 2) can be readily extended to include

[13] Even if the investments are not observable, sellers will invest suboptimally, since they do not enjoy the full benefit from the investment — they enjoy the benefit from using a remediated asset pre-sale, but they do not capture the benefit that buyers enjoy from using the remediated asset post-sale.

MPCD. Let π_S^{MPCD} denote the payoff of a silent seller in the MPCD regime. The no-investigation payoff is $\pi_{NI}^{MPCD}(\alpha) = \pi_S^{MPCD}$. Next consider the investigation payoff. In the Improvement case, the expected profit of a type α seller who investigates is $\pi_I^{MPCD}(\alpha) = \alpha(\nu_H + \Delta\tilde{\nu}) + (1-\alpha)\pi_S^{MPCD}$. In the Remediation case, the expected profit of a type α seller who investigates is $\pi_I^{MPCD}(\alpha) = \alpha\nu_H + (1-\alpha)\pi_S^{MPCD}$. The $\pi_I^{MPCD}(\alpha)$ functions are similar to those derived for VD, subject to the different π_S, since in both regimes a seller who investigates and finds ν_L will remain silent and get π_S. We can now calculate the private value of information. In the Improvement case, we have: $I^{MPCD}(\alpha) = \pi_I^{MPCD}(\alpha) - \pi_{NI}^{MPCD}(\alpha) = \alpha(\nu_H + \Delta\tilde{\nu} - \pi_S^{MPCD})$. And in the Remediation case, we have: $I^{MPCD}(\alpha) = \pi_I^{MPCD}(\alpha) - \pi_{NI}^{MPCD}(\alpha) = \alpha(\nu_H - \pi_S^{MPCD})$.

As with MD and VD, the expected payoff of a silent seller plays a central role in the analysis. This expected value depends on the equilibrium "mix" of silent sellers. Let θ^{MPCD}, defined as in Section 2.3, denote the share of non-investigating sellers among all silent sellers. And let $1 - \theta^{MPCD}$ denote the share of investigating sellers (who found ν_L) among all silent sellers. An investigating seller who gets good news will disclose pre-contract. An investigating seller who gets bad news will only disclose post-contract. In the Improvement case, the buyer will invest only if the seller reveals good news pre-contract. Therefore, the expected payoff of a silent seller is $\pi_S^{MPCD} = \theta^{MPCD} E[\bar{\nu}(\alpha) | \alpha \leq \hat{\alpha}^{MPCD}] + (1 - \theta^{MPCD})\nu_L$. In the Remediation case, the buyer will wait and invest only if the post-contract disclosure reveals bad news. Therefore, the expected payoff of a silent seller is $\pi_S^{MPCD} = \theta^{MPCD} E[\bar{\nu}(\alpha) | \alpha \leq \hat{\alpha}^{MPCD}] + (1-\theta^{MPCD})(\nu_L + \Delta\tilde{\nu})$.

We next consider the incentives to investigate and find that MPCD leads to excessive investigation by high-α sellers and, in the Remediation case, also to insufficient investigation by low-α sellers. More precisely -

Lemma 4:

(a) *In the Improvement case, $\hat{\alpha}^{MPCD} < \hat{\alpha}^*$; MPCD induces inefficient investigation, by sellers with $\alpha \in (\hat{\alpha}^{MPCD}, \hat{\alpha}^*)$.*

(b) *In the Remediation case, $\hat{\alpha}^{MPCD}$ can be either smaller than or greater than $\hat{\alpha}^*$. If $\hat{\alpha}^{MPCD} < \hat{\alpha}^*$, then MPCD induces inefficient investigation, by sellers with $\alpha > \hat{\alpha}^*$ and inefficiently fails to induce investigation by sellers with $\alpha < \hat{\alpha}^{MPCD}$. If $\hat{\alpha}^{MPCD} > \hat{\alpha}^*$, then MPCD induces inefficient investigation by sellers with $\alpha > \hat{\alpha}^{MPCD}$, and inefficiently fails to induce investigation by sellers with $\alpha < \hat{\alpha}^*$.*

In both the Improvement case and the Remediation case, the social welfare level is:

$$W^{MPCD} = E[\bar{\nu}(\alpha)] + \int_{\hat{\alpha}^{MPCD}}^{1} (I^*(\alpha) - k) f(\alpha) d\alpha$$

The welfare function with MPCD is identical to the welfare function with MD, subject to the different investigation threshold. In the Improvement case, value-enhancing investment occurs when the seller gets good news and discloses pre-contract. In the Remediation case, value-enhancing investment occurs when the seller gets bad news and discloses post-contract. Either way, all information, including bad news, is revealed and we get optimal investment, conditional on investigation. As with MD, the inefficiency with MPCD results from the inefficient investigation decision.

Comparing the welfare level with mandatory post-contract disclosure to the first-best welfare level, we obtain the following result.

Proposition 4: In both the Improvement case and the Remediation case, Mandatory Post-Contract Disclosure does not achieve the first-best social welfare level: $W^{MPCD} < W^*$.

Like MD and VD, MPCD does not attain the first-best. It is useful to consider MPCD, because it can outperform the two standard rules. We begin the comparison of the three rules by considering the private value of information. We obtain the intuitive result that the private value of information with voluntary disclosure is larger than the private value of information with mandatory post-contract disclosure which is larger than the private value of information with mandatory (pre-contract) disclosure. These results, presented in terms of the investigation threshold, are stated in the following lemma:

Lemma 5: In both the Improvement case and the Remediation case, $\hat{\alpha}^{VD} < \hat{\alpha}^{MPCD} < \hat{\alpha}^{MD}$.

We next compare welfare levels. In the Improvement case, MPCD is more efficient than VD, and less efficient than MD. In the Remediation case, MPCD is clearly more efficient than VD, and can be either more or less efficient than MD. Specifically, when $\hat{\alpha}^{MPCD} > \hat{\alpha}^*$, MPCD is less efficient than MD, as it simply induces more inefficient investigation. When $\hat{\alpha}^{MPCD} < \hat{\alpha}^*$, MPCD can be more efficient than MD, as it induces some efficient investigation by sellers with $\alpha \in (\hat{\alpha}^{MPCD}, \hat{\alpha}^*)$. These results are summarized in the following proposition.

Proposition 5:

(a) **In the Improvement case, MPCD is more efficient than VD, and less efficient than MD.**

(b) **In the Remediation case, MPCD is more efficient than VD, and can be either more or less efficient than MD.**

We see that, under certain conditions, the new, MPCD rule is more efficient than the two standard rules. In particular, MPCD is always more efficient than VD. Proposition 5, which considers MPCD in the framework of the basic model, states that MPCD can be more efficient than MD only in the Remediation case. In the heterogenous investigation costs extension (see Section 4.1 above), MPCD can be more efficient than MD also in the Improvement case.

6 Concluding Remarks

6.1 Investigation by the Buyer

This paper studies the seller's investigation decision and how it is affected by different disclosure rules. In some cases, the buyer can also investigate (pre-contract). A full analysis of this sequential investigation game is beyond the scope of this paper. We can, however, offer a few observations: If investigation by the seller and investigation by the buyer are substitutes, it is efficient that only one party investigate. Therefore, the possibility of investigation by the buyer introduces a new type of efficiency cost: duplicative investigation. This problem arises with VD, where an investigating seller might remain silent and thus trigger duplicative investigation by the buyer.

Perhaps more fundamentally, when both Seller and Buyer can investigate (and investigation by the seller and investigation by the buyer are substitutes), we must first determine which party should optimally investigate. If the timing of the investigation is not crucial, then the party with the lower investigation costs should investigate. In many cases, however, early investigation is desirable. In the Improvement case, early investigation is desirable, when some investment opportunities are time sensitive. And in the Remediation case, early investigation is desirable, when remediation costs increase with time. Therefore, investigation by the seller — perhaps years before the sale — is often more efficient. In addition, the seller will generally have private information that allows for more efficient, focused investigation. For example, a seller who suspects underground water will investigate this particular problem, whereas an investigation by an uninformed buyer

would have to be broader and more expensive. The timing and information considerations provide further justification for our focus on the seller's investigation decision.

6.2 Liability Rules

What if a buyer who learns, post sale, that the value of the asset is ν_L can rescind the contract, regardless of whether the seller knew or investigated? This alternative rule resembles a strict liability (SL) regime. With SL, when the value of the asset is ν_L, the buyer will eventually learn this information and then, using the threat of rescission, force the seller to return any payment above the asset's true value. This means that the seller has no reason to withhold bad news and, in the Remediation case, the seller has an affirmative reason to disclose bad news — to enable remediation investment (and thus get a higher price, accounting for liability risk). And so an investigating seller discloses everything, as with MD. But MD does not induce optimal investigation, because of the asymmetric information problem. SL overcomes the asymmetric information problem by allowing the buyer to rescind the contract whenever he learns the bad news, even if the information arrives long after the contract was signed. Thus, SL induces optimal investigation.

Consider the Remediation case (a similar analysis applies to the Improvement case). The expected profit of a type α seller who investigates, but does not disclose bad news, is: $\pi_I^{SL}(\alpha) = \alpha\nu_H + (1-\alpha)\nu_L = \bar{\nu}(\alpha)$. If the seller investigates and discloses bad news (as well as good news), her expected profit is: $\pi_I^{SL}(\alpha) = \alpha\nu_H + (1-\alpha)(\nu_L + \Delta\tilde{\nu}) = \bar{\nu}(\alpha) + (1-\alpha)\Delta\tilde{\nu}$. Therefore, a seller who investigates will disclose bad news and get: $\pi_I^{SL}(\alpha) = \bar{\nu}(\alpha) + (1-\alpha)\Delta\tilde{\nu}$. A seller who chooses to investigate will disclose both good and bad news, even if there is no legal duty to disclose. (There is no point in withholding bad news from the buyer, since the buyer will eventually learn that the value of the asset is ν_L and rescind the contract unless the price is reduced accordingly.) The expected profit of a type α seller who does not investigate is $\pi_{NI}^{SL}(\alpha) = \alpha\nu_H + (1-\alpha)\nu_L = \bar{\nu}(\alpha)$. Therefore, the private value of information is $I^{SL}(\alpha) = \pi_I^{SL}(\alpha) - \pi_{NI}^{SL}(\alpha) = (1-\alpha)\Delta\tilde{\nu}$. We see that $I^{SL}(\alpha) = I^*(\alpha)$. Strict liability achieves the first-best investigation levels and the first-best welfare levels.

But a strict liability rule is not without cost. SL, in essence, forces the seller to provide a broad warranty — a warranty that would also cover problems and risks that the seller could not have discovered through investigation. Such a mandatory warranty intervenes with the contractually specified risk allocation or, more precisely, prevents the parties from allocating risk as they see fit. (A similar analysis applies to voluntary warranties. See Section 1.4 above.)

In addition to the strict liability rule, it is also possible to envision a negligence rule, where the buyer's

right to rescind the contract arises only if the seller was negligent in her decision to remain uninformed (or if the seller investigated but failed to disclose bad news). Doctrinally, such a rule would impose liability on a seller who "should have known" about the asset's condition.[14] In our framework, liability would be imposed on a non-investigating (or non-disclosing) type-α seller, when $I^*(\alpha) > k$. In theory, such a negligence rule can achieve the first-best investigation levels and the first-best welfare levels. In practice, however, courts are unlikely to have the information required to implement a negligence rule. Specifically, the optimal due care standard, $I^*(\alpha)$, is a function of the seller's type, which will generally be non-verifiable.

[14] *See, e.g.,* Ralbovsky v. Lamphere, 731 F. Supp. 79 (N.D.N.Y 1990) (holding that a seller of a used car reporting that the mileage shown on the odometer is true, may be held liable in the absence of actual knowledge that an odometer reading is false, if he reasonably should have known that the odometer reading was incorrect); Easton v. Strassburger, 199 Cal. Rptr. 383, 388 (Cal. App. 1st Dist. 1984) (holding that a broker is under a duty to disclose material facts which he should have known); Bradbury v. Rentz, Ohio App. LEXIS 9780 (1984) ("a failure to state a fact is equivalent to a fraudulent concealment when the seller knew of or in the exercise of reasonable diligence should have known of the presence of this material fact, and further knew or should have known that this material fact may have affected the action of the buyer"). *But see* Jacobson v. Sweeney, 82 F. Supp. 2d 458, 462 n.2 (D. Md. 2000) ("Real estate agents owe property buyers a duty, in some circumstances, to disclose defects of which they know or should have known, but recent cases have limited this duty to the disclosure of material facts known to the seller's agent"); Eric T. Freyfogle, *Real Estate Sales and the New Implied Warranty of Lawful Use*, 71 Cornell L. Rev. 1, 25–28 (1985) ("Sellers generally need disclose only matters of which they have some degree of personal knowledge").

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Appendix

The Appendix contains the proofs of Lemma 1, Lemma 2 and Lemma 3. The proofs of Proposition 1, Proposition 2, Proposition 3, Proposition 3a, Proposition 4, Proposition 5 and Lemma 5 follow immediately from the analysis in the body of the paper and are, therefore, omitted.

Proof of Lemma 1

(a) The Improvement case: we have $I^{MD}(\hat{\alpha}^{MD}) = \bar{\nu}(\hat{\alpha}^{MD}) + \hat{\alpha}^{MD}\Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] = k$, which after some reorganizing becomes: $\hat{\alpha}^{MD}\Delta\tilde{\nu} = k + E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] - \bar{\nu}(\hat{\alpha}^{MD})$. Comparing with the equation that defines the first-best investigation threshold, $\hat{\alpha}^*\Delta\tilde{\nu} = k$, and noting that $E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] < \bar{\nu}(\hat{\alpha}^{MD})$, we obtain: $\hat{\alpha}^{MD} < \hat{\alpha}^*$.

(b) The Remediation case: We have $I^{MD}(\hat{\alpha}^{MD}) = \bar{\nu}(\hat{\alpha}^{MD}) + (1 - \hat{\alpha}^{MD})\Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] = k$, which, after some reorganizing, becomes: $(1-\hat{\alpha}^{MD})\Delta\tilde{\nu} = k+E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]-\bar{\nu}(\hat{\alpha}^{MD})$. Comparing with the equation that defines the first-best investigation threshold, $(1 - \hat{\alpha}^*)\Delta\tilde{\nu} = k$, and noting that $E(\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] < \bar{\nu}(\hat{\alpha}^{MD})$, we obtain: $\hat{\alpha}^{MD} > \hat{\alpha}^*$.

QED

Proof of Lemma 2

(a) The Improvement case: We have $I^{VD}(\hat{\alpha}^{VD}) = \hat{\alpha}^{VD}(\nu_H + \Delta\tilde{\nu} - \pi_S^{VD}) = k$. Comparing with the equation that defines the first-best investigation threshold, $\hat{\alpha}^*\Delta\tilde{\nu} = k$, and noting that $\nu_H > \pi_S^{VD}$, we obtain: $\hat{\alpha}^{VD} < \hat{\alpha}^*$.

(b) The Remediation case: $\hat{\alpha}^{VD}$ can be either higher or lower than $\hat{\alpha}^*$. (It is easy to construct an example where $\hat{\alpha}^{VD} < \hat{\alpha}^*$ and an example where $\hat{\alpha}^{VD} > \hat{\alpha}^*$.) Moreover, the relative magnitude of $\hat{\alpha}^{VD}$, as compared to $\hat{\alpha}^*$, is less important, since any investigation is wasteful in a VD regime.

QED

Proof of Lemma 3

(a) The Improvement case: We prove that $\hat{\alpha}^{VD} < \hat{\alpha}^{MD}$ by showing that the private value of information is higher with VD. Consider —

$$I^{MD}(\hat{\alpha}^{MD}) = \bar{\nu}(\hat{\alpha}^{MD}) + \hat{\alpha}^{MD}\Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

$$I^{VD}(\hat{\alpha}^{VD}) = \hat{\alpha}^{VD}(\nu_H + \Delta\tilde{\nu} - \pi_S^{VD})$$

Dealing with the multiple equilibrium problem as explained in the body of the paper, we show that $I^{VD}(\hat{\alpha}^{MD}) > I^{MD}(\hat{\alpha}^{MD})$: After substituting the expressions derived for $I^{MD}(\hat{\alpha}^{MD})$ and $I^{VD}(\hat{\alpha}^{MD})$, the condition becomes:

$$\hat{\alpha}^{MD}(\nu_H + \Delta\tilde{\nu} - \pi_S^{VD}) > \bar{\nu}(\hat{\alpha}^{MD}) + \hat{\alpha}^{MD}\Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

Evaluated at $\hat{\alpha}^{MD}, \pi_S^{VD} = \theta^{VD}E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] + (1-\theta^{VD})\nu_L < E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$. We thus know that: $I^{VD}(\hat{\alpha}^{MD}) > \hat{\alpha}^{MD}(\nu_H + \Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}])$. Therefore, it is enough to prove that: $\hat{\alpha}^{MD}(\nu_H + \Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]) > I^{MD}(\hat{\alpha}^{MD})$. Or:

$$\hat{\alpha}^{MD}(\nu_H - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]) > \bar{\nu}(\hat{\alpha}^{MD}) - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

Substituting $\bar{\nu}(\hat{\alpha}^{MD}) = \hat{\alpha}^{MD}\nu_H + (1-\hat{\alpha}^{MD})\nu_L$, the condition becomes:

$$\hat{\alpha}^{MD}(\nu_H - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]) > \hat{\alpha}^{MD}\nu_H + (1-\hat{\alpha}^{MD})\nu_L - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

Since $E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] > \nu_L$, this condition is always satisfied.

(b) The Remediation case: We prove that $\hat{\alpha}^{VD} < \hat{\alpha}^{MD}$ by showing that the private value of information is higher with VD. Consider —

$$I^{MD}(\hat{\alpha}^{MD}) = \bar{\nu}(\hat{\alpha}^{MD}) + (1-\hat{\alpha}^{MD})\Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

$$I^{VD}(\hat{\alpha}^{VD}) = \hat{\alpha}^{VD}(\nu_H - \pi_S^{VD})$$

Dealing with the multiple equilibrium problem as explained in the body of the paper, we show that

$I^{VD}(\hat{\alpha}^{MD}) > I^{MD}(\hat{\alpha}^{MD})$: After substituting the expressions derived for $I^{MD}(\hat{\alpha}^{MD})$ and $I^{VD}(\hat{\alpha}^{MD})$, the condition becomes:

$$\hat{\alpha}^{MD}(\nu_H - \pi_S^{VD}) > \bar{\nu}(\hat{\alpha}^{MD}) + (1 - \hat{\alpha}^{MD})\Delta\tilde{\nu} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

Recall from Section 2.3 that $\pi_S^{VD} > \nu_L + \Delta\tilde{\nu}$, or $\pi_S^{VD} - \nu_L > \Delta\tilde{\nu}$ (otherwise, VD converges to MD). Therefore, it is enough to prove that:

$$\hat{\alpha}^{MD}(\nu_H - \pi_S^{VD}) > \bar{\nu}(\hat{\alpha}^{MD}) + (1 - \hat{\alpha}^{MD})(\pi_S^{VD} - \nu_L) - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

Or:

$$\hat{\alpha}^{MD}\nu_H > \bar{\nu}(\hat{\alpha}^{MD}) + \pi_S^{VD} - (1 - \hat{\alpha}^{MD})\nu_L - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

Substituting $\bar{\nu}(\hat{\alpha}^{MD}) = \hat{\alpha}^{MD}\nu_H + (1 - \hat{\alpha}^{MD})\nu_L$, the condition becomes:

$$\hat{\alpha}^{MD}\nu_H > \hat{\alpha}^{MD}\nu_H + \pi_S^{VD} - E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$$

Or:

$$E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] > \pi_S^{VD}$$

Evaluated at $\hat{\alpha}^{MD}, \pi_S^{VD} = \theta^{VD}E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}] + (1 - \theta^{VD})\nu_L < E[\bar{\nu}(\alpha)|\alpha \leq \hat{\alpha}^{MD}]$.

QED

Proof of Lemma 4

(a) The Improvement case: Similar to the proof of Lemma 2(a).

(b) The Remediation case: It is easy to construct an example where $\hat{\alpha}^{MPCD} < \hat{\alpha}^*$ and and example where $\hat{\alpha}^{MPCD} > \hat{\alpha}^*$.

QED