

Multi-Dimensional Information with Specialized Lenders*

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Abstract

We study specialized lending in a credit market competition model with multi-dimensional information. Two banks, equipped with similar data processing systems, possess “hard” signals regarding the borrower’s quality. However, the specialized bank gains an additional advantage through further interactions with the borrower, allowing it to access “soft” signals. In equilibrium, both lenders use hard signals to screen loan applications, and the specialized lender prices the loan based on its soft signal conditional on making a loan. This private-information-based pricing helps us deliver the empirical regularity that loans by specialized lenders have lower rates and better ex-post performance. Our multi-dimensional information framework enables us to discern between broader and more precise data, thereby capturing the emerging trend in fintech lending where traditionally subjective information becomes more objective and concrete. We finally endogenize the specialized lending with information acquisition, and discuss its various economic implications.

JEL Classification: G21, L13, L52, O33, O36

Keywords: Banking competition, Winner’s curse, Specialization, Big data, Information acquisition

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1 Introduction

Banks are crucial intermediaries in modern economies, serving as the main conduit between savers and borrowers. One of their primary functions is to choose in which borrowers to invest, and as it has long been recognized by the literature (e.g., Broecker, 1990; Riordan, 1993; Hauswald and Marquez, 2003), competition among informed financial intermediaries in the credit market is central to the stability and efficiency of financial systems.

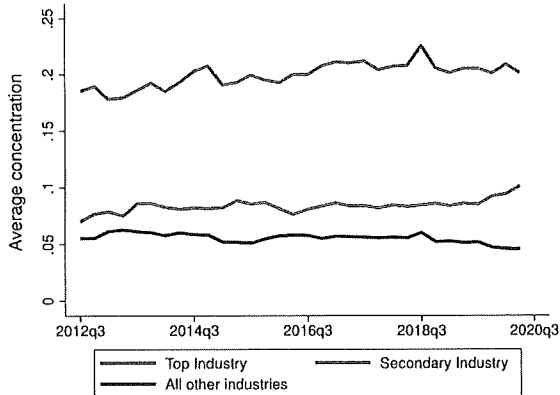
Of significant importance, banks hold a diverse array of lending-related information, including financial data on customers, collateral evaluations, and market and economic trends, not to mention state-of-the-art data analytics. In practice, many lenders also specialize in certain industries and companies by providing customized financial services and pricing, often by diligently collecting and analyzing information about individual firms and sector-specific business practices. Moreover, the banking industry’s evolution towards richer information categories aligns with the trend of big data technology, which transforms qualitative or subjective assessments into quantifiable and objective metrics, known as “hardening soft information” (e.g., Hardik, 2023).

Despite the remarkable technological advancement that could significantly impact the industrial landscape of the banking sector, the prevailing literature on information-based credit market competition predominantly focuses on binary signal realizations, overlooking the nuances of the aforementioned intricate economics.¹ To bridge this gap, our paper introduces multi-dimensional information into an otherwise classic credit market competition model similar to Broecker (1990), which allows us to study specialized lending and its equilibrium implications on credit allocation.

We begin by presenting some key empirical facts that motivate our theoretical analyses. Panel (a) of Figure 1, which we take from Blikle, Parlato, and Saunders (2021), shows that banks specialize their lending to specific industries—i.e., they have a “top” industry which accounts for the largest share of their loans. On average, a bank’s “top” industry accounts for twice as much of the bank’s lending portfolio as the second more preferred industry. What is more, as shown in Panel B of Figure 1, banks offer better terms—more specifically, lower loan rates—to firms in their top industry, and are better at identifying high-quality loans—hence less likely to be non-performing—within their top “specialized” industry. These salient empirical patterns, which seem to have strengthened over time since 2012, suggest that specialized banks can “undercut” the non-specialized opponent lenders in their specialized industries.

As we mention above, the existing literature on bank competition with adverse selection has predominantly focused on settings with binary signals. There, each lender actively competes only upon receiving the positive realization, offering interest rates that are outcomes of a completely randomized mixed strategy. Consequently, the bank strength is mechanically tied to the *ex-ante*

¹One notable exception is Riordan (1993) who studies the setting in Broecker (1990) with N symmetric lenders whose signal realizations are smooth; see literature review for more details. For the research question of specialized lenders, asymmetric information technology is crucial.



(a) Specialized lending



(b) Difference in loan rates and performance

Figure 1: **Specialization, loan rates, and performance.** We plot the average loan portfolio concentration, measured as the share of C&I lending to one two-digit industry, and the average difference in loan rates and performance in a bank’s top industry and all other industries. In Panel 1a, data is split into the average bank’s “top” industry, its secondary industry, and all other industries. A bank’s top industry is defined as the two-digit NAICS code industry into which a bank has invested the largest share of its portfolio. Panel 1b plots (left scale) the average difference between the loan rates extended to firms in a bank’s top industry and those offered to firms in other industries. Panel 1b also plots (right scale) the average difference between the share of non-accruing (non-performing) loans in a bank’s top industry and the same share in other industries. Source: Blickle, Parlatore, and Saunders (2021).

precision of each lender’s signal, leading to a stark information rent effect under which the stronger lender charges higher interest rates *ex post*. This is counterfactual in light of Figure 1b.

Specialization in lending and its empirical regularity shown in Figure 1a suggest that private-information-based loan pricing is important in credit market competition. In our model outlined in Section 2, a specialized bank competes with a non-specialized bank. Each lender has a “hard” information signal on the loan quality from data processing. Moreover, the specialized lender has access to an additional signal coming from “soft” information about the borrower, based on which the lender decides on the offered interest rate. These two signals may represent either two distinct fundamental states—say “hard” and “soft” respectively—or just one single fundamental state that dictates the overall quality of the project. Our preferred setting is the former: by emphasizing the multi-dimensional information sources in credit market competition, later in an extension of the model, we study an increase in the breadth of the “hard” signal by allowing it to cover more underlying states.

We assume that, while the “hard” signal is binary and is “decisive” in that each lender makes an offer only if it receives a positive hard signal, the “soft” signal—which differentiates our paper from existing models (Broecker, 1990; Marquez, 2002)—is continuous which guides the fine-tuned interest rate offering. Besides analytical convenience, this loan-making rule of the specialized bank

matches well with the lending practices observed in the real world. Essentially, in our model, the specialized bank acquires two signals, one being “principal” while the other being “supplementary;” the former determines whether to lend while the latter affects the detailed pricing terms.²

In Section 3, we fully characterize in closed form the competitive credit market equilibrium with specialized lending, with the specialized bank’s interest rate schedule *decreasing* in its soft signal. In fact, since the successful project’s payoff is capped, our specialized bank—even conditional on a positive hard signal—withdraws itself from the competition after receiving a sufficiently unfavorable soft signal. In contrast, the non-specialized bank behaves just like in Broecker (1990) with interest rate offering fully randomized. Therefore by incorporating both hard and soft information, our model delivers the key result of private-information-based pricing.³

Our model features a unique credit market equilibrium, which can fall into two distinct categories depending on whether the non-specialized bank makes zero profits or not as a result of competition. In the first category of equilibria, the winner’s curse dominates and pushes the non-specialized “weak” bank to earn zero profits—therefore we call it a zero-weak equilibrium. In this case, the non-specialized bank randomly withdraws from the loan market when receiving a positive hard signal, which increases the specialized lender’s monopoly power over borrowers. This information rent enjoyed by the specialized bank pushes it to make less aggressive offers than the non-specialized bank, as in He, Huang, and Zhou (2023). In the second category of equilibria, the winner’s curse is less severe and the non-specialized bank makes a positive profit in equilibrium (therefore always participates upon a positive hard signal)—we call it a positive-weak equilibrium. The private-information-based pricing effect tends to dominate in this case, as the specialized bank with less monopoly power makes more aggressive offers to get good borrowers.

In Section 4, we show the latter private-information-based pricing is crucial in delivering the empirical regularity that loans of specialized lenders have lower rates, which we simply call “negative interest rate wedge.” First, canonical credit competition models a la Broecker (1990), which feature the information rent effect only, give rise to the counterfactual implications that the specialized bank’s loans have higher rates. In contrast, the private-information-based pricing in our model delivers a lower interest on loans granted by specialized banks, and as discussed earlier, a negative interest rate wedge is more likely to occur in the positive-weak equilibrium where pricing based on private information takes precedence (though, we formally show that a positive profit for the non-specialized bank is not a prerequisite for achieving this result).

²Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit).

³Conceptually, this is similar to Milgrom and Weber (1982), in which the informed buyer who privately observes a continuum of signal realizations in a common value auction bids monotonically based on its own private information; see literature review for more details. In addition, one could extend the range of quoted interest rates by borrowers to include infinity and interpret $r = \infty$ as “rejection/withdrawal;” this way the lenders in the classic credit market competition model in Broecker (1990) and Hauswald and Marquez (2003) also have private-information-based pricing. However, Figure 1b is constructed based on interest rates of granted loans, and therefore loan rejection with $r = \infty$ cannot help explain the empirical regularity of lower interest rates of loans granted by specialized lenders.

As one of the main applications of our model, we conduct comparative statics of model predictions on information technology. We vary two precision parameters in our baseline model, one for the hard signals (for both lenders) and the other for the soft signal (only for the specialized lender). When signal precision increases, either leveling the playing field across two lenders regarding hard information or strengthening the specialized lender’s soft information advantage—it is harder for our model to deliver a negative interest wedge. Intuitively, each precision makes the non-specialized lender relatively “weaker;” this makes a zero-weak equilibrium—in which the information rent effect tends to dominate—more likely to arise.

The information technology advancement in the past decades is richer than the higher information quality (captured by greater signal precision, either hard or soft, in our model). More precisely, an increasing information breadth/span witnessed in the last decade could be the driving force of a stronger negative interest rate wedge observed in the data. To this end, Section 4.3 extends our baseline model to allow for multiple fundamental states.⁴ A hard signal that covers increasingly more underlying states thus captures the notion of “hardening soft information,” facilitated by big data and machine learning technology in recent years. Interestingly, opposite to the implication of a greater signal precision, an enlarged span of hard information offers non-specialized lenders a certain edge against their specialized competitors, thus helping explain the time trend of interest rate wedge documented in Blickle, Parlato, and Saunders (2021).

We close our paper by studying two extensions in Section 5. First, we show that our equilibrium characterization is robust to a general information structure. Second, we endogenize the information structure in the baseline model by considering two ex-ante symmetric banks that compete on two firms/industries. Lenders can invest in the hard information technology (which has a lump-sum fixed cost and produces a binary signal of borrower quality in either firm); in addition, they can acquire firm-specific soft information (which is a continuous signal and costly for each firm) and hence becoming specialized. We provide conditions that support the “symmetric” specialization equilibrium where, as in our baseline model, each industry supports one specialized lender and another non-specialized lender.

Literature Review

Lending market competition and common-value auctions. Our paper is built on Broecker (1990) who studies lending market competition with screening tests with symmetric lenders (i.e., with the same screening abilities). Hauswald and Marquez (2003) study the competition between an inside bank that can conduct credit screenings and an outside bank without such access. He, Huang, and Zhou (2023) consider competition between asymmetric lenders with different screening abilities under open banking when borrowers control access to data. Asymmetric credit market competition

⁴The model with multiple fundamental states, when admits a multiplicative structure, is isomorphic to our baseline when lenders still receive two signals each covering different ranges of these underlying states, and therefore admits the same solution.

can also naturally arise from the bank-customer relationship, as a bank knows its existing customers better than a new competitor does.⁵ In these models, for analytical tractability it is often assumed that private screening yields a binary signal and lenders participate in bidding only following the positive signal realization. In contrast to these papers, our paper considers competition between asymmetric lenders with multiple information sources.

Fundamentally speaking, credit market competition is an application of common-value auctions, and notably, the auction literature typically allows for general signal distributions (other than the binary signal in the aforementioned papers).⁶ For instance, Riordan (1993) extends the N -symmetric-lender model in Broecker (1990) to a setting with continuous private signals. In terms of modeling, our framework can be viewed as a combination of Broecker (1990) (hard information) and Milgrom and Weber (1982) (soft information). It is worth highlighting that lenders are each privately informed with hard information and hence break the Blackwell ordering of two lenders in Milgrom and Weber (1982).⁷ However, the economics revealed by a setting with multi-dimensional information can be fundamentally different, as highlighted by the distinction between information precision and information span discussed in Section 4.3.

The nature of information in bank lending. Berger and Udell (2006) provides a comprehensive framework of the two fundamental types of bank lending technology, i.e., relationship lending and transactions lending, in the SME lending market.⁸ A fundamental difference between these two types of lending is related to the role played by information as highlighted by Stein (2002).⁹ Recently, based on Harte Hanks data, He, Jiang, Xu, and Yin (2023) shows a significant rise in IT investment within the U.S. banking sector over the past decade, particularly among large banks. They also establish a causal link between communication IT investments and the enhancement of banks' capacity for generating and transmitting soft information, which motivates our modeling of the soft signal as the outcome of interactions with borrowers.

Specialization in lending. There is a growing literature documenting specialization in bank lending; the early work includes Acharya, Hasan, and Saunders (2006). Paravisini, Rappoport, and Schnabl

⁵This idea was explored by a two-period model in Sharpe (1990) where asymmetric competition arises in the second period (with the corrected analysis of a mixed-strategy equilibrium offered by Von Thadden (2004)). A similar analysis is present in Rajan (1992).

⁶The early papers on this topic include Milgrom and Weber (1982) and Engelbrecht-Wiggans, Milgrom, and Weber (1983), and later papers such as Hausch (1987); Kagel and Levin (1999) explore information structures where each bidder has some private information, which is the information structure adopted in Broecker (1990).

⁷More precisely, one bidder knows strictly more than the other bidder. In this setting, one can show that the under-informed bidder always makes zero profit; see also Engelbrecht-Wiggans, Milgrom, and Weber (1983).

⁸Relatedly, Bolton, Freixas, Gambacorta, and Mistrulli (2016) study the joint determination of relationship lending and transactions lending. They find that firms that rely more on relationship banking are better able to weather a crisis than firms that rely on transaction banking, suggesting a higher capital requirement for relationship banks.

⁹Along these lines, Liberti and Mian (2009) find empirically that greater hierarchical distance leads to less reliance on subjective information and more on objective information. Paravisini and Schoar (2016) document that credit scores, which serve as "hard information," improve the productivity of credit committees, reduce managerial involvement in the loan approval process, and increase the profitability of lending.

(2023) show that Peruvian banks specialize their lending across export markets benefiting borrowers who obtain credit from their specialized banks. Based on data for US stress-tested banks, Blicable, Parlatore, and Saunders (2021) document that specialization is linked with lower interest rates and better performance in the industry of specialization, pointing to a strong link between specialization in lending and informational advantages. Our paper contributes to this literature by providing a framework that can rationalize these patterns allowing us to understand the economic mechanisms behind them and their implications more deeply.

Fintech. Our paper connects to the growing literature on fintech disruption.¹⁰ Empirical studies document the use of alternative data in fintech lending, which is consistent with our emphasis on the increasing span of hard information.¹¹ In particular, Huang, Zhang, Li, Qiu, Sun, and Wang (2020) show that unconventional data from the Alibaba platform, such as business transactions, customer ratings, and consumption patterns improve credit assessment. Our paper emphasizes that the recent development of cashless payments increases the scope of firms that could be assessed by hard information (Ghosh, Vallee, and Zeng, 2022), and perhaps more importantly, the combination of payments and big data technology enlarges the span of hard information.

The remainder of the paper is organized as follows. Section 2 presents the baseline model. Section 2.3 characterizes the credit market equilibrium and Section 4 explores the economic implications of technology advancement within our framework. We present extensions of the model in Section 5 and conclude in Section 6.

2 Model Setup

We first introduce the general model setting in Section 2.1. We then specialize the setting to a multiplicative structure for uncertainty in Section 2.2, which renders great analytical tractability.

2.1 General Setting

We consider a credit market competition model with two dates, $t = 0, 1$, risk-neutral agents, and one good. There are two ex-ante symmetric lenders (banks), indexed by $j \in \{A, B\}$. In the baseline model, we consider only one borrower (firm); in the extension, we introduce a second firm.

Project. At $t = 0$, the firm needs to borrow one dollar to invest in a (fixed-scale) risky project that pays a random cash flow y at $t = 1$. The cash flow realization y depends on the project's

¹⁰See Berg, Fuster, and Puri (2021); Vives (2019), for instance, for a review of fintech companies competing with traditional banks in originating loans.

¹¹Examples of alternative data include phone device and spelling (Berg, Burg, Gombović, and Puri, 2020), mobile phone logs (Agarwal, Alok, Ghosh, and Gupta, 2020). Along the line of our model with different dimensions of information, Huang (2023) developed a theoretical framework wherein the importance of information concerning underlying qualities varies between collateral-backed bank lending and revenue-based fintech lending such as Square.

quality denoted by $\theta \in \{0, 1\}$. For simplicity, we assume that

$$y = \begin{cases} 1 + \bar{r} & \text{when } \theta = 1 \\ 0 & \text{when } \theta = 0, \end{cases} \quad (1)$$

where $\bar{r} > 0$ is exogenously given, i.e., only the good project has a positive NPV. We will later refer to \bar{r} as the interest rate cap or the return of a good project. The project’s quality θ is the firm’s private information at $t = 0$, and the prior probability of a good project is $q \equiv \mathbb{P}(\theta = 1)$.

Credit market competition. At date $t = 0$, each bank j can choose to make a take-it-or-leave-it offer to the borrower firm or to make no offer (i.e., exit the lending market). An offer consists of a fixed loan amount of one and an interest rate r . The borrower firm accepts the offer with the lowest rate if it receives multiple offers.

Information technology. Although the project qualities are unobservable, banks have access to information about the borrower’s project quality before choosing whether to make an offer. We assume that both lenders have access to “hard” data (say financial and operating data), which they can process to produce a *hard-information*-based private signal h^j for the firm. We call these information “hard” signals. For simplicity, we assume that these hard signals are binary, i.e., $h^j \in \{H, L\}$, with a realization H (L) being a positive (negative) signal; and that, conditional on the (relevant) state, hard signals are independent across lenders. This hard signal structure is the same as the one in Broecker (1990), and it captures the coarseness with which much of the hard information is used in practice.¹²

Additionally, we endow Bank A with a signal s , which captures the bank being “specialized” in the firm. This assumption represents the major departure from the existing literature. Our preferred interpretation of this additional signal is as a *soft-information*-based private signal, which is collected after due diligence or face-to-face interactions with the borrower after on-site visits. We assume that the firm-specific soft signal s is continuous, and its distribution is characterized by the Cumulative Distribution Function (CDF) $\Phi(s)$ and probability density function (pdf) $\phi(s)$. Besides mathematical convenience, the continuous distribution captures soft information resulting from research tailored to the particular borrower and, therefore, allows for a more granular assessment of the borrower’s quality.

Remark 1. Endogenous information structure. In our main analysis, we take the lenders’ information technologies—specifically, Bank A being the specialized lender—as given. Section 5.2 endogenizes this “asymmetric” information technology in a “symmetric” setting with two firms, a and b ,

¹²For example, credit scores are binned in five ranges even though scores are computed at a much granular level and go from 300 to 850.

where Bank A (B) endogenously becomes specialized by acquiring both “hard” and “soft” signals of the firm a (b), while non-specialized Bank B (A) only acquires the “hard” signal of the firm a (b). There, the key difference between hard and soft information is that a lender j only needs to invest once—say installing IT equipment and software—to get two hard signals, one for each firm, while soft information needs to be collected individually for each firm.

The information structure is incomplete unless we specify the correlations between the fundamental states and the two types of signals, to which we turn next.

2.2 The Setting with a Multiplicative Structure

Our main analysis focuses on the specific setting with a multiplicative structure for the state θ , say $\theta = \theta_h \theta_s$ with one state being “hard” and the other being “soft.” We generalize this setting to multiple multiplicative states (characteristics) in Section 4.3; and Section 5.1 shows that the main results are robust to a non-multiplicative environment.

Hard and soft fundamental states. One major simplification in our main setting is the complete independence between soft and hard signals, achieved by introducing two multiplicative binary states $\{\theta_s, \theta_h\}$, soft and hard, that jointly determine the project’s success θ , that is,

$$\theta \equiv \theta_s \theta_h \equiv \begin{cases} 1, & \text{when } \theta_s = \theta_h = 1, \\ 0, & \text{when either } \theta_s = 0 \text{ or } \theta_h = 0. \end{cases} \quad (2)$$

As a result, the prior probability of the state being “1” is $q = q_h q_s$, where

$$q_h \equiv \mathbb{P}(\theta_h = 1) \quad \text{and} \quad q_s \equiv \mathbb{P}(\theta_s = 1).$$

The conditional distribution of the signals reflects the information technology. As hard information signals are binary, without loss of generality we assume that for $j \in \{A, B\}$,

$$\mathbb{P}(h^j = H | \theta_h = 1) = \alpha_u, \quad \mathbb{P}(h^j = L | \theta_h = 0) = \alpha_d, \quad (3)$$

where $1 - \alpha_u$ and $1 - \alpha_d$ capture the probabilities of Type I and Type II errors, respectively. Implicitly we impose that lenders have the same technology to process hard information, an assumption that we relax later in Section 5.1. The bad-news signal structure in He, Huang, and Zhou (2023) corresponds to $\alpha_u = 1$ and a symmetric signal structure has $\alpha_u = \alpha_d = \alpha \in (0.5, 1]$ as in Hauswald and Marquez (2003). Our main numerical illustration focuses on the latter case, although the equilibrium characterization does not rely on any specific structure.

For the continuous soft signal, without loss of generality, we directly work with the posterior

probability of the soft state being good $\theta_s = 1$ given the soft signal realization, i.e.,

$$s = \Pr[\theta_s = 1 | s] \in \mathcal{S} \equiv [0, 1]. \quad (4)$$

Recall that the pdf of s is $\phi(s)$, so we have $\int_0^1 s \phi(s) ds \equiv q_s$ due to prior consistency. Although our theoretical characterization works for a general density function $\phi(\cdot)$, most of our numerical illustrations use the specification that Bank A 's soft signal is a noisy version of the underlying soft state θ_s , with the signal-to-noise ratio being captured by the precision parameter τ (for more details, see Section 3.3).

The specialized Bank A has both hard and soft signals $\{h^A, s\}$ while Bank B only has a hard signal h^B . Throughout we assume that the hard signal is “decisive” for participation: Bank j participates if and only if it receives $h^j = H$. For the specialized Bank A , the hard signal serves as “pre-screening,” in the sense that the bank rejects the borrower upon receiving an L signal, while upon an H signal it makes a pricing decision based on its soft signal s .

Remark 2. Principal and supplementary signals and relation to the literature. The equilibrium loan-making rule of the specialized bank is practically relevant and new to the theoretical literature. Essentially, the specialized bank has two signals, one being “principal” which determines whether to lend, and the other being “supplementary” which helps its loan pricing.¹³ This is in sharp contrast to the existing literature mentioned in the introduction where lenders make loan offers randomly only conditional on the most favorable realization of their (binary) signals. By decoupling the lender’s *ex-post* loan assessment from its *ex-ante* technology strength, our setting naturally delivers the empirical regularity of lower observed loan rates extended by specialized banks.

Remark 3. Correlated hard signals. One widely-acknowledged aspect of information technology advancement is that the lenders’ hard information signals become more correlated. For example, the open banking regulation enables sharing financial data with potential lenders under customer consent (He, Huang, and Zhou, 2023; Babina, Buchak, and Gornall, 2022), and as a result, lenders’ assessments become more alike. A simple modification of our framework captures this effect. Suppose that with probability $\rho_h \in [0, 1]$, lenders receive the same signal realization $h^c \in \{H, L\}$ and $\mathbb{P}(h^c = H | \theta_h = 1) = \mathbb{P}(h^c = L | \theta_h = 0) = \alpha$; while with probability $1 - \rho_h$, each lender receives an independent hard signal according to Eq. (3). Our main analysis focuses on the baseline model with $\rho_h = 0$, although Section 4.3 considers the comparative statics with respect of ρ_h .

Parametric assumptions. It is useful to introduce notation for the joint distribution of hard signals. We use subscript $\{h^A h^B\}$ to denote the events of the corresponding hard signal realizations. This subscript takes a value from the set $\{HH, HL, LH, LL\}$, where HL says Bank A 's hard information signal is H and Bank B 's hard information signal is L .

¹³Alternatively, the principal signal represents the result of a credit screening test, while the supplementary signal serves the role of internal ratings (of borrowers who are qualified for credit).

We denote by $p_{h^A h^B}$ the joint probability of any collection of hard signal realization. For instance, $p_{HH} \equiv \mathbb{P}(h^A = H, h^B = H) = q_h \alpha_u^2 + (1 - q_h)(1 - \alpha_d)^2$. Similarly, we denote by $\mu_{h^A h^B}$ the posterior probability of the hard state being one conditional on the hard signal realization, i.e., $\mu_{h^A h^B} \equiv \mathbb{P}(\theta_h = 1 | h^A, h^B)$. For example, $\mu_{HH} = \frac{q_h \alpha_u^2}{q_h \alpha_u^2 + (1 - q_h)(1 - \alpha_d)^2}$. Under the multiplicative structure, given the collection of signals $\{HH, s\}$, the posterior probability of project success is

$$\mathbb{P}(\theta = 1 | h^A = H, h^B = H, s) = \mu_{HH} \cdot s \quad (5)$$

To ensure that the pre-screening hard signal is “decisive,” throughout the paper we impose the following parameter restrictions.

Assumption 1. (Strength of the hard signal)

a) Bank A rejects the borrower upon an L hard signal, regardless of any soft signal s :

$$q_h(1 - \alpha_u)\bar{r} < (1 - q_h)\alpha_d. \quad (6)$$

b) Bank B is willing to participate if and only if its hard signal $h^B = H$:

$$q_h \alpha_u q_s \bar{r} > q_h \alpha_u (1 - q_s) + (1 - q_h)(1 - \alpha_d); \quad (7)$$

Assumption 1 says that the hard signal has to be sufficiently strong (informative) to serve as pre-screening of loan applications for both lenders. Condition (6), states that it is not profitable for Bank A to participate in competition upon receiving a hard signal L even when the soft signal reveals that the soft fundamental θ_s is good with certainty. This condition implies that Bank B, which only has the hard signal and is uncertain about the realization of the soft fundamental, also chooses not to compete upon receiving $h^B = L$. Condition (7) states that upon $h^B = H$, Bank B is willing to lend at the highest possible interest rate if it is the monopolist lender. This condition also implies that Bank A, which also receives a soft signal, is willing to lend at the highest interest rate if it is the monopolist lender upon $h^A = H$ if it also observes high enough realizations of its soft signal.

2.3 Credit Market Equilibrium Definition

We now formally define the credit market equilibrium with specialized lending.

Bank strategies. Given our assumptions, in equilibrium, each lender makes a potential offer only upon receiving a positive hard signal. Conditional the hard signal, we define the space of interest rate offers to be $\mathcal{R} \equiv [0, \bar{r}] \cup \infty$. Here, \bar{r} is the exogenous maximum interest rate imposed in Section 2.1 and ∞ captures the strategy of not making an offer. We denote Bank A’s pure strategy by $r^A(s) : \mathcal{S} \rightarrow \mathcal{R}$, which induces a distribution of its interest offerings denoted by

$F^A(r) \equiv \Pr(r^A \leq r)$ according to the underlying distribution of the soft signal. (We now take as given that Bank A uses pure strategy, though we formally prove this result in Proposition 1.) Below we show that the endogenous support of the equilibrium interest rates offered when making an offer is a sub-interval of $[0, \bar{r}]$. Therefore, with a slight abuse of terminology, we refer to that sub-interval as the “support” of the interest rate distribution even though loan rejection ($r = \infty$) could also occur along the equilibrium path.

Bank B randomizes its interest rate offerings conditional on a positive hard signal in equilibrium. In this case, we use $F^B(r) \equiv \Pr(r^B \leq r)$ to denote the cumulative distribution of its interest rate offerings. Note that since the domain of offers includes $r = \infty$ which captures rejection, it is possible that $F^B(\bar{r}) = \mathbb{P}(r^B < \infty | h^B = H) \leq 1$.

The borrower picks the lowest interest rate possible if multiple loan offers are ever available. For instance, conditional on both banks receiving positive hard signals, if Bank B quotes r^B , then its winning probability $1 - F^A(r^B)$ equals the probability that Bank A with soft signal s offers a rate that is higher than r^B , which includes the event that Bank A rejects the borrower ($r^A(s) = \infty$), presumably because of an unfavorable soft signal. Upon ties, which occurs when $r^A = r^B < \infty$, borrowers randomly choose the lender with probability one half, although the details of the tie-breaking rule do not matter as ties occur as zero-measure events in equilibrium. When $r^A = r^B = \infty$, no bank wins the competition as they both reject the borrower.

The following lemma summarizes the discussion above and shows that resulting equilibrium strategies in our setting are still well-behaved as established in the literature (Engelbrecht-Wiggans, Milgrom, and Weber (1983); Broecker (1990)). The key steps of proof are standard, though we make certain adjustments due to the presence of both discrete and continuous signals.

Lemma 1. (*Equilibrium Structure*) *Under Assumption 1, in any credit market equilibrium,*

1. *A lender j rejects the borrower upon $h^j = L$ for $j \in \{A, B\}$;*
2. *Upon $h^j = H$, the lender may participate:*
 - i) *Bank A uses a pure strategy $r^A(s) : \mathcal{S} \rightarrow \mathcal{R}$ which induces a distribution $F^A(\cdot)$;*
 - ii) *Bank B offers interest rate based on an endogenous cumulative distribution function $F^B(r) : \mathcal{R} \rightarrow [0, 1]$;*
3. *The two distributions $F^j(\cdot)$, $j \in \{A, B\}$ share a common support $[\underline{r}, \bar{r}]$ (besides ∞ as rejection). Over $[\underline{r}, \bar{r}]$ both distributions are smooth, i.e. no gap and atomless, so that they admit well-defined density functions. At most only one lender can have a mass point at \bar{r} .*

Bank profits and optimal strategies. Conditional on $h^A = H$ and s , Bank A 's profit $\pi^A(r|s)$, when competing with its opponent lender B by quoting $r \in [\underline{r}, \bar{r}]$, equals

$$\pi^A(r|s) \equiv \underbrace{p_{HH}}_{h^A=H, h^B=H} \underbrace{[1 - F^B(r)]}_{A \text{ wins}} [\mu_{HH}s(1+r) - 1] + \underbrace{p_{HL}}_{h^A=H, h^B=L} [\mu_{HL}s(1+r) - 1], \text{ for } r \in [\underline{r}, \bar{r}]. \quad (8)$$

Bank A can also choose to exit by quoting $r = \infty$, in which case $\pi^A(\infty|s) = 0$. We then denote Bank A 's optimal interest rate offering by $r^A(s) \equiv \arg \max_{r \in \mathcal{R}} \pi^A(r|s)$.

Recall that Bank A cannot observe the realization of Bank B 's hard signal when making an offer. With probability p_{HH} , both banks get favorable hard signals H , and Bank A wins with probability $1 - F^B(r)$ if it offers r , whereas with probability p_{HL} Bank B receives a low hard signal and Bank A faces no competition for the borrower. Moreover, whether Bank B participates in the loan market affects Bank A 's expected quality of the borrower, which is captured by $\mu_{HH}s$ and $\mu_{HL}s$. Importantly, since Bank B randomizes its strategy upon $h^B = H$, from the perspective of Bank A winning the price competition against Bank B is not informative about borrower quality.

This last observation is in sharp contrast with the problem of the non-specialized Bank B . A standard winner's curse ensues because the outcome of competition against the specialized Bank A is informative about θ_s . More specifically, besides the possibility of competitor's unfavorable hard information as mentioned above, the non-specialized lender B who wins the price competition also infers $r^A(s) > r^B$ so Bank A 's soft information is unfavorable. Taking these inferences into account, Bank B 's lending profits when quoting r are

$$\pi^B(r) \equiv \underbrace{p_{HH}}_{h^A=H} \underbrace{[1 - F^A(r)]}_{B \text{ wins}} \mathbb{E} [\mu_{HH}\theta_s(1+r) - 1 | r \leq r^A(s)] + \underbrace{p_{LH}}_{h^A=L} [\mu_{LH}q_s(1+r) - 1]. \quad (9)$$

Bank B 's strategy $F^B(\cdot)$ maximize its expected payoff

$$\max_{F^B} \int \pi^B(r) dF^B(r). \quad (10)$$

As a standard equilibrium property with mixed strategies, profit-maximizing Bank B is indifferent between any action on its support.

Equilibrium definition. We now define the credit market equilibrium with specialized lending.

Definition 1. In any industry, a credit market equilibrium between a specialized lender (A) and non-specialized lender B is a collection of strategies $\{r^A(s); F^B(\cdot)\}$ such that $r^A(s)$ maximizes Bank A 's objective in (8) for any $s \in \mathcal{S}$, and $F^B(\cdot)$ solves Bank B 's problem in (10).

3 Credit Market Equilibrium Characterization

To characterize the credit market equilibrium, we first take the equilibrium non-specialized Bank B 's profit π^B as given and solve for the other equilibrium objects. Lemma 2 then solves for π^B , which completes the construction.

3.1 Solve for Pricing Strategies of Lenders

Solve for $r^A(s)$. Following Milgrom and Weber (1982), we start by solving for Bank A 's equilibrium strategy $r^A(s)$. Suppose that $r^A(s)$ is decreasing, which will be verified later. From Bank B 's problem, we know that it makes a constant profit π^B from any interest rate quotes; and if $F^B(\bar{r}) < 1$ so that it chooses to reject the borrower upon H with some probability, we have $\pi^B = 0$. Then, when Bank B quotes $r = r^A(s)$, conditional on $h^A = H$ Bank B understands that it only wins the customer when A 's soft signal is below s . Bank B , therefore, updates the belief about the borrower's quality accordingly—its posterior for the soft state is $\int_0^s t\phi(t) dt$.

On the other hand, conditional on $h^A = L$, Bank B wins the borrower for sure. Plugging $r^B = r^A(s)$ in Bank B 's lending profits in Eq. (9), we have the following indifference condition:

$$\pi^B = \underbrace{\left[p_{HH}\mu_{HH} \int_0^s t\phi(t) dt + p_{LH}\mu_{LH}q_s \right]}_{B's \text{ customers who repay}} \left(1 + r^A(s) \right) - \underbrace{(p_{HH}\Phi(s) + p_{LH})}_{B's \text{ lending amount}}, \quad (11)$$

which holds for any $r^B = r^A(s) \in [\underline{r}, \bar{r}]$. It immediately follows that

$$r^A(s) = \frac{\pi^B + p_{HH}\Phi(s) + p_{LH}}{p_{HH}\mu_{HH} \int_0^s t\phi(t) dt + p_{LH}\mu_{LH}q_s} - 1, \quad \text{when } s \in [\hat{s}, 1], \quad (12)$$

where \hat{s} is the highest realization of the soft signal such that Bank A quotes \bar{r} :¹⁴

$$\hat{s} \equiv \sup \left\{ s \mid r^A(s) = \bar{r} \right\}. \quad (13)$$

For worse signal realizations, we further define $x \leq \hat{s}$ as the threshold such that $\pi^A(\bar{r} \mid x) = 0$. It is worth highlighting that $x = \hat{s}$ could occur along the equilibrium path. Then it is straightforward to show that $r^A(s) = \bar{r}$ for $s \in [x, \hat{s}]$, and $r^A(s) = \infty$ for $s \in [0, x]$.

As shown in Proposition 1 below, the conjectured strategy $r^A(s)$, which is strictly decreasing, gives the unique equilibrium. Define its inverse function $s^A(r)$, which is also decreasing, as¹⁵

$$s^A(r) \equiv r^{A(-1)}(r) \quad \text{for } r \in [\underline{r}, \bar{r}]; \quad (14)$$

¹⁴Recall the convention that $\sup \{\emptyset\} = \inf S = 0$.

¹⁵The function $s^A(r)$ is decreasing even over the entire range $\mathcal{R} \equiv [0, \bar{r}] \cup \infty$ including rejection by quoting ∞ .

and when $r = \infty$ we have $s^A(\infty) = [0, x)$. Then, the two relevant cutoffs for Bank A 's strategy can be succinctly written as $\hat{s} = \sup s^A(\bar{r})$, i.e., the highest signal that Bank A quotes \bar{r} , and $x = \sup s^A(\infty)$, i.e., the highest signal that Bank A rejects the borrower. It is worth noting that \hat{s} may coincide with x , and we take the convention that $r^A(x) = \bar{r}$.

Solve for $F^B(\cdot)$. We now turn to Bank B 's strategy. In equilibrium, B 's strategy needs to support $r^A(\cdot)$ in (12) to be Bank A 's optimal strategy. Bank A 's first-order-condition (FOC) that maximizes its objective in (8), which balances the lower probability of winning against the higher payoff from served borrowers, is

$$p_{HH} \left(-\frac{dF^B(r)}{dr} \right) [\mu_{HH}s(1+r) - 1] + \left\{ p_{HH} [1 - F^B(r)] \mu_{HH}s + p_{HL}\mu_{HL}s \right\} = 0. \quad (15)$$

Bank A 's equilibrium strategy $r^A(s)$ satisfies (15) for all $s \in [\hat{s}, 1]$, which helps us pin down $F^B(\cdot)$.

From Bank B 's perspective, by quoting $r = r^A(s)$, the corresponding marginal borrower type (soft signal) is $s^A(r)$. Writing everything in terms of r ; when Bank B marginally cuts its quote by dr , it gets $\phi(s^A(r))(-s^{A'}(r))dr$ additional borrowers of quality $\mu_{HH}s^A(r)$ if there is competition, which occurs with probability p_{HH} . This gain is exactly offset by the marginal lower payoff from the borrowers who are already served. Therefore, Bank B ' FOC is

$$\underbrace{p_{HH} \left[\phi(s^A(r)) \cdot (-s^{A'}(r)) \right]}_{\text{additional borrowers}} \left[\mu_{HH}s^A(r)(1+r) - 1 \right] = \underbrace{p_{HH}\mu_{HH} \int_0^{s^A(r)} t\phi(t) dt + p_{LH}\mu_{LH}q_s}_{\text{existing borrowers}}. \quad (16)$$

Using the expression for $\mu_{HH}s^A(r)(1+r) - 1$ in Bank B 's FOC (16) in Eq. (15) which captures Bank A 's FOC, we have

$$\frac{dF^B(r)}{dr} \left[\frac{p_{HH}\mu_{HH} \int_0^{s^A(r)} t\phi(t) dt + p_{LH}\mu_{LH}q_s}{\phi(s^A(r))s^{A'}(r)} \right] + p_{HH} [1 - F^B(r)] \mu_{HH}s^A(r) + p_{HL}\mu_{HL}s^A(r) = 0.$$

One can show that the above equation yields the following ODE, which pins down $F^B(\cdot)$:

$$\frac{d}{dr} \left\{ \frac{p_{HH}\mu_{HH} [1 - F^B(r)] + p_{HL}\mu_{HL}}{p_{HH}\mu_{HH} \int_0^{s^A(r)} t\phi(t) dt + p_{LH}\mu_{LH}q_s} \right\} = 0. \quad (17)$$

Here is the intuition behind the differential Eq. (17). At any interest rate r , both lenders are competing for the same marginal borrowers with quality $\mu_{HH} \cdot s^A(r)$, that yield an expected profit of $\mu_{HH} \cdot s^A(r) \cdot (1+r) - 1$. This term shows up in both lenders' optimization conditions, i.e., (15) for Bank A and (16) for Bank B . We denote by $Q^j(r)$ the size of the effective customer base of

Bank $j \in \{A, B\}$ when it offers interest rate r . Then,

$$\begin{aligned} Q^A(r) &= p_{HH}\mu_{HH} \left[1 - F^B(r)\right] s^A(r) + p_{HL}\mu_{HL}s^A(r), \\ Q^B(r) &= p_{HH}\mu_{HH} \int_0^{s^A(r)} t\phi(t) dt + p_{LH}\mu_{LH}q_s. \end{aligned}$$

Q^A and Q^B differ in that Bank A observes s while Bank B only knows that it gets borrowers with $s < s^A(r)$ (if $h^A = H$) or prior q_s (if $h^A = L$). For Bank A , the marginal effect of price cutting on customer size is $\frac{1}{\mu_{HH}} \left[\frac{Q^A(r)}{s^A(r)}\right]'$, where the division inside the bracket adjusts for the quality of the soft fundamental of the marginal borrower. Then, Bank A 's optimal pricing strategy must satisfy

$$\underbrace{\left[\frac{Q^A(r)}{\mu_{HH}s^A(r)}\right]'}_{\text{MB on marginal borrowers}} dr \cdot \underbrace{\left[\mu_{HH}s^A(r)(1+r) - 1\right]}_{\text{MC on existing borrowers}} = \underbrace{Q^A(r)dr}_{\text{MC on existing borrowers}} \Leftrightarrow \frac{\mu_{HH}s^A(r)}{\mu_{HH}s^A(r)(1+r) - 1} = \frac{\left[\frac{Q^A(r)}{s^A(r)}\right]'}{\frac{Q^A(r)}{s^A(r)}}, \quad (18)$$

which is equivalent to Eq. (15)). On the other hand, for Bank B , which does not observe s , the marginal effect on customer size is $\frac{1}{\mu_{HH}} \frac{Q^{B'}(r)}{s^A(r)}$, implying an optimality condition of

$$\underbrace{\frac{Q^{B'}(r)}{\mu_{HH}s^A(r)} dr \cdot \left[\mu_{HH}s^A(r)(1+r) - 1\right]}_{\text{MB on marginal borrowers}} = \underbrace{Q^B(r)dr}_{\text{MC on existing borrowers}} \Leftrightarrow \frac{\mu_{HH}s^A(r)}{\mu_{HH}s^A(r)(1+r) - 1} = \frac{Q^{B'}(r)}{Q^B(r)}, \quad (19)$$

which is exactly Eq. (16).¹⁶ Combining (18) and (19), we have:

$$\frac{\left[\frac{Q^A(r)}{s^A(r)}\right]'}{\frac{Q^A(r)}{s^A(r)}} = \frac{Q^{B'}(r)}{Q^B(r)} \Leftrightarrow \frac{d}{dr} \left[\frac{Q^A(r)/s^A(r)}{Q^B(r)} \right] = 0, \quad (20)$$

which is exactly our key ODE in Eq. (17).

The boundary condition $F^B(\underline{r}) = 0$ defines the lower-end support of the offered interest rate. Combining this bound with the ODE in Eq. (17) one can derive

$$1 - F^B(r) = \frac{\int_0^{s^A(r)} t\phi(t) dt}{q_s}, \text{ for } r \in (\underline{r}, \bar{r}) \quad (21)$$

¹⁶Readers might notice the important difference between the two lenders' marginal effects of cutting their prices on the quantity. For Bank A which observes the soft signal realization directly, its pricing decision should not affect its quality; this is why we scale Q^A first by s and then take derivative, i.e., $\left[\frac{Q^A(r)}{s^A(r)}\right]'$. In contrast, without observing s directly, Bank B 's price cutting affects its inferred quality of the borrower (that it wins over Bank A). Therefore we take the derivative of $Q^B(r)$, which includes the quality of its borrowers, and then scale by the quality of marginal borrowers to avoid double counting.

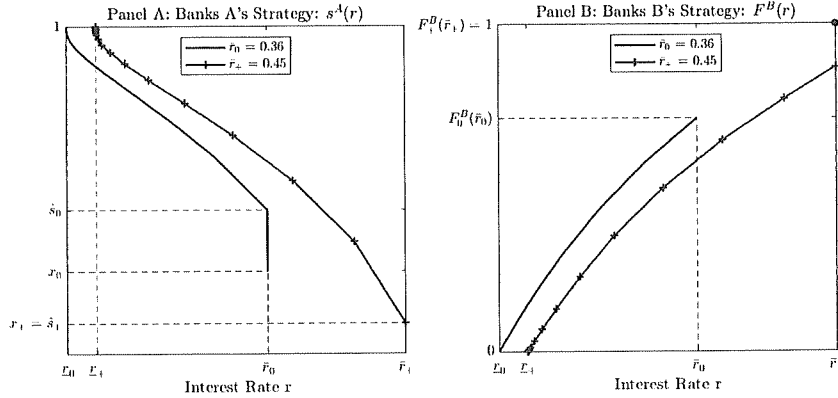


Figure 2: **Equilibrium strategies** $s^A(r)$ for Bank A (left) and $F^B(r)$ for Bank B (right). In both panels, strategies under \bar{r}_+ (i.e., positive-weak equilibrium) are depicted in red with “+” markers while strategies with \bar{r}_0 (i.e., zero-weak equilibrium) are depicted in blue. In the zero-weak equilibrium, Bank A (but not Bank B) has a point mass at \bar{r}_0 while in the positive-weak equilibrium, Bank B (but not Bank A) has a point mass at \bar{r}_+ . Parameters: $q_h = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.7$, and $\tau = 1$.

as we have focused on the interior of the strategy space.¹⁷ It is clear that $F^B(r) < 1$ for $r \in [\underline{r}, \bar{r})$, because $F^B(\bar{r}^-) = \frac{1}{q_s} \int_{s^A(\bar{r}^-)=\hat{s}}^1 t\phi(t)dt < 1$; and Bank B’s strategy on the boundary \bar{r} depends on whether it is profitable in equilibrium: it either places a mass of $1 - F^B(\bar{r}^-) = \frac{1}{q_s} \int_0^{\hat{s}} t\phi(t) dt > 0$ on \bar{r} if $\pi^B > 0$, or quotes $r = \infty$ (i.e., withdraws) if $\pi^B = 0$. Finally, we observe that parameters on the hard signals do not enter $F^B(\cdot)$ in (21) directly; but as shown later they do affect $F^B(\cdot)$ indirectly via the endogenous lower bound \underline{r} .

Illustration of lenders’ pricing strategies. Figure 2 illustrates the equilibrium strategies for both lenders for two cases, $\pi^B > 0$ and $\pi^B = 0$ indicated by the subscripts “+” and “0,” respectively. The exogenous parameter that drives the different profits for Bank B is the interest rate cap \bar{r} , which we denote by $\bar{r}_+ > \bar{r}_0$ depending on the equilibrium type. As one would expect, the greater the borrower surplus the higher the lender’s profits. For ease of exposition, both figures are plotted against interest rate r , so that the left panel is the inverse function $s^A(r)$ of Bank A’s quoting strategy $r^A(s)$ (which is decreasing), while the right panel plots $F^B(r)$ which is Bank B’s CDF for its interest rate offerings. We also plot the corresponding cutoff signals \hat{s} , at which Bank A’s strategy hits \bar{r} , and x , at which Bank A exits.

While we discuss the equilibrium strategies in more detail after providing a full characterization of the equilibrium, Figure 2 highlights a key difference between the two types of equilibrium that can arise, one with $\pi^B = 0$ —the zero-weak equilibrium as the weak bank earns no profits—and the other with $\pi^B > 0$ —the positive-weak equilibrium as the weak bank earns positive profits. As

¹⁷In deriving (21) we have used the fact that the two lenders share the same hard information technology. This implies that the identity of the lender who receives high/low hard signal is irrelevant and hence $p_{LH}\mu_{LH} = p_{HL}\mu_{HL}$.

shown in Figure 2, in the case in which $\pi^B = 0$, Bank A has a point mass at \bar{r}_0 (corresponding to $s \in (x_0, \hat{s}_0)$) but Bank B does not, while in the case of $\pi^B = 0$ the opposite holds. This reflects the competition at the interest rate cap and it is the exact manifestation of point c) in Lemma 1 (otherwise, lenders will undercut each other at this point).

3.2 Solving for the Equilibrium Profit of Bank B

In the last step, we solve for the equilibrium profits for Bank B , π^B , which then pins down the entire equilibrium. Define s_A^{BE} as the soft signal realization under which Bank A quotes \bar{r} and breaks even (therefore the superscript “ BE ”). Formally, using $\pi^A(\cdot)$ given in (8) and using the strategic response of Bank B in Eq. (21),¹⁸ s_A^{BE} is the unique solution to the following equation

$$\pi^A(\bar{r} | s_A^{BE}) = p_{HH} \frac{\int_0^{s_A^{BE}} t \phi(t) dt}{q_s} \cdot [\mu_{HH} s_A^{BE} (1 + \bar{r}) - 1] + p_{HL} [\mu_{HL} s_A^{BE} (1 + \bar{r}) - 1] = 0, \quad (22)$$

which admits a unique solution inside the interval $(0, 1)$.¹⁹ We define s_B^{BE} following a similar logic as follows. Consider the case in which Bank B quotes the maximum rate \bar{r} . Then, the potential winner’s curse implies that Bank B only wins the borrower when either Bank A ’s hard signal is $h^A = L$ or its soft signal is sufficiently unfavorable, i.e., $s < s_B^{BE}$. The break-even condition for Bank B uniquely defines s_B^{BE} , as follows.

$$0 = \pi^B(\bar{r}) = p_{HH} \left[\mu_{HH} \left(\int_0^{s_B^{BE}} t \phi(t) dt \right) (1 + \bar{r}) - \Phi(s_B^{BE}) \right] + p_{LH} [\mu_{LH} q_s (1 + \bar{r}) - 1]. \quad (23)$$

Lemma 2 below shows that the relative ranking between s_B^{BE} and s_A^{BE} fully determines π^B and \hat{s} in equilibrium, with both being fully characterized explicitly. Intuitively, the equilibrium crucially depends on which lender quoting \bar{r} hits zero profits first when the soft signal goes down from the top. If $s_A^{BE} < s_B^{BE}$ then Bank B hits zero profit first, and this supports the equilibrium of $\pi^B = 0$ with $\hat{s} = s_B^{BE}$; otherwise we have $\pi^B > 0$ with $\hat{s} = s_A^{BE}$.

Lemma 2. *Given s_A^{BE} defined in (22), the equilibrium Bank B profit is*

$$\pi^B = \max \left\{ \left[p_{HH} \mu_{HH} \int_0^{s_A^{BE}} t \phi(t) dt + p_{LH} \mu_{LH} q_s \right] (1 + \bar{r}) - \left(p_{HH} \Phi(s_A^{BE}) + p_{LH} \right), 0 \right\}.$$

When $s_B^{BE} < s_A^{BE}$ we are in the positive-weak equilibrium in which the weak Bank B makes a positive profit, and $x = \hat{s} = s_A^{BE}$. Otherwise, when $s_B^{BE} \geq s_A^{BE}$ we are in the zero-weak equilibrium

¹⁸Technically speaking Bank A quotes \bar{r}^- so that $1 - F^B(\bar{r}^-) = \frac{1}{q_s} \int_0^{s_A^{BE}} t \phi(t) dt$, as (21) requires $r \in [\underline{r}, \bar{r}]$.

¹⁹Note $\pi^A(\bar{r} | s_A^{BE})$ as a function of s_A^{BE} is strictly increasing. Moreover, we have $\pi^A(\bar{r} | s_A^{BE} = 0) < 0$ and $\pi^A(\bar{r} | s_A^{BE} = 1) = p_{HH} [\mu_{HH} (1 + \bar{r}) - 1] + p_{HL} [\mu_{HL} (1 + \bar{r}) - 1] > 0$; the latter is implied by that Bank A is willing to make an offer given $h^A = H$.

where Bank B earns zero profits, with $x < \hat{s} = s_B^{BE}$.

To understand the result, note that s_B^{BE} is the highest soft signal under which Bank A's offer hits \bar{r} , given $\pi^B = 0$.²⁰ Moreover, recall that s_A^{BE} is the level of soft signal under which Bank A just breaks even when quoting \bar{r} . Then if $s_B^{BE} < s_A^{BE}$, Bank A hits zero profit first, implying that it will lose money upon receiving a soft signal $s = s_B^{BE} < s_A^{BE}$. Combining these two pieces, we know that quoting \bar{r} at s_B^{BE} , under the assumption of $\pi^B = 0$, must be off-equilibrium for Bank A. Therefore in equilibrium $\pi^B > 0$ and Bank A withdraws itself upon $s < x = \hat{s} = s_A^{BE}$. If on the other hand $s_B^{BE} \geq s_A^{BE}$, we are in the alternative scenario where $\hat{s} = s_B^{BE}$ and $\pi^B = 0$; Bank A who is making a positive profit at s_B^{BE} will keep quoting \bar{r} for $s < s_B^{BE}$, until $s < x$ upon which it exits.

3.3 Credit Market Equilibrium

We now present the main result of our paper. The credit market equilibrium, which is fully characterized analytically, not only helps us understand the observed pattern on interest rates when some lenders are specialized but also allows us to study the implications of the evolution of information technologies.

Credit market equilibrium characterization. The next proposition summarizes the credit market equilibrium with specialized lending.

Proposition 1. (Credit Market Equilibrium) *In the unique equilibrium, Bank A follows a pure strategy as in Definition 1. In this equilibrium, lenders reject borrowers upon a low hard signal realization $h^j = L$ for $j \in \{A, B\}$. Otherwise (i.e., when $h^j = H$), their strategies are characterized as follows, with the equilibrium π^B given in Lemma 2.*

1. Bank A with soft signal s offers

$$r^A(s) = \begin{cases} \min \left\{ \frac{\pi^B + p_{HH}\Phi(s) + p_{LH}}{p_{HH}\mu_{HH} \int_0^s t\phi(t)dt + p_{LH}\mu_{LH}q_s} - 1, \bar{r} \right\} & \text{for } s \in [x, 1], \\ \infty, & \text{for } s \in [0, x). \end{cases} \quad (24)$$

The equation pins down $\underline{r} = r^A(1)$. If $s \in (\hat{s}, 1]$ where $\hat{s} = \sup s^A(\bar{r})$, $r^A(\cdot)$ is strictly decreasing with its inverse function $s^A(\cdot) = r^{A(-1)}(\cdot)$.

²⁰Note that (23) can be rewritten as $s_B^{BE} = \arg_{s \in S} \sup \left\{ r^A(s; \pi^B = 0) = \frac{p_{HH}\Phi(s) + p_{LH}}{p_{HH}\mu_{HH} \left(\int_0^s t\phi(t)dt \right) + p_{LH}\mu_{LH}q_s} - 1 \geq \bar{r} \right\}$. (Recall we take the convention that $\arg \sup \emptyset = \inf S = 0$.)

2. Bank B makes an offer with cumulative probability given by ($\mathbf{1}_{\{X\}} = 1$ if X holds)

$$F^B(r) = \begin{cases} 1 - \frac{\int_0^{s^A(r)} t\phi(t)dt}{q_s}, & \text{for } r \in [\underline{r}, \bar{r}), \\ 1 - \mathbf{1}_{\{\pi^B=0\}} \cdot \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s}, & \text{for } r = \bar{r}. \end{cases} \quad (25)$$

When $\pi^B = 0$, $F^B(\bar{r}) = F^B(\bar{r}^-)$ is the probability that Bank B makes the offer (and with probability $\frac{1}{q_s} \int_0^{\hat{s}} t\phi(t) dt$ it withdraws by quoting $r^B = \infty$); when $\pi^B > 0$, $F^B(\bar{r}) = 1$ and there is a point mass of $\frac{1}{q_s} \int_0^{\hat{s}} t\phi(t) dt$ at \bar{r} .

The proof for Proposition 1 mainly covers two issues. First, we prove that the FOC conditions used in the equilibrium construction detailed in Section 3 are sufficient to ensure global optimality. Second, somewhat surprisingly, thanks to the endogenous adjustment of π^B and \underline{r} , we never need to “iron” a la Myerson (1981) at the interior part of the range for equilibrium interest rates. In fact, in our model, Bank A never bunches its quotes—except at \bar{r} when the zero-weak equilibrium ensues. (This is consistent with point 3 in Lemma 1 that states that Bank B will undercut if Bank A bunches at some interior interest rate.)

Properties of credit market equilibrium. Figure 3 illustrates the main properties of the credit market equilibrium with specialized lenders. For the purpose of exposition, we assume that Bank A 's soft signal s is obtained from observing a noisy version of θ_s , i.e., $\theta_s + \epsilon$, so that

$$s = \mathbb{E}[\theta_s | \theta_s + \epsilon]. \quad (26)$$

Here, $\epsilon \sim \mathcal{N}(0, 1/\tau)$ indicates a white noise, with the precision parameter τ capturing the signal to noise ratio of Bank A 's soft information technology.

The top two panels in Figure 3 plot both lenders' pricing strategies conditional on making an offer; they are different presentations of Figure 2, with Panel A plotting Bank A 's $r^A(s)$ as a function of s (instead of its inverse function $s^A(r)$) and Panel B plotting the density dF^B/dr for Bank B . As we explained above, $r^A(s)$ decreases in s —that is to say, when the specialized Bank A receives a more favorable soft signal about credit quality, it bids more aggressively with a lower rate to win the borrower over the competitor Bank B . This strategic response to exploit the competitor bank is weakened when the private assessment of credit quality is low, leading Bank A to scale back. In fact, Bank A rejects the borrower when $s < x$. In contrast, as shown in Panel B, the competitor Bank B randomizes as it only observes the hard signal.

Panel C plots the two soft signal cut-offs for Bank A , i.e., \hat{s} at which it starts quoting \bar{r} and x at which it starts rejecting the borrower. Panel D plots the expected profits— $\mathbb{E}(\pi^A)$ and π^B —for two lenders. Both panels are plotted against the exogenous interest rate cap \bar{r} .

Recall that \bar{r} , which is the return of the good project, captures the surplus in competition. Thus,

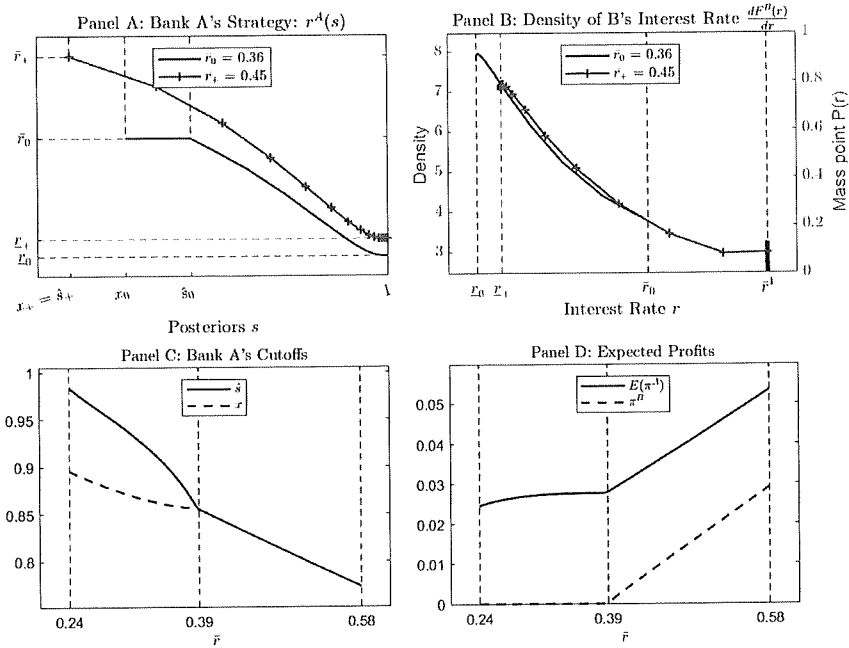


Figure 3: **Equilibrium strategies and profit.** In the top two panels, we plot equilibrium strategies for both lenders. Panel A depicts $r^A(s)$ as a function of s and Panel B plots $dF^B(r)/dr$ for as a function r ; strategies with \bar{r}_+ are depicted in red with markers while strategies with \bar{r}_0 are depicted in blue. Panel C depicts Bank A's thresholds $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$, and Panel D depicts the expected profits for two lenders. Parameters: $q_h = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.7$, and $\tau = 1$.

a higher total surplus gives rise to less fierce competition, and as a result, both lenders—including the weak lender B —are making profits upon a favorable hard signal H . This immediately explains Panel D, which shows that π^B turns strictly positive for sufficiently high \bar{r} . Put differently, the model features a positive-(zero-) weak equilibrium when \bar{r} is relatively high (low).

For a better illustration, consider the competition at interest rate \bar{r} . In the positive-weak equilibrium (high \bar{r} 's), the non-specialized Bank B places a point mass on this interest rate, enjoying some “local monopoly power” in competition as it is the only lender when Bank A rejects the borrower upon $s < \hat{s} = x$. This is possible because when the project's surplus (captured by \bar{r}) is sufficiently large, the nonspecialized Bank B is still profitable by quoting \bar{r} despite the winner's curse.²¹ In contrast, in the positive-weak equilibrium (low \bar{r} 's), the specialized Bank A is the monopolistic lender who places a point mass on this interest rate (when $s \in (x, \hat{s})$, as shown in Panel C) while the nonspecialized Bank B withdraws.

²¹Bank B who quotes \bar{r} gets the borrower too if Bank A receives an unfavorable hard signal $h^A = L$. Despite this winner's curse, the surplus is sufficiently high so that the nonspecialized Bank B is still profitable by quoting \bar{r} .

4 Economic Implications on Information Technology Advancement

We now discuss the model's implications from the perspective of information technology. We first show that the private-information-based pricing featured in our model generates the empirical regularity observed in Figure 1b. We then extend our baseline multiplicative setting with two states to the one with many states, and show that the expanded scope of hard information, which is distinct from an enhanced quality of hard information, aligns more consistently with the time-series pattern displayed in Figure 1b.

4.1 Specialized Lending: Loan Performance and Interest Rate

An econometrician observes the granted bank loans that are accepted by borrowers. Put it differently, the loans that we use to calculate loan quality and interest rates are already on the book of lenders who won the bidding competition.

In our credit market competition setting, when Bank A makes a loan offer ($r^A < \infty$), it would be accepted by the borrower if $r^A < r^B \leq \infty$, i.e., either there is no offer from Bank B (when $h^B = L$), or Bank A 's rate is lower. Therefore the theoretical counterparts of the objects of interest in Figure 1b are i) the (better) performance of the loans granted by the specialized lender relative to those granted by the nonspecialized lender:

$$\underbrace{\mathbb{E} [\theta = 1 | r^A < r^B \leq \infty]}_{\text{quality of } A\text{'s loan}} - \underbrace{\mathbb{E} [\theta = 1 | r^B < r^A \leq \infty]}_{\text{quality of } B\text{'s loan}} > 0, \quad (27)$$

and ii) the (lower) interest rates of granted loans charged by the specialized lender relative to those by the nonspecialized lender:

$$\Delta r \equiv \underbrace{\mathbb{E} [r^A | r^A < r^B \leq \infty]}_{\text{interest rate of } A\text{'s loan}} - \underbrace{\mathbb{E} [r^B | r^B < r^A \leq \infty]}_{\text{interest rate of } B\text{'s loan}} < 0. \quad (28)$$

In Figure 1b, we plot the within-bank differences; it is the wedge of loan qualities or rates in the same bank's specialized and non-specialized industries. They indeed match the theoretical counterparts in (27) and (28): once we endogenize the information acquisition decisions in Section 5.2, the Bank A who specialized in industry a also serves as the non-specialized lender in industry b (which plays the same role as Bank B in industry a).

The positive loan quality wedge in (27), i.e., better-informed lenders are with higher quality loans, is driven by the information advantage of specialized lenders and hence a robust prediction of any information-based models. The following analysis thus focuses on the interest rate wedge (28) between two lenders.

4.2 Information Rent vs. Private-information-based Pricing

We now discuss two important economic forces in typical credit market competition models that drive the interest rate wedge (28).

Canonical models: information rent In canonical credit competition models a la Broecker (1990), information technology is parameterized as the signal precision, which captures the lenders’ ability to screen out uncreditworthy borrowers. Within this framework, the most natural way to capture “specialized lending” is by imposing asymmetric screening abilities, along the line of Marquez (2002); He, Huang, and Zhou (2023). Specifically, let lender $j \in \{A, B\}$ receive a signal regarding θ with the following binary distributions

$$\mathbb{P}(h^j = H | \theta = 1) = \mathbb{P}(h^j = L | \theta = 0) = \alpha^j,$$

with $\alpha^A > \alpha^B$ so that the specialized Bank A has a higher precision than the nonspecialized Bank B .²² As emphasized in the Introduction, here only quantity decisions (i.e., whether to lend or not) are based on the signal realizations while pricing decisions (offered interest rates) are randomized.

As Bank A ’s private signal is more precise, the weak lender B is more concerned about the winner’s curse, i.e., picking up a “lemon” that is assessed as L and rejected by the competitor lender. This in turn results in an information rent and monopoly power for Bank A : when $\alpha^A - \alpha^B > 0$ is sufficiently large, Bank B randomly withdraws even after receiving a favorable signal $h^B = H$, in which case Bank A is a monopolist.

This economic force, which we term as *information rent*, drives the specialized Bank A to have a higher loan rate than Bank B , opposite to the empirical regularity documented in Figure 1b. Proposition 2, combined with empirically relevant primitives calibrated in Appendix A.3,²³ implies that in canonical models Bank A ’s loans would have higher rates even when $\alpha^B \uparrow \alpha^A$. Presumably, the information rent effect is stronger when the gap in information technology, i.e., $\alpha^A - \alpha^B > 0$ is larger.²⁴ The formal theoretical result in Proposition 2 therefore allows us to argue that canonical models generate counterfactual empirical implications on rates.

²²This setting is nested in our model by shutting down the soft information possessed by Bank A , either simply setting the space of soft signal to be degenerate $\mathcal{S} = \{q_s\}$ or shrinking the breadth of soft information (to zero) as modeled in Section 4.3), and allowing for asymmetric hard signal informativeness among two lenders.

²³We calibrate q and α based on two empirical moments in the U.S. banking industry. First, according to this Federal Reserve report the non-performing loan (NPL) ratio is about 2%; second, Yates (2020) reports that the approval rate for business C&I loans ranges from 55% (small firms) to 80% (large firms). Matching to these two moments in Appendix A.3 we show that the implied parameters violate $q < 1 - \alpha + \alpha^2$ in Proposition 2. For instance, taking an approval rate of 70%, we obtain $q = 0.9629$ and $\alpha = 0.716$, which violate $q < 1 - \alpha + \alpha^2$. Note that our conclusion is independent of the parameter value of \bar{r} , which is harder to gauge. (One could set $\bar{r} = 36\%$ according to the usury law in many states that caps interest rates, but it only applies to consumer loans.)

²⁴Although we have not been able to prove this claim formally, it is confirmed in all of our numerical exercises.

Proposition 2. (Counterfactual Prediction in Canonical Models) *Suppose that $\alpha^A = \alpha$ and $\alpha^B \uparrow \alpha$. Then (28) holds (so that the interest rates of loans granted by the stronger bank are lower than those by the weaker bank) only if $\frac{1}{1+\bar{\tau}} \leq q < 1 - \alpha + \alpha^2$. The empirical relevant parameters on q and α , as shown in Appendix A.3, violate this condition.*

Private-information-based pricing By introducing Bank A 's informed rates offering, our model naturally generates the empirical regularity that “specialized banks feature lower observed rates” shown in Figure 1b. As illustrated by Panel A in Figure 2, the specialized Bank A who receives a more favorable soft signal about credit quality bids more aggressively (i.e., offers a lower rate) to win the borrower over the competitor Bank B . In fact, Bank A rejects the borrower when its soft signal falls below a certain threshold (i.e., $s < x$). In a positive-weak equilibrium, Bank B then enjoys some “local monopoly power” by having a point mass at $\bar{\tau}$ and is the only lender when Bank A rejects the customer upon $s < x$.²⁵

A special case of uniformly distributed soft signal To establish the robustness of our main result, in Proposition 3 we present an interesting special case where the two aforementioned effects—information rent and private-information-based pricing—equalize, and the lenders have the same interest rates on their granted loans. This special case corresponds to degenerate hard information and a uniformly distributed soft signal.

Proposition 3. (A Special Case of Uniform Distribution in Our Model) *Suppose that the hard signals are degenerate (either $\alpha^A = \alpha^B = 1$ or $\frac{1}{2}$), and the soft signal follows a uniform distribution $s \sim U[0, 1]$. When $\bar{\tau} = \infty$, we have $\Delta r = 0$ and $\pi^B = 0$.*

There are two important implications of this proposition. First, starting from this benchmark, any tilting toward the new force of private-information-based pricing—e.g., tilting more masses toward favorable soft signals—would generate the desired empirical regularity. Second, although our numerical examples show that $\Delta r < 0$ often arises in positive-weak equilibria, $\pi^B > 0$ is not necessary: the previous point provides one counter-example with $\pi^B = 0$ (under degenerate hard information.)

Information technologies and interest rate wedge We now study the comparative statics of the interest rate wedge Δr with respect to information technologies in our model, which are captured by two distinct parameters: the hard signal precision α that affects both lenders equally, and the soft signal precision τ that benefits only the specialized lender.

²⁵In contrast, in canonical models, even if the weak bank may earn profits given a high borrower surplus (say large $q, \bar{\tau}$), it never enjoys the “local” monopoly power—the strong bank never withdraws upon H while the weak bank never has a point mass at $\bar{\tau}$ (as the signal only determines participation).

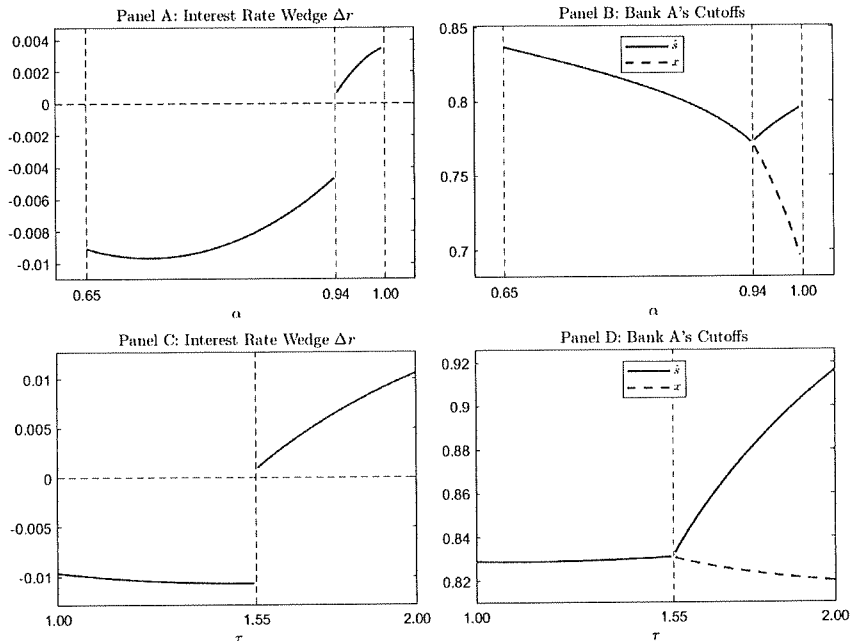


Figure 4: **Interest rate wedge and equilibrium strategies.** Panel A and Panel C depict $\Delta r = \mathbb{E}[r^A | r^A < r^B \leq \infty] - \mathbb{E}[r^B | r^B < r^A \leq \infty]$ as a function of α and τ . Panel B and Panel D depict Bank A's strategy cutoffs $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$ as functions of α and τ , where \hat{s} is in dashed line and x is in solid line. The positive-weak equilibrium arises when α or τ lies below certain value so that \hat{s} and x diverge. Parameters: $\bar{r} = 0.45$, $q_h = 0.8$, $q_s = 0.9$, $\tau = 1$ (top two panels) and $\alpha_u = \alpha_d = \alpha = 0.7$ (bottom two panels).

Figure 4 plots the interest rate wedge and Bank A's two cutoffs as a function of the information technology parameters. The general pattern is that when information technology improves—regardless of α (Panel A and B) or τ (Panel C and D)—the credit market competition is more likely to be in the zero-weak equilibrium where the nonspecialized Bank B is sufficiently “weak” and hence makes zero profits. This is intuitive because i) a higher hard signal precision α levels the playing field on hard information and hence effectively enlarges the soft information advantage of the specialized bank, and ii) a higher soft signal precision τ directly boosts the specialized bank's soft information advantage. Since the effect of private-information-based pricing tends to dominate in a positive-weak equilibrium, a sufficiently low information technology helps deliver a negative interest rate wedge, as shown in Panel A and C in Figure 4.

The comparative statics of increasing α or τ seems inconsistent with Figure 1b. There, we observe not only a negative interest rate wedge but also its discernible decreasing trend over the years, during which we witness remarkable advancement in information technology in the banking industry. We, however, believe that conducting comparative statics on precision parameters (α or τ) is an overly simplified approach and overlooks the numerous recent crucial innovations in infor-

mation technology. As we show next, our model, once incorporating multi-dimensional information, enables us to investigate alternative comparative statics, thereby providing a more profound insight into the observed pattern.

4.3 Information Span and Hard vs. Soft Information

A generalization with multiple characteristics To provide a framework for studying the relative importance of hard information versus soft information, consider the following simple extension of the main model. Suppose that the success of the project θ depends on N characteristics, again in a multiplicative way:

$$\theta = \prod_{n=1}^N \theta_n. \quad (29)$$

For each $n \in \{1, \dots, N\}$, we assume that $\{\theta_n\}$ follows independent Bernoulli distributions, i.e., $\theta_n = 1$ with probability $q_n \in [0, 1]$. When $N = 2$ the specification in Eq. (30) degenerates to the baseline model in Section 2 with hard and soft states (i.e., $\theta_h = \theta_1$ and $\theta_s = \theta_2$).

Suppose that, as the baseline model in Section 2, lenders can only access one hard signal and (potentially) one soft signal. More specifically, we assume that there exists a threshold N_h such that the signal collected from hard information is about characteristics below N_h while the signal collected from soft information is above. That is to say,

$$\theta = \prod_{n=1}^N \theta_n = \overbrace{\prod_{n=1}^{N_h} \theta_n}^{\theta_h} \cdot \overbrace{\prod_{n=N_h+1}^N \theta_n}^{\theta_s}. \quad (30)$$

The relative ranking of characteristics plays no role in our analysis, and it is clear that our previous analysis in Section 2.3 applies to the general case of $N > 2$ and $N_h < N$, once we specify

$$q_h = \prod_{n=1}^{N_h} q_n, \quad \text{and} \quad q_s = \prod_{n=N_h+1}^N q_n. \quad (31)$$

This is because θ_h , as a sufficient statistics of $\{\theta_n; n \in \{1, \dots, N_h\}\}$, takes the value of 1 with probability q_h ; a similar statement holds for θ_s which is a sufficient statistics of $\{\theta_n : n \in \{N_h + 1, \dots, N\}\}$. Given this, the extension maps exactly to the multiplicative setting in Section 2.2.

Now we take the hard characteristics cutoff N_h^η as a parameter. When N_h^η varies, say $N_h^\eta = N_h + N_\eta$ with $N_\eta \in \mathbb{Z}$, the information content covered by the hard signal varies.²⁶ Fixing the borrower quality θ ; the larger the N_η , the greater the hard signal's information content, as $\theta_h^\eta = \prod_{n=1}^{N_h+N_\eta} \theta_n$ covers more characteristics of the borrower. This way, the relative importance of the hard signal can be parameterized by N_h , which is directly linked to q_h and q_s as suggested by Eq. (31).

²⁶For $N_h^\eta \in [0, N]$ to be well defined we require $-N_h \leq N_\eta \leq N - N_h$.

To operationalize this idea, it is notationally more convenient to work with a continuum of characteristics. Consider the limit of the discrete characteristics outlined above with $n \in [0, 2]$. Suppose that the success probability of characteristic n is $\Pr(\theta_n = 1) = e^{(\ln q_n) \cdot dn} < 1$ with $q_n \in (0, 1)$.²⁷ Then, given independence, the prior probability of project success is

$$q = \exp\left(\int_0^2 \ln q_n dn\right) = \overbrace{\exp\left(\int_0^{N_h} \ln q_n dn\right)}^{q_h} \cdot \overbrace{\exp\left(\int_{N_h}^2 \ln q_n dn\right)}^{q_s}. \quad (32)$$

Our baseline model, similar to Eq. (31), can be recovered by setting q_h and q_s as in Eq. (32).

To study the relative importance of hard information, note when the hard information cutoff N_η varies we can define q_h^η and q_s^η as follows so that $q = q_h^\eta q_s^\eta$ still holds:

$$q = \underbrace{\exp\left(\int_0^{N_h} \ln q_n dn\right)}_{q_h} \cdot \overbrace{\exp\left(\int_{N_h}^{N_h+N_\eta} \ln q_n dn\right)}^{q_h^\eta} \cdot \underbrace{\exp\left(\int_{N_h+N_\eta}^2 \ln q_n dn\right)}_{q_s} \cdot \overbrace{\exp\left(\int_{N_h+N_\eta}^2 \ln q_n dn\right)}^{q_s^\eta}. \quad (33)$$

Information span and hardening soft information The framework developed here allows us to investigate the concept of “hardening soft information.” Introduce a new parameter η , which summarizes the span of hard information in a more succinct way: (the Greek letter η corresponds to the letter h)

$$\eta \equiv \exp\left(\int_{N_h}^{N_h+N_\eta} (-\ln q_n) dn\right). \quad (34)$$

The information span η is a monotone (decreasing) transformation of N_η ; and the model parameterized by η admits the solution given in Section 2.3 with a modified prior pair $\{q_h^\eta, q_s^\eta\}$:

$$q_h^\eta = \frac{q_h}{\eta}, \quad \text{and} \quad q_s^\eta = q_s \eta. \quad (35)$$

Note that $\eta > 1$ if and only if $N_\eta > 0$, i.e., when the hard signal encapsulates more characteristics. All else equal, the larger η , the broader the span of hard information, and the greater the hard signal’s information content (and capturing more of information that was soft previously).

Our model, by incorporating multi-dimensional information, highlights the distinction between the information span η and information technology parameters (say α). The former measures the scope/breadth of hard information while the latter measures the quality of hard information. Both are significant parts of the astonishing technological advancement in the past decades but with important differences. When the computer was introduced, it was faster and easier to process and compile bank statements. This improvement in processing made information more precise but did

²⁷We have $\Pr(\theta_n = 1) < 1$; this is because $e^{(\ln q_n) \cdot dn} \approx 1 + \ln q_n \cdot dn < 1$ if $q_n \in (0, 1)$.

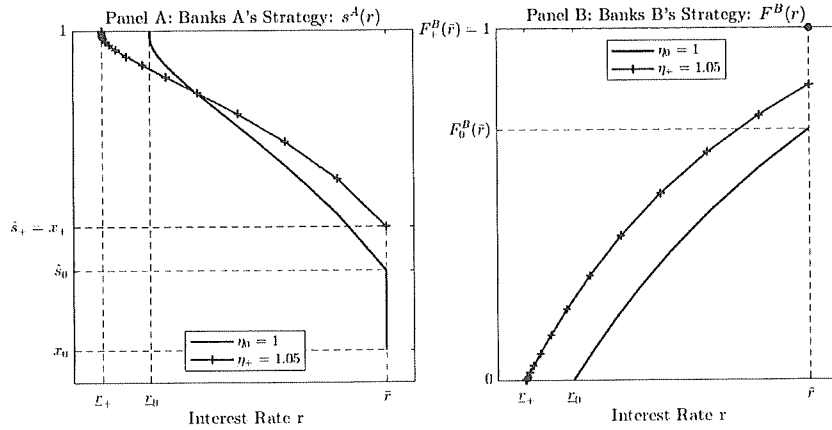


Figure 5: **Equilibrium strategies** $s^A(r)$ for Bank A (left) and $F^B(r)$ for Bank B (right). In both panels, strategies under η_+ (i.e., positive-weak equilibrium) are depicted in red with “+” markers while strategies with η_0 (i.e., zero-weak equilibrium) are depicted in blue. In the zero-weak equilibrium, Bank A (but not Bank B) has a point mass at \bar{r}_0 while in the positive-weak equilibrium, Bank B (but not Bank A) has a point mass at \bar{r}_+ . Parameters: $\bar{r} = 0.36$, $q_h = 0.8/\eta$, $q_s = 0.9\eta$, $\alpha_u = \alpha_d = \alpha = 0.7$, and $\tau = 1$.

not change its scope much. However, the use of “big data”, a distinctive trend in information technology during the last decade, has changed what can be inferred from hard information (think of Amazon predicting consumer preferences). In fact, as many scholars have argued, big data technology has expedited the process of “hardening soft information” by converting subjective or qualitative data (soft information) into more objective or quantifiable (hard) metrics; for recent evidence in the banking industry, see for example in Hardik (2023). By incorporating multi-dimensional information, our model allows us to study the distinction between these two economic forces, which, as we explain shortly, have distinctive economic implications regarding credit market competition.

Figure 5 plots two lenders’ equilibrium strategies as a function of r , when we vary η . Similar to Figure 2, we pick two levels of η so that one generates a zero-weak equilibrium while the other generates a positive-weak equilibrium. We observe that a higher η , i.e., when hard information span gets broader, a positive-weak equilibrium ensues (red lines with markers). In contrast to Figure 2 where we vary the exogenous good project returns, \bar{r} is fixed in Figure 5. Given a higher η , the endogenous lower bound \underline{r} adjusts downward, and more fierce competition leads to a wider range of observed interest rates. We explain the intuition in the next section.

Economic implications of information span η To understand the comparative statics of η , consider two polar cases. When $\eta \downarrow q_h$, which corresponds to the case of $N_\eta = -N_h < 0$ so the entire vector of characteristics is collected by the soft signal, our model is reduced to Milgrom and Weber (1982) so that only one of the two competing lenders is informed (has soft information). In

this limiting case, the nonspecialized Bank B is totally uninformed and hence earns zero profits in equilibrium. In contrast, when $\eta \uparrow \frac{1}{q_s}$, which corresponds to the case where $N_\eta = 2 - N_h > 0$ so the entire vector of characteristics is collected by the hard signal, our model is reduced to Broecker (1990) so that two banks who are competing on hard information are symmetric. From Broecker (1990) we know that both banks are making positive profits when \bar{r} is large enough.

The numerical example in Figure 5 is consistent with the above discussion on the two limiting cases. In fact, the next proposition formally establishes a desirable monotonicity, so that a positive-weak equilibrium arises if and only if when η is sufficiently high.

Proposition 4. (*Monotonicity of Information Span*) *There exists a cutoff $\hat{\eta}$ such that when $\eta \geq \hat{\eta}$, we are in the positive-weak equilibrium with $\pi^B > 0$ and while when $\eta < \hat{\eta}$ we are in the zero-weak equilibrium with $\pi^B = 0$.*

Figure 6 plots the model implied interest rate wedge against the information span η (Panel A), together with Bank A 's two endogenous cutoffs (\hat{s} and x , Panel B). We observe a different pattern from the one that emerges by changing information technology parameters (α and τ) in Figure 4, as the interest rate wedge turns negative when η rises. This is easy to understand given Proposition 4, since a higher η gives rise to positive-weak equilibria where the private-information-based pricing effect tends to dominate the information rent effect (Section 4.2).

The striking difference between the information span η and signal precision parameters (say α or τ) is worth highlighting. Take the hard information in our model as an example. As we have emphasized, α measures the quality of hard information while η captures the scope/breadth of hard information, and the latter has increased dramatically in the past decade—e.g., Amazon uses consumers' footprints to predict their preferences, reflecting the trend of “hardening soft information” where qualitative or subjective data are converted to objective or quantifiable (hard) metrics. A greater hard information precision α , which leads to an opposite prediction regarding interest rate wedge as shown in Figure 4, will miss this empirically important effect of “hardening soft information.”

4.4 Correlated Hard Signals

For a further illustration of the potentially different aspects of information technology advancement, Panels C and D in Figure 6 provide comparative statics with respect to the correlation $\rho_h \in [0, 1]$ of hard signals across two lenders. Recall from Remark 3 in Section 2.2 that our model can be easily extended to allow for correlated hard signals, as follows: with probability $\rho_h \in [0, 1]$ lenders receive the same binary signal realization $h^c \in \{H, L\}$, while with probability $1 - \rho_h$ each lender receives an independent binary hard signal. This captures the recent technology trend that the lenders' hard information signals become increasingly correlated; for instance, open banking regulation studied

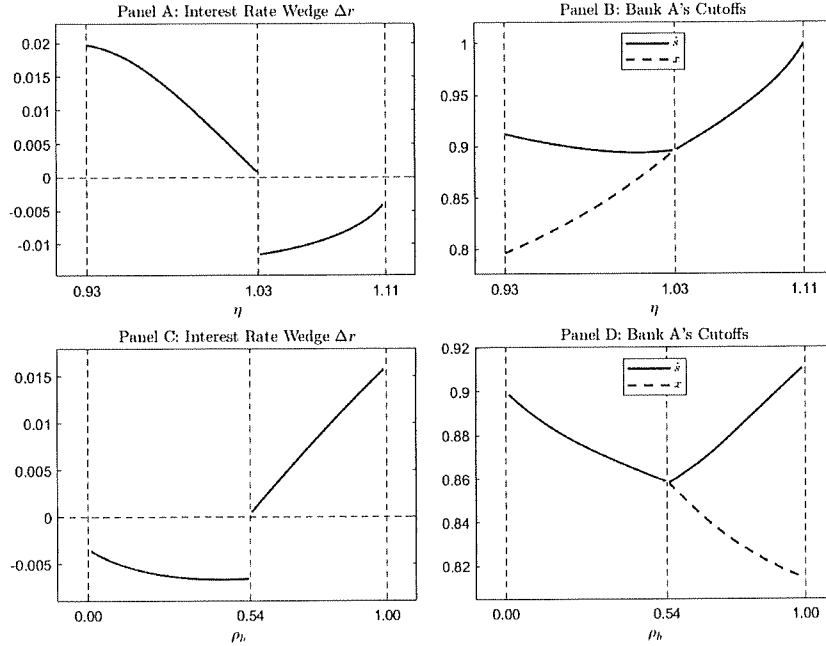


Figure 6: **Interest rate wedge and equilibrium strategies.** Panel A and Panel C depict $\Delta r = \mathbb{E}[r^A | r^A < r^B \leq \infty] - \mathbb{E}[r^B | r^B < r^A \leq \infty]$ as a function of η and ρ_h . Panel B and Panel D depict Bank A's strategy cutoffs $\hat{s} = \sup s^A(\bar{r})$ and $x = \sup s^A(\infty)$ as functions of η and ρ_h , where \hat{s} is in dashed line and x is in solid line. The positive-weak equilibrium arises when η exceeds a certain value or ρ_h falls below a certain value. Parameters: $\bar{r} = 0.36$, $\alpha_u = \alpha_d = \alpha = 0.7$, $\tau = 1$, $\rho_h = 0$, $q_h = 0.8/\eta$, $q_s = 0.9\eta$ (Panel A and B) and $q_h = 0.8$, $q_s = 0.9$ (Panel C and D).

in He, Huang, and Zhou (2023) and Babina, Buchak, and Gornall (2022). We provide a detailed analysis of this extension in Appendix A.6.

We observe in the bottom two panels on Figure 6 that a larger ρ_h leads a zero-weak equilibrium more likely to occur, hence is unable to generate the desired empirical regularity of $\Delta r < 0$. In the extreme case in which $\rho_h = 1$, the hard signal becomes a public signal, and Bank B who becomes effectively uninformed ends up with zero profit (Milgrom and Weber, 1982). It is also worth mentioning that, from the perspective of our model, the economic implications of ρ_h , which is more about data sharing, are qualitatively similar to that of changes in signal precision studied in Figure 4 but opposite to the ones focusing on information span investigated in this section.

5 Model Extensions and Discussions

In this section, we consider several model extensions, including general information structure as well as endogenous information acquisition before engaging in the credit market competition.

5.1 General Information Structure

The multiplicative setting has two key advantages in our main model. First, it captures the commonly observed lending practice; for example, the computer-based hard information signal is usually used as pre-screening and decisive for loan granting, while the soft information collected by the specialized bank tailors interest rate terms (see Remark 2). The multiplicative structure, by making the “hard” state decisive in project success, makes such lending strategies more likely to arise in equilibrium. In addition, the multiplicative structure also makes it tractable in introducing multiple characteristics as in Section 4.3, which allows us to analyze the breadth of different information technologies. However, our solution method applies beyond the multiplicative setting.

In this part, we solve for the credit market equilibrium under a general information structure. We impose two major assumptions. First, conditional on the project’s state θ , signals are independent across hard and soft and across lenders. Second, lenders only participate when the hard signal realization is H , with parameter restrictions in the same spirit as Assumption 1 but tailored for the general information structure. We keep the presentation minimal in the main text and provide a detailed analysis in the Appendix A.8.

Consider a general soft signal $z \sim \phi_z(z)$ for $z \in [\underline{z}, \bar{z}]$ where both \underline{z} and \bar{z} can be unbounded. Denote by $\mu_{h^A h^B}(z) \equiv \mathbb{P}(\theta = 1 | h^A, h^B, z)$ the posterior probability density for $\theta = 1$, i.e., the state of the project being successful. Without loss of generality, we assume that $\mu_{HH}(z)$ strictly increases in z (as we can always use $\mu_{HH}(z)$ as signal; recall the posterior s serves as the signal in the baseline model given in Section 2). This implies that just as in the baseline, there exists \hat{z} at which Bank A starts to quote \bar{r} , and z_x below which it starts rejecting borrowers. Let $\bar{\mu}_{h^A h^B} \equiv \mathbb{P}(\theta = 1 | h^A, h^B)$ denote the posterior probability of $\theta = 1$ based on hard signals.

Denote further by $p_{h^A h^B}(z) \equiv \mathbb{P}(h^A, z, h^B)$ and $\bar{p}_{h^A h^B} \equiv \mathbb{P}(h^A, h^B)$, and let $\alpha_u^j \equiv \mathbb{P}(h^j = H | \theta = 1)$ for $j \in \{A, B\}$, and $\phi_z(z | \theta = 1)$ be the density of z conditional on $\theta = 1$. The following proposition summarizes the credit market competition equilibrium with specialized lenders under this general information structure.

Proposition 5. (Credit Market Equilibrium under General Information Structure) *Lender $j \in \{A, B\}$ rejects the borrower (by quoting $r = \infty$) upon $h^j = L$; when $h^j = H$, lender j may make offers from a common support $[\underline{r}, \bar{r}]$ (or reject) with the following properties.*

1. Bank A who observes a soft signal z offers

$$r^A(z) = \begin{cases} \min \left\{ \frac{\pi^B + \int_{\underline{z}}^z p_{HH}(t) dt + \bar{p}_{LH}}{\int_{\underline{z}}^z p_{HH}(t) \cdot \mu_{HH}(t) dt + \bar{p}_{LH} \bar{\mu}_{LH}} - 1, \bar{r} \right\}, & \text{for } z \in [z_x, \bar{z}] \\ \infty, & \text{for } z \in [\underline{z}, z_x]. \end{cases} \quad (36)$$

This equation pins down $\underline{r} = r^A(\bar{z})$, $\hat{z} = \sup \{z : r^A(z) = \bar{r}\}$, and $z_x = \sup \{z : r^A(z) = \infty\}$.

2. Bank B makes an offer by randomizing its rate according to:

$$F^B(r) = \begin{cases} \frac{\alpha_u^A}{\alpha_u^B} \left[1 - \int_{\underline{z}}^{z^A(r)} \phi_z(t|\theta=1) dt \right], & \text{for } r \in [\underline{r}, \bar{r}), \\ 1 - \mathbf{1}_{\{\pi^B=0\}} \cdot \left\{ 1 - \frac{\alpha_u^A}{\alpha_u^B} \left[1 - \int_{\underline{z}}^{\bar{z}} \phi_z(t|\theta=1) dt \right] \right\}, & \text{for } r = \bar{r}. \end{cases} \quad (37)$$

3. The endogenous non specialized Bank B 's profit π^B is determined similarly to Lemma 2, with detailed expression provided in Appendix A.8.

Proposition 5 shows that the simple equilibrium structure does not depend on our multiplicative setting. Following the same logic as in the baseline model, lenders' customer quantities change proportionately with interest rates in equilibrium. To see this, when cutting interest rate at $r \in [\underline{r}, \bar{r})$, both lenders are competing for the same marginal borrowers, whose revenue should equal a unit loss from each lender's existing customer base so that lenders are indifferent and use a mixed strategy. For the existing customers, only the good type of customers who repay the loan matter. As a result, as long as soft and hard information are independent conditional on the project being successful, their effects on equilibrium strategies are separable, and a simple characterization as in Proposition 5 ensues.

5.2 Information Acquisition and Endogenous Specialization

Although the information structure is likely to be fixed in the short run, in the long run banks choose what type of information they want to have about borrowers. For example, banks can invest in equipment that allows them to analyze their existing transaction data more efficiently (hard information), or spend resources gathering information about specific borrowers (soft information). In this section, we look at the lender's information acquisition problem and derive conditions under which the information structure we study in the previous sections is an equilibrium outcome.

Setting and information acquisition technologies We introduce another borrower firm—which we call b —in addition to the borrower firm a in our baseline model. We may equally interpret a and b as different industries.

There are two types of technologies that respectively relate to “hard” information and “soft” information. For the “hard” information technology, a lender j invests once in equipment at a cost of κ_h , which allows the lender to process data (say financial and operating data) and produce a *hard information* based private signal $h_i^j \in \{H, L\}$ for each firm $i = a, b$, independently (across two lenders and two firms). This captures the idea that hard information is collected via standardized and transferable data such as credit reports and income statements, so once the IT equipment, software, and APIs are installed, credit analysis is easy to implement on multiple firms and the information generated is also standardized and coarse.

For the “specialized/soft” information technology, a lender needs to collect soft information on firms one by one. Due to symmetry, we omit the indexation for firm i from now on. Lender A specializes in firm a if it spends κ_s to acquire a *soft information* based private signal s^A , whose smooth distribution is characterized by the CDF $\Phi(s)$ and pdf $\phi(s)$ for $s \in \mathcal{S} \equiv [0, 1]$. If a bank wants to acquire soft information about both firms, it needs to pay $2\kappa_s$.

We are interested in the following equilibrium: Bank A (B) endogenously specializes in firm a (b)—i.e., acquires both hard and soft signals on firm a (b)—and competes with the other non-specialized Bank B (A) who only acquires hard signal on firm a (b). The baseline model analyzed in Section 2.3 is the subgame for either firm following the equilibrium information acquisition strategies.

Incentive compatibility conditions Banks are making their information acquisition decisions simultaneously. Moreover, we assume that information acquisition is observable when banks enter the credit market competition game. This implies that a lender’s deviation from the proposed equilibrium information acquisition will lead to a different information structure in the credit market competition. Hence, to examine the incentives of banks to acquire each type of information, we need to define the bank’s lending profits in all possible subgames following a deviation.

Denote by $\Pi_j^i(I_A^h, I_A^s, I_B^s, I_B^h)$ the expected lending profits of bank j in industry i when the information structure in industry i is given by $(I_A^h, I_A^s, I_B^h, I_B^s)$, where I_j^h and I_j^s take value of one if bank j acquired hard information and soft information in industry i , respectively, and zero otherwise. The symmetry on industries implies that a bank’s expected lending profits in industry i only depend on the information structure in that industry and not on the industry itself, i.e.,

$$\Pi_j^a(I_A^h, I_A^s, I_B^s, I_B^h) = \Pi_j^b(I_A^h, I_A^s, I_B^h, I_B^s). \quad (38)$$

Therefore, we drop index i from the expected lending profits. What is more, we focus on Bank A ’s incentives in what follows since the no deviation conditions for banks A and B are symmetric.

Bank A can deviate along three dimensions: it can choose not to acquire hard information, it can choose not to acquire soft information about industry a , and it can choose to acquire soft information in industry b . Bank A ’s incentives to deviate along these dimensions will depend on the costs of acquiring information. As one would expect, the lower the cost of acquiring hard information, the more likely Bank A has incentives to acquire hard information and not deviate along this dimension. For deviations along the soft information dimension, the cost of acquiring soft information has to be low enough such that it is worth acquiring soft information in industry a and having an informational advantage over Bank B in this industry but high enough such that it is not worth acquiring soft information in industry b to stop being the less informed lender. This intuition can be formally stated in the following incentive compatibility constraints. Bank A does

not want to deviate by

1. not acquiring hard information

$$\begin{aligned} & \Pi_A \left(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 0 \right) - \Pi_A \left(I_A^h = 0, I_A^s = 1, I_B^h = 1, I_B^s = 0 \right) + \\ & \Pi_A \left(I_A^h = 1, I_A^s = 0, I_B^h = 1, I_B^s = 1 \right) - \Pi_A \left(I_A^h = 0, I_A^s = 0, I_B^h = 1, I_B^s = 1 \right) \geq \kappa_h; \end{aligned} \quad (\text{H})$$
2. not acquiring hard information nor soft information in industry a

$$\begin{aligned} & \Pi_A \left(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 0 \right) - \Pi_A \left(I_A^h = 0, I_A^s = 0, I_B^h = 1, I_B^s = 0 \right) + \\ & \Pi_A \left(I_A^h = 1, I_A^s = 0, I_B^h = 1, I_B^s = 1 \right) - \Pi_A \left(I_A^h = 0, I_A^s = 0, I_B^h = 1, I_B^s = 1 \right) \geq \kappa_h + \kappa_s; \end{aligned} \quad (\text{NI})$$
3. not acquiring soft information in industry a

$$\Pi_A \left(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 0 \right) - \Pi_A \left(I_A^h = 1, I_A^s = 0, I_B^h = 1, I_B^s = 0 \right) \geq \kappa_s; \quad (\text{Sa})$$
4. and, acquiring soft information in industry b

$$\Pi_A \left(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 1 \right) - \Pi_A \left(I_A^h = 1, I_A^s = 0, I_B^h = 1, I_B^s = 1 \right) \leq \kappa_s. \quad (\text{NSb})$$

Consistent with the intuition above, Constraints (H) and (NI) impose an upper bound on κ_h so that Bank A has incentives to acquire hard information. Analogously, Constraints (NI) and (Sa) impose an upper bound on κ_s so that Bank A wants to acquire soft information in industry a , while Constraint (NSb) imposes a lower bound on κ_s to assure Bank A does not want to acquire soft information in industry b .

Deviation payoffs. Our goal is to show that there exist costs κ_h and κ_s such that the conditions above hold for some parameterization. To do so, we need to characterize the deviation payoffs. We provide the expressions for $\Pi_A \left(I_A^h, I_A^s, I_B^h, I_B^s \right)$ in Appendix A.7.

Note that an uninformed bank will make zero profits (Milgrom and Weber, 1982; Engelbrecht-Wiggans, Milgrom, and Weber, 1983), i.e,

$$\Pi_A \left(I_A^h = 0, I_A^s = 0, I_B^h = 1, I_B^s = 0 \right) = \Pi_A \left(I_A^h = 0, I_A^s = 0, I_B^h = 1, I_B^s = 1 \right) = 0.$$

Then it follows that Constraint (NI) is equivalent to the participation constraint of Bank A . Moreover, this condition implies that for any cost of acquiring soft information κ_s such that (Sa) is satisfied, we can always find a cost of hard information κ_h small enough to satisfy (H) and (NI). Therefore, there will be an equilibrium with specialized lenders as long as κ_s satisfies the bounds imposed by (Sa) and (NSb). For this to be the case, it is enough to find parameters such that the benefits from acquiring soft information to become the more informed lender are greater than the benefits from acquiring soft information to stop being the less informed lender. This is confirmed in Figure 7 in Appendix A.7, which depicts the range of information acquisition costs κ_h and κ_s so that the conjectured information structure with a specialized lender and the ensuing lending competition indeed form an equilibrium.

6 Concluding Remarks

One of banks' main roles in the economy is producing information to allocate credit. In this paper, we show that the nature of information produced by banks affects the credit market equilibrium and the degree of competition among banks. More specifically, we explore how multi-dimensional information determines credit market outcomes in the presence of specialized lenders.

By considering soft and hard information, we can explain empirical patterns in bank lending specialization unexplained by canonical models where information technology is solely characterized by signal precision (one-dimensional). Moreover, our model with multiple sources of uncertainty and information allows us to differentiate between the quality and breadth of information. This distinction is crucial in understanding the changing landscape in the credit market due to technological advances related to data gathering and processing that lead to the hardening of soft information.

From a modeling perspective, including a continuously distributed signal within a credit market equilibrium enables us to examine private-information-based pricing, a practically pertinent aspect with crucial importance for the banking sector. Furthermore, by incorporating both soft and hard information—which reflects potentially many more underlying states—among asymmetric lenders, our paper markedly advances the field of auction literature involving such lenders in which each lender possesses private information (in contrast to Milgrom and Weber (1982) where one bidder knows strictly more than the other). We fully characterize the equilibrium in closed form and anticipate broader applications based on our framework and the solution methodology.

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A Technical Appendices

A.1 Credit Competition Equilibrium

Proof of Lemma 1

Proof. Point 1 directly follows from Assumption 1. In this proof, we show that the distributions of interest rate offered are well behaved. We postpone the proof of Bank A using a pure strategy to the second point discussion of Lemma 3 which shows the monotonicity of $r^A(s)$.

Specifically, we show that the two lenders' interest rate distributions have the following properties:

- a) they share the same lower bound $\underline{r} > 0$ and the same upper bound \bar{r} in their supports;
- b) they have no gaps in their supports;
- c) they have no mass points except that one of them can have one at \bar{r} .

Note that Property b) implies Property a), because if a bank's interest rate offering has a larger lower bound or a smaller upper bound interest rate than its competitor's, this is one example of gaps in the first bank's support.

To show Property b), suppose that, say, the support of F^B has a gap $(r_1, r_2) \subset [\underline{r}, \bar{r}]$.²⁸ Then F^A should have no weight in this interval either, as any $r^A(s) \in (r_1, r_2)$ will lead to the same demand for Bank A and so a higher r will be more profitable. At least one lender does not have a mass point at r_1 (it is impossible that both distributions have a mass point at \bar{r}_1), under which its competitor has a profitable deviation by revising r_1 to $r \in (r_1, r_2)$ instead. Contradiction.

To show Property c), suppose that one distribution, say F^B has a mass point at $\tilde{r} \in [\underline{r}, \bar{r}]$. Then Bank A would not quote any $r^A(s) \in [\tilde{r}, \tilde{r} + \epsilon]$ and it would strictly prefer quoting $r^A = \tilde{r} - \epsilon$ instead. In other words, the support of F^A must have a gap in the interval $[\tilde{r}, \tilde{r} + \epsilon]$. This contradicts with Property b) which we have shown. Finally it is impossible that both distributions have a mass point at \bar{r} .

A.2 Proof of Proposition 1

Proof. This part proves that Bank A 's equilibrium interest rate quoting strategy as a function of soft signal $r^A(s)$ is always decreasing; this implies that the FOC that helps us derive Bank A 's strategy also ensures the global optimality.

²⁸The same argument follows if the support of F^A has a gap in the conjectured equilibrium, and then for Bank B , any quotes within the gap lead to the same demand of the same posterior quality of customers, where Bank B updates its belief from Bank A 's strategy.

Write Bank A 's value $\Pi^A(r, s)$ as a function of its interest rate quote and soft signal, in the event of $h^A = H$ and s . (We use π to denote the equilibrium profit but Π for any strategy.) Recall that Bank A solves the following problem:

$$\max_r \Pi^A(r, s) = \underbrace{p_{HH}}_{h^A=H, h^B=H} \underbrace{[1 - F^B(r)]}_{A \text{ wins}} [\mu_{HH}s(1+r) - 1] + \underbrace{p_{HL}}_{h^A=H, h^B=L} [\mu_{HL}s(1+r) - 1] \quad (39)$$

with the following FOC:

$$0 = \Pi_r^A(r(s), s) = \underbrace{p_{HH} \left[-\frac{dF^B(r)}{dr} \right]}_{\text{lost customer}} \underbrace{\left[\mu_{HH}s(1+r) - 1 \right]}_{\text{customer return}} + \underbrace{p_{HH} [1 - F^B(r)]}_{\text{customer}} \underbrace{\mu_{HH}s}_{\text{MB of customer}} + p_{HL}\mu_{HL}s. \quad (40)$$

One useful observation is that on the support, it must hold that $\mu_{HH}s(1+r) - 1 > 0$; otherwise, $\mu_{HL}s(1+r) - 1 < \mu_{HH}s(1+r) - 1 \leq 0$, implying that Bank A 's profit is negative (so it will exit).

Lemma 3. *Consider s_1, s_2 in the interior domain with corresponding interest rate quote r_1 and r_2 . The marginal value of quoting r_2 for type $s = s_1$ is*

$$\Pi_r^A(r_2, s_1) = \frac{s_2 - s_1}{\mu_{HH}s_2(1+r_2) - 1} \left\{ p_{HH} [1 - F^B(r_2)] \mu_{HH} + p_{HL}\mu_{HL} \right\}$$

and its sign depends on the sign of $s_2 - s_1$.

Proof. Evaluating the FOC of type s_1 but quoting r_2 :

$$\Pi_r^A(r_2, s_1) = p_{HH} \left[-\frac{dF^B(r_2)}{dr} \right] [\mu_{HH}s_1(1+r_2) - 1] + p_{HH} [1 - F^B(r_2)] \mu_{HH}s_1 + p_{HL}\mu_{HL}s_1. \quad (41)$$

FOC at type s_2 yields

$$\Pi_r^A(r_2, s_2) = p_{HH} \left[-\frac{dF^B(r_2)}{dr} \right] [\mu_{HH}s_2(1+r_2) - 1] + p_{HH} [1 - F^B(r_2)] \mu_{HH}s_2 + p_{HL}\mu_{HL}s_2 = 0,$$

or

$$\frac{dF^B(r_2)}{dr} = \frac{p_{HH} [1 - F^B(r_2)] \mu_{HH}s_2 + p_{HL}\mu_{HL}s_2}{p_{HH} [\mu_{HH}s_2(1+r_2) - 1]}. \quad (42)$$

Plugging in this term to (41), $\Pi_r^A(r_2, s_1)$ becomes

$$\begin{aligned} & -\frac{\mu_{HH}s_1(1+r_2)-1}{\mu_{HH}s_2(1+r_2)-1} \left\{ p_{HH} [1-F^B(r_2)] \mu_{HH}s_2 + p_{HL}\mu_{HL}s_2 \right\} + p_{HH} [1-F^B(r_2)] \mu_{HH}s_1 + p_{HL}\mu_{HL}s_1 \\ & = \left[s_1 - \frac{\mu_{HH}s_1(1+r_2)-1}{\mu_{HH}s_2(1+r_2)-1} \cdot s_2 \right] \left\{ p_{HH} [1-F^B(r_2)] \mu_{HH} + p_{HL}\mu_{HL} \right\} \\ & = (s_2 - s_1) \cdot \frac{p_{HH} [1-F^B(r_2)] \mu_{HH} + p_{HL}\mu_{HL}}{\mu_{HH}s_2(1+r_2)-1}, \end{aligned}$$

which is the claimed marginal benefit of quoting r_2 for type s_1 . Its sign depends on $s_2 - s_1$ because the denominator is positive as we noted right after Eq. (40). \square

Lemma 3 has three implications. First, as long as $r^A(\cdot)$ is (strictly) increasing in some segment, then Bank A would like to deviate in this segment. To see this, suppose that $r_1 > r_2$ when $s_1 > s_2$ for s_1, s_2 arbitrarily close. Because Lemma 1 has shown that Bank A 's strategy is smooth, r_2 is arbitrarily close to r_1 . Then $\Pi_r^A(r_2, s_1) < 0$, implying that the value is convex and the Bank A at s_1 (who in equilibrium is supposed to quote r_1) would like to deviate further.

Second, the monotonicity implied by Lemma 3 helps us show that Bank A uses a pure strategy, thereby completing the proof of Part 2 in Lemma 1. To see this, for any $s_1 > s_2$ that induce interior quotes $r_1, r_2 \in [\underline{r}, \bar{r})$, however close, in equilibrium we must have $\sup r^A(s_1) < \inf r^A(s_2)$ by monotonicity. Combining this with Part 3 of Lemma 1, i.e., the induced distribution $F^A(\cdot)$ is atomless except for at \bar{r} and has no gaps, we know that Bank A must adopt a pure strategy in the interior of $[\underline{r}, \bar{r})$, or for $s \leq \hat{s}$. Finally, the following argument shows pure strategy for $s < \hat{s}$: i) randomize over $s = 0$ is a zero-measure set; and ii) on $s > \hat{s}$ Bank A can either quote \bar{r} or ∞ , which, generically, gives different values (and hence rules out randomization).

Third, if $r^A(\cdot)$ is decreasing globally over \mathcal{S} , then the FOC is sufficient to ensure global optimality. Consider a type s_1 who would like to deviate to $\check{r} > r_1$; then

$$\Pi^A(\check{r}, s_1) - \Pi^A(r_1, s_1) = \int_{r_1}^{\check{r}} V_r^A(r, s_1) dr.$$

Given the monotonicity of $r(s)$, we can find the corresponding type $s(r)$ for $r \in [r_1, \check{r}]$. From Lemma 3 we know that

$$\Pi_r^A(r, s_1) = (s(r) - s_1) \frac{p_{HH} [1-F^B(r)] \mu_{HH} + p_{HL}\mu_{HL}}{\mu_{HH}s(r)(1+r)-1}$$

which is negative given $s(r) < s_1$. Therefore the deviation gain is negative. Similarly we can show a negative deviation gain for any $\check{r} < r_1$.

Next we show that $r^A(\cdot)$ is single-peaked over the space of $\mathcal{S} = [0, 1]$.

Lemma 4. *Given any exogenous $\pi^B \geq 0$, $r^A(\cdot)$ single-peaked over $S = [0, 1]$ with a maximum point.*

Proof. When $r \in [\underline{r}, \bar{r}]$, the derivative of $r^A(s)$ with respect to s is

$$\frac{dr^A(s)}{ds} = \frac{p_{HH}\phi(s) \left(\overbrace{p_{HH}\mu_{HH} \left[\int_0^s t\phi(t) dt - s\Phi(s) \right]}^{M_1(s) < 0, \text{ and } M_1'(s) < 0} + \overbrace{p_{LH}\mu_{LH}q_s - (\pi^B + p_{LH})\mu_{HH}s}^{M_2(s) ? 0, \text{ but } M_2'(s) < 0} \right)}{(p_{HH}\mu_{HH} \int_0^s t\phi(t) dt + p_{LH}\mu_{LH}q_s)^2}.$$

As $\int_0^s t\phi(t) dt < s\Phi(s)$, the first term in the bracket $M_1(s) < 0$, and

$$M_1'(s) = -p_{HH}\mu_{HH}\Phi(s) < 0.$$

For $M_2(s) = p_{LH}\mu_{LH}q_s - (\pi^B + p_{LH})\mu_{HH}s$, it has an ambiguous sign, but is decreasing in s . This implies that $M_1(s) + M_2(s)$ decreases with s . It is easy to verify that $M_1(0) + M_2(0) > 0$ and $M_1(1) + M_2(1) < 0$. Therefore $r^A(s)$ first increases and then decreases, therefore single-peaked. \square

Suppose that the peak point is \tilde{s} ; then Bank A should simply charge $r(s) = \tilde{r}$ for $s < \tilde{s}$ for better profit. This is the standard “ironing” technique and we therefore define the following ironed strategy formally (here, we also take care of the capping $r \leq \bar{r}$):

$$r_{ironed}^A(s) \equiv \sup_{t \in [s, 1]} \min(r^A(t), \bar{r}).$$

By definition $r_{ironed}^A(s)$ is monotonely decreasing.

We now argue that in equilibrium, π^B and \underline{r} adjust so that $r^A(\cdot)$ is always monotonely decreasing over $[x, 1]$. (Since we define $r^A(s) = \infty$ for $s < x$, monotonicity over the entire signal space $[0, 1]$ immediately follows.) There are two subcases to consider.

1. Suppose that $\tilde{r} = \bar{r}$. In this case, $r^A(s)$ in Eq. (12) used in Lemma 3 and 4 does not apply to $s < \tilde{s}$ because the equation is defined only over the left-closed-right-open interval $[\underline{r}, \bar{r}]$. Instead, $r^A(s)$ in this region is determined by Bank A 's optimality condition: as r^A does not affect the competition from Bank B (which equals $F^B(\bar{r}^-)$), Bank A simply sets the maximum possible rate $r^A(r) = \bar{r}$ unless it becomes unprofitable (for $s < x$). (This is our zero-weak equilibrium with $\pi^B = 0$, and there is no “ironing” in this case.)
2. Suppose that $\tilde{r} < \bar{r}$; then bank A quotes \tilde{r} for all $s < \hat{s}$. But this is not an equilibrium—Bank A now leaves a gap in the interval $[\tilde{r}, \bar{r}]$, contradicting with point 3) in Lemma 1 (there, we rule out gaps in equilibrium). Intuitively, Bank B always would like to raise its quotes inside

$[\bar{r}, \bar{r}]$ to \bar{r} ; there is no “ironing” in this case. (This is our positive-weak equilibrium with $\pi^B > 0$.)

□

Proof of Lemma 2

Proof. First, we argue that equilibrium $\hat{s} \equiv \arg \sup_s \{s : r^A(s) \geq \bar{r}\}$ either equals \hat{s}_A^{BE} or \hat{s}_B^{BE} . To see this, if $\pi^B = 0$, we have $\hat{s} = \hat{s}_B^{BE}$ by construction. If $\pi^B > 0$, then Bank B always makes an offer upon H, i.e., $F^B(\bar{r}) = 1$. We also know that $F^B(\bar{r}^-) = 1 - \frac{\int_0^{s^A(r)=\bar{r}^+} t\phi(t)dt}{q_s} < 1$, because Bank A must reject the borrower when s realizes as close to 0 and $\hat{s} > 0$. Hence, $F^B(r)$ has a point mass at \bar{r} . It follows that $F^A(r)$ is open at \bar{r} : $\hat{s} = x$ and $\pi^A(r^A(\hat{s})|\hat{s}) = 0$, which is exactly the definition of \hat{s}_A^{BE} and so $\hat{s} = \hat{s}_A^{BE}$.

Now we prove the claim in this lemma. In the first case of $\hat{s}_B^{BE} < \hat{s}_A^{BE}$, we have $\hat{s} \leq \hat{s}_A^{BE}$ and thus Bank A’s probability of winning when quoting $r^A = \bar{r}$ is at most $\frac{\int_0^{\hat{s}_A^{BE}} t\phi(t)dt}{q_s} \geq \frac{\int_0^{\hat{s}} t\phi(t)dt}{q_s} = 1 - F^B(\bar{r}^-)$. The definition of \hat{s}_A^{BE} says that Bank A upon \hat{s}_A^{BE} breaks even when quoting $r^A(\hat{s}_A^{BE}) = \bar{r}$ and facing this most favorable winning probability $\frac{\int_0^{\hat{s}_A^{BE}} t\phi(t)dt}{q_s}$. Then upon a worse soft signal $\hat{s}_B^{BE} < \hat{s}_A^{BE}$, Bank A must reject the borrower because offering \bar{r} leads to losses, which rules out $\hat{s} = \hat{s}_B^{BE}$. According to our earlier observation of $\hat{s} = \hat{s}_B^{BE}$ or \hat{s}_A^{BE} , we have $\hat{s} = \hat{s}_A^{BE}$ and $\pi^B > 0$ in this case, where π^B could be characterized from Eq. (11) at $r = \bar{r}$.

In the second case of $\hat{s}_B^{BE} \geq \hat{s}_A^{BE}$, we have $\hat{s} \leq \hat{s}_B^{BE}$ and thus Bank B’s probability of winning when quoting $r^B = \bar{r}$ is at most $\Phi(\hat{s}_B^{BE}) \geq \Phi(\hat{s}) = 1 - F^A(\bar{r}^-)$. The definition of \hat{s}_B^{BE} says that Bank B breaks even when quoting $r^B = \bar{r}$ and facing this most favorable winning probability $\Phi(\hat{s}_B^{BE})$. Then if the competition from A were more aggressive, say $1 - F^A(\bar{r}^-) = \Phi(\hat{s}_A^{BE})$, Bank B would make a loss when quoting \bar{r} , so $\hat{s} = \hat{s}_A^{BE}$ cannot support an equilibrium. Hence, in this case, $\hat{s} = \hat{s}_B^{BE}$ and $\pi^B = 0$. From the definition of \hat{s}_A^{BE} , Bank A’s equilibrium break-even condition $0 = \pi^A(\bar{r}|x)$, and the fact that $\hat{s}_B^{BE} \geq \hat{s}_A^{BE}$ in this case, we have

$$\begin{aligned} 0 &= \frac{p_{HH} \int_0^{\hat{s}_A^{BE}} t\phi(t) dt}{q_s} [\mu_{HH}\hat{s}_A^{BE}(1+\bar{r}) - 1] + p_{HL} [\mu_{HL}\hat{s}_A^{BE}(1+\bar{r}) - 1] \\ &= \frac{p_{HH} \int_0^{\hat{s}_B^{BE}} t\phi(t) dt}{q_s} [\mu_{HH}x(1+\bar{r}) - 1] + p_{HL} [\mu_{HL}x(1+\bar{r}) - 1] \\ &\geq \frac{p_{HH} \int_0^{\hat{s}_A^{BE}} t\phi(t) dt}{q_s} [\mu_{HH}x(1+\bar{r}) - 1] + p_{HL} [\mu_{HL}x(1+\bar{r}) - 1]. \end{aligned}$$

Hence, $x \leq \hat{s}_A^{BE} \leq \hat{s}_B^{BE} = \hat{s}$.

□

A.3 Proof of Proposition 2 and Calibration

Lemma 5. For any $r \in [\underline{r}, \bar{r})$, we have

$$\frac{F^B(r)}{F^A(r)} = \frac{\alpha^A}{\alpha^B}, \quad \frac{dF^B(r)/dr}{dF^A(r)/dr} = \frac{\alpha^A}{\alpha^B}$$

i.e., Bank A offers higher interest rates than Bank B in the sense of F.O.S.D..

Proof. For any $r \in [\underline{r}, \bar{r})$, lenders' profit functions are

$$\pi^A = \underbrace{p_{HH}}_{B \text{ gets H}} \underbrace{(1 - F^B(r))}_{\text{wins}} [\mu_{HH}(r+1) - 1] + \underbrace{p_{HL}}_{B \text{ gets L}} [\mu_{HL}(r+1) - 1], \quad (43)$$

$$\pi^B = \underbrace{p_{HH}}_{A \text{ gets H}} \underbrace{(1 - F^A(r))}_{\text{wins}} [\mu_{HH}(r+1) - 1] + \underbrace{p_{LH}}_{A \text{ gets L}} [\mu_{LH}(r+1) - 1]. \quad (44)$$

These two equations imply that

$$\frac{F^B(r)}{F^A(r)} = \frac{p_{HH} [\mu_{HH}(r+1) - 1] + p_{HL} [\mu_{HL}(r+1) - 1] - \pi^A}{p_{HH} [\mu_{HH}(r+1) - 1] + p_{LH} [\mu_{LH}(r+1) - 1] - \pi^B}. \quad (45)$$

And, evaluating Eq. (43), (44) at $r = \underline{r}$ and using $F^A(\underline{r}) = F^B(\underline{r}) = 1$ gives lenders' profits:

$$\begin{aligned} \pi^A(\underline{r}) &= p_{HH} [\mu_{HH}(\underline{r}+1) - 1] + p_{HL} [\mu_{HL}(\underline{r}+1) - 1], \\ \pi^B(\underline{r}) &= p_{HH} [\mu_{HH}(\underline{r}+1) - 1] + p_{LH} [\mu_{LH}(\underline{r}+1) - 1]. \end{aligned}$$

Using these in Eq. (45), we have

$$\frac{F^B(r)}{F^A(r)} = \frac{(p_{HH}\mu_{HH} + p_{HL}\mu_{HL})(r - \underline{r})}{(p_{HH}\mu_{HH} + p_{LH}\mu_{LH})(r - \underline{r})} = \frac{\mathbb{P}(h^A = H, \theta = g)}{\mathbb{P}(h^B = H, \theta = g)} = \frac{\alpha^A}{\alpha^B}.$$

Here, $F^B(r) = \frac{\alpha^A}{\alpha^B} F^A(r)$ immediately implies that $\frac{dF^B(r)/dr}{dF^A(r)/dr} = \frac{\alpha^A}{\alpha^B}$. \square

Lemma 6. $\mathbb{E}[r^A | r^A < r^B \leq \infty] \geq \mathbb{E}[r^B | r^B < r^A \leq \infty]$ is equivalent to the following inequality

$$\begin{aligned} & \frac{\mathbb{P}(x^A = H) \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} F^B(r) dr + p_{HH} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) - p_{HH} \bar{r} \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2}{p_{HH} \left[1 - F^B(\bar{r}) + \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2 \right] + p_{HL}} \\ & \leq \frac{\mathbb{P}(x^B = H) \int_{\underline{r}}^{\bar{r}} F^B(r) dr + p_{HH} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) - p_{HH} \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2 \bar{r}}{p_{HH} \left[F^B(\bar{r}) - \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2 \right] + p_{LH} F^B(\bar{r})}. \end{aligned}$$

Proof. The expected rate of a lender's loan is

$$\mathbb{E} \left[r^A \mid r^A < r^B \leq \infty \right] \triangleq \frac{\underbrace{p_{HH}}_{\text{B gets H}} \int_{\underline{r}}^{\bar{r}} \underbrace{[1 - F^B(r)]}_{\text{A wins}} r dF^A(r) + \underbrace{p_{HL}}_{\text{B gets L}} \int_{\underline{r}}^{\bar{r}} r dF^A(r)}{p_{HH} \int_{\underline{r}}^{\bar{r}} [1 - F^B(r)] dF^A(r) + p_{HL}}, \quad (46)$$

$$\mathbb{E} \left[r^B \mid r^B < r^A \leq \infty \right] \triangleq \frac{\underbrace{p_{HH}}_{\text{A gets H}} \int_{\underline{r}}^{\bar{r}} \underbrace{[1 - F^A(r)]}_{\text{B wins}} r dF^B(r) + \underbrace{p_{LH}}_{\text{A gets L}} \int_{\underline{r}}^{\bar{r}} r dF^B(r)}{p_{HH} \int_{\underline{r}}^{\bar{r}} [1 - F^A(r)] dF^B(r) + p_{LH} F^B(\bar{r})}. \quad (47)$$

In the first step, we rewrite the equations as functions of $dF^B(r)$ and dr which are continuous at \bar{r} . Using integration by parts and Lemma 5, we have

$$\int_{\underline{r}}^{\bar{r}} r dF^A(r) = r F^A(r) \Big|_{\underline{r}}^{\bar{r}} - \int_{\underline{r}}^{\bar{r}} F^A(r) dr = \bar{r} - \int_{\underline{r}}^{\bar{r}} F^A(r) dr = \bar{r} - \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} F^B(r) dr.$$

In the last step, although Lemma 5 does not apply at $r = \bar{r}$, it is of zero measure. Similarly, the probability of Bank A winning in competition is

$$\begin{aligned} \int_{\underline{r}}^{\bar{r}} [1 - F^B(r)] dF^A(r) &= \int_{\underline{r}}^{\bar{r}} dF^A(r) - \int_{\underline{r}}^{\bar{r}} F^B(r) dF^A(r) \\ &\stackrel{\text{integration by parts}}{=} 1 - \left[F^B(\bar{r}) - \int_{\underline{r}}^{\bar{r}} F^A(r) dF^B(r) \right] \\ &\stackrel{F^A = \frac{\alpha^B}{\alpha^A} F^B}{=} 1 - F^B(\bar{r}) + \int_{\underline{r}}^{\bar{r}} \frac{\alpha^B}{\alpha^A} F^B(r) dF^B(r) \\ &= 1 - F^B(\bar{r}) + \frac{\alpha^B}{2\alpha^A} \left(F^B(\bar{r}) \right)^2, \end{aligned}$$

and thus the probability of Bank B winning is the residual

$$\int_{\underline{r}}^{\bar{r}} [1 - F^A(r)] dF^B(r) = F^B(\bar{r}) - \frac{\alpha^B}{2\alpha^A} \left(F^B(\bar{r}) \right)^2.$$

Similarly,

$$\begin{aligned} \int_{\underline{r}}^{\bar{r}} F^B(r) r dF^A(r) &= \int_{\underline{r}}^{\bar{r}^-} F^B(r) r dF^A(r) + F^B(\bar{r}) \bar{r} [1 - F^A(\bar{r}^-)] \\ &\stackrel{F^A = \frac{\alpha^B}{\alpha^A} F^B, F^B(\bar{r}^-) = F^B(\bar{r})}{=} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) + F^B(\bar{r}) \bar{r} \left(1 - \frac{\alpha^B}{\alpha^A} F^B(\bar{r}) \right) \end{aligned}$$

Plug these terms into Eq. (46) and (47), and we have

$$\begin{aligned}\mathbb{E}\left[r^A \mid r^A < r^B \leq \infty\right] &= \frac{\mathbb{P}\left(h^A = H\right) \int_{\underline{r}}^{\bar{r}} r dF^A(r) - p_{HH} \int_{\underline{r}}^{\bar{r}} F^B(r) r dF^A(r)}{p_{HH} \left[1 - F^B(\bar{r}) + \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2\right] + p_{HL}} \\ &= \bar{r} - \frac{\mathbb{P}\left(h^A = H\right) \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} F^B(r) dr + p_{HH} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) - p_{HH} \bar{r} \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2}{p_{HH} \left[1 - F^B(\bar{r}) + \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2\right] + p_{HL}},\end{aligned}$$

for Bank B,

$$\begin{aligned}\mathbb{E}\left[r^B \mid r^B < r^A \leq \infty\right] &= \frac{\mathbb{P}\left(h^B = H\right) \int_{\underline{r}}^{\bar{r}} r dF^B(r) - p_{HH} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r)}{p_{HH} \left[F^B(\bar{r}) - \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2\right] + p_{LH} F^B(\bar{r})} \\ &= \bar{r} - \frac{\mathbb{P}\left(h^B = H\right) \int_{\underline{r}}^{\bar{r}} F^B(r) dr + p_{HH} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) - p_{HH} \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2 \bar{r}}{p_{HH} \left[F^B(\bar{r}) - \frac{\alpha^B}{2\alpha^A} (F^B(\bar{r}))^2\right] + p_{LH} F^B(\bar{r})}.\end{aligned}$$

Therefore, $\mathbb{E}\left[r^A \mid r^A < r^B \leq \infty\right] \geq \mathbb{E}\left[r^B \mid r^B < r^A \leq \infty\right]$ is equivalent to the stated inequality. \square

Lemma 7. *In the case of $q > \frac{1}{1+\bar{r}}$, when $\alpha^B \uparrow \alpha^A$, there exists a threshold $\hat{\alpha}(\alpha^A) < \alpha^A$ so that when $\alpha^B > \hat{\alpha}(\alpha^A)$ we have $F^B(\bar{r}) = 1$.*

Proof. Let $\alpha^B = \alpha^A - \epsilon$. Bank B's profit could be pinned down by setting $r = \bar{r}^-$,

$$\begin{aligned}\pi^B &= p_{HH} \left[1 - F^A(\bar{r}^-)\right] [\mu_{HH}(\bar{r} + 1) - 1] + p_{LH} [\mu_{LH}(\bar{r} + 1) - 1] \\ &\geq \underbrace{p_{LH} (\mu_{LH}(\bar{r} + 1) - 1)}_{F^A(\bar{r}^-) \leq 1} \\ &\stackrel{\alpha^B = \alpha^A - \epsilon}{=} \underbrace{q(1 - \alpha^A)(\alpha^A - \epsilon)\bar{r} - (1 - q)\alpha^A(1 - (\alpha^A - \epsilon))}_{\alpha^B = \alpha^A - \epsilon} \\ &= (1 - \alpha^A)\alpha^A [q\bar{r} - (1 - q)] - \epsilon [q(1 - \alpha^A)\bar{r} + (1 - q)\alpha^A].\end{aligned}$$

Hence, when $\epsilon < \frac{(1 - \alpha^A)\alpha^A [q\bar{r} - (1 - q)]}{q(1 - \alpha^A)\bar{r} + (1 - q)\alpha^A}$, or equivalently, when

$$\alpha^B > \hat{\alpha}(\alpha^A) = \alpha^A - \frac{(1 - \alpha^A)\alpha^A [q\bar{r} - (1 - q)]}{q(1 - \alpha^A)\bar{r} + (1 - q)\alpha^A},$$

we have $\pi^B > 0$ and $F^B(\bar{r}) = 1$. \square

Proof of Proposition 2

Proof. There are two cases depending on whether $q < \frac{1}{1+\bar{r}}$, i.e., whether the project has a negative NPV at prior.

The first case of $q < \frac{1}{1+\bar{r}}$ is easier. When $\alpha^B \uparrow \alpha^A$ and $\alpha^A - \alpha^B = o\left(q - \frac{1}{1+\bar{r}}\right)$, Bank B 's signal distributions and strategies approach that of Bank A except at $r = \bar{r}$ (Lemma 5):

$$F^B(r) \uparrow F^A(r) \quad \text{for any } r \in [\underline{r}, \bar{r}), \quad \text{and} \quad F^B(\bar{r}) < 1 = F^A(\bar{r}).$$

Then from Lemma 6,

$$\begin{aligned} \frac{\bar{r} - \mathbb{E}\left[r^A \mid r^A < r^B \leq \infty\right]}{\bar{r} - \mathbb{E}\left[r^B \mid r^B < r^A \leq \infty\right]} &= \frac{p_{HH} \left[F^B(\bar{r}) - \frac{1}{2} (F^B(\bar{r}))^2 \right] + p_{LH} F^B(\bar{r})}{p_{HH} \left[1 - F^B(\bar{r}) + \frac{1}{2} (F^B(\bar{r}))^2 \right] + p_{HL}} \\ &\leq \underbrace{\frac{\frac{1}{2} p_{HH} + p_{LH}}{\frac{1}{2} p_{HH} + p_{HL}}}_{\text{RHS set } F^B(\bar{r})=1} = 1, \end{aligned}$$

where the last inequality holds because the ratio is increasing in $F^B(\bar{r})$. Hence, $\mathbb{E}\left[r^A \mid r^A < r^B \leq \infty\right] \geq \mathbb{E}\left[r^B \mid r^B < r^A \leq \infty\right]$ always holds in this case.

Now consider the second case $q \geq \frac{1}{1+\bar{r}}$. When $\alpha^B \rightarrow \alpha^A$, since $\mathbb{E}\left[r^A \mid r^A < r^B \leq \infty\right]$ decreases while $\mathbb{E}\left[r^B \mid r^B < r^A \leq \infty\right]$ increase in $F^B(\bar{r})$, it is sufficient to show that the equivalent inequality in Lemma 6 holds under $F^B(\bar{r}) = 1$, i.e.,

$$\begin{aligned} &\frac{\mathbb{P}\left(h^A = H\right) \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} F^B(r) dr + p_{HH} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) - p_{HH} \bar{r} \frac{\alpha^B}{2\alpha^A}}{p_{HH} \frac{\alpha^B}{2\alpha^A} + p_{HL}} \\ &\leq \frac{\mathbb{P}\left(h^B = H\right) \int_{\underline{r}}^{\bar{r}} F^B(r) dr + p_{HH} \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) - p_{HH} \frac{\alpha^B}{2\alpha^A} \bar{r}}{p_{HH} \left(1 - \frac{\alpha^B}{2\alpha^A}\right) + p_{LH}}, \end{aligned} \quad (48)$$

where both the LHS and RHS are positive. When $q > \frac{1}{1+\bar{r}}$, recall that Lemma 7 shows $F^B(\bar{r}) = 1$ as $\alpha^B \rightarrow \alpha^A$ under $q > \frac{1}{1+\bar{r}}$ and so the inequality is also necessary.

Denote by $N \triangleq \int_{\underline{r}}^{\bar{r}} F^B(r) dr > 0$, and $M \triangleq \bar{r} \frac{\alpha^B}{2\alpha^A} - \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r)$. $M > 0$ because

$$\int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) < \bar{r} \int_{\underline{r}}^{\bar{r}} F^A(r) dF^B(r) = \bar{r} \int_{\underline{r}}^{\bar{r}} \frac{\alpha^B}{\alpha^A} F^B(r) dF^B(r) = \bar{r} \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} d\left(\frac{F^B(r)^2}{2}\right) = \bar{r} \frac{\alpha^B}{2\alpha^A}.$$

Collect terms in the key inequality (48), we have

$$\begin{aligned} & \left\{ \left[p_{HH} \left(1 - \frac{\alpha^B}{2\alpha^A} \right) + p_{LH} \right] (p_{HH} + p_{HL}) \frac{\alpha^B}{\alpha^A} - \left(p_{HH} \frac{\alpha^B}{2\alpha^A} + p_{HL} \right) (p_{HH} + p_{LH}) \right\} N \\ & \leq p_{HH} \left[p_{HH} \left(1 - \frac{\alpha^B}{2\alpha^A} \right) + p_{LH} - \left(p_{HH} \frac{\alpha^B}{2\alpha^A} + p_{HL} \right) \right] M \end{aligned} \quad (49)$$

Let $\alpha^B = \alpha^A - \epsilon$ and calculate the coefficients. Note that as $\alpha^B = \alpha^A - \epsilon$, we have $p_{HL} - p_{LH} = (2q - 1)\epsilon$.²⁹ The coefficient on the LHS of (49):

$$\begin{aligned} & \left[p_{HH} \left(1 - \frac{\alpha^B}{2\alpha^A} \right) + p_{LH} \right] (p_{HH} + p_{HL}) \frac{\alpha^B}{\alpha^A} - \left(p_{HH} \frac{\alpha^B}{2\alpha^A} + p_{HL} \right) (p_{HH} + p_{LH}) \\ & = \left(\frac{p_{HH}}{2} + \frac{\epsilon}{2\alpha^A} p_{HH} + p_{LH} \right) (p_{HH} + p_{HL}) \left(1 - \frac{\epsilon}{\alpha^A} \right) - \left(\frac{p_{HH}}{2} - \frac{\epsilon}{2\alpha^A} p_{HH} + p_{HL} \right) (p_{HH} + p_{LH}) \\ & = -\frac{p_{HH}}{2} (2q - 1)\epsilon + \frac{\epsilon}{2\alpha^A} p_{HH}^2 - \frac{\epsilon}{2\alpha^A} p_{LH} p_{HH} - \frac{\epsilon}{\alpha^A} p_{LH} p_{HL} \end{aligned}$$

The coefficient on the RHS of (49):

$$\begin{aligned} p_{HH} \left[p_{HH} \left(1 - \frac{\alpha^B}{2\alpha^A} \right) + p_{LH} - \left(p_{HH} \frac{\alpha^B}{2\alpha^A} + p_{HL} \right) \right] & = \frac{\epsilon}{\alpha^A} p_{HH}^2 - p_{HH} (p_{HL} - p_{LH}) \\ & = \frac{\epsilon}{\alpha^A} p_{HH}^2 - p_{HH} (2q - 1)\epsilon \end{aligned}$$

Plug the coefficients back into the inequality (49), so we need to show that

$$\begin{aligned} 0 & \leq \left\{ \frac{\epsilon}{\alpha^A} p_{HH}^2 - p_{HH} (2q - 1)\epsilon \right\} M - \left\{ -\frac{p_{HH}}{2} (2q - 1)\epsilon + \frac{\epsilon}{2\alpha^A} p_{HH}^2 - \frac{\epsilon}{2\alpha^A} p_{LH} p_{HH} - \frac{\epsilon}{\alpha^A} p_{LH} p_{HL} \right\} N \\ & = \left[(2q - 1) - \frac{p_{HH}}{\alpha} \right] \frac{p_{HH} (N - 2M)}{2} \epsilon + \left(\frac{1}{2} p_{LH} p_{HH} + p_{LH} p_{HL} \right) \frac{N}{\alpha} \epsilon. \end{aligned}$$

²⁹We have $p_{HL} = q\alpha^A(1 - \alpha^B) + (1 - q)\alpha^B(1 - \alpha^A)$ and $p_{LH} = q(1 - \alpha^A)\alpha^B + (1 - q)\alpha^A(1 - \alpha^B)$ and then therefore $p_{HL} - p_{LH} = q(\alpha^A - \alpha^B) + (1 - q)(\alpha^B - \alpha^A) = (2q - 1)\epsilon$.

Note that

$$\begin{aligned}
N - 2M &= \int_{\underline{r}}^{\bar{r}} F^B(r) dr - 2 \left(\bar{r} \frac{\alpha^B}{2\alpha^A} - \int_{\underline{r}}^{\bar{r}} F^A(r) r dF^B(r) \right) \\
&= \int_{\underline{r}}^{\bar{r}} F^B(r) dr - 2 \left(\bar{r} \frac{\alpha^B}{2\alpha^A} - \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} F^B(r) r dF^B(r) \right) \\
&= \int_{\underline{r}}^{\bar{r}} F^B(r) dr - 2 \left(\bar{r} \frac{\alpha^B}{2\alpha^A} - \frac{\alpha^B}{2\alpha^A} \bar{r} + \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} \frac{(F^B(r))^2}{2} dr \right) \\
&= \int_{\underline{r}}^{\bar{r}} F^B(r) dr - \frac{\alpha^B}{\alpha^A} \int_{\underline{r}}^{\bar{r}} (F^B(r))^2 dr > 0.
\end{aligned}$$

Therefore, one sufficient condition is

$$2q - 1 \geq \frac{p_{HH}}{\alpha} = \frac{q\alpha^2 + (1-q)(1-\alpha)^2}{\alpha}$$

collecting terms, it requires $q \geq 1 - \alpha + \alpha^2$. Since $1 - \alpha + \alpha^2$ increases in α for $\alpha \in (\frac{1}{2}, 1)$, this imposes a simple condition that prior needs to be sufficiently good and information technology α cannot be too high. \square

Calibration For calibration, we rely on two empirical moments in the U.S. banking industry to gauge the magnitudes of q and α . First, this website on Federal Reserve reports the NPL ratio to be about 2%; second, Yates (2020) shows that the approval rate for business C&I loans is from 55% (small) to 80% (large).

We gauge q and α from the limiting case where Bank B 's information technology α^B approaches that of Bank A , i.e., $\alpha^B \rightarrow \alpha^A = \alpha$. Depending on the primitives, Bank B may either make zero or positive profit in the unique equilibrium, which we call zero-weak or positive weak in analogous to our main equilibrium characterization with multi-dimensional information.

Recall that in the beginning of proof of Proposition 2 we have shown that condition ??) fails in the zero-weak case (i.e., if and only if $q < \frac{1}{1+\bar{r}}$ where Bank B makes zero profit). Therefore we only need to consider the positive-weak case.

In this case lenders are symmetric: upon H each lender makes interest rate with randomized strategy, with a winning probability of 0.5. Therefore we can write down the NPL ratio and

approval rate of, say Bank A ,

$$2\% = \frac{\mathbb{P}(\theta = 0 | r^A < r^B < \infty)}{\mathbb{P}(r^A < r^B < \infty)} = \frac{(1-q) \left[\frac{(1-\alpha)^2}{2} + \alpha(1-\alpha) \right]}{(1-q) \left[\frac{(1-\alpha)^2}{2} + \alpha(1-\alpha) \right] + q \left[\frac{\alpha^2}{2} + \alpha(1-\alpha) \right]},$$

$$y = \mathbb{P}(h^A = H) = q\alpha + (1-q)(1-\alpha), \text{ for } y \in [0.55, 0.80].$$

which allows us to solve for the pair (q, α) . For instance, when $y = 0.7$ one can solve for $q = 0.9629$ and $\alpha = 0.716$, which satisfies the proposed sufficient condition $q > 1 - \alpha + \alpha^2$. The same result holds for $y = 0.55$ (so that $q = 0.9771$ and $\alpha = 0.5524$) or $y = 0.8$ (so that $q = 0.9349$ and $\alpha = 0.8449$).

A.4 Proof of Proposition 3

Proof. Based on the credit competition equilibrium in Proposition 1, the expected rates of a lender's issued loan are:

$$\mathbb{E} \left[r^A \mid r^A < r^B \leq \infty \right] = \frac{\underbrace{p_{HH}}_{h^B=H} \int_x^1 \underbrace{\left[1 - F^B(r^A(t)^-) \right]}_{A \text{ wins}} r^A(t) \phi(t) dt + \underbrace{p_{HL}}_{h^B=L} \int_x^1 r^A(t) \phi(t) dt}{p_{HH} \int_x^1 \left[1 - F^B(r^A(t)^-) \right] \phi(t) dt + p_{HL} \int_x^1 \phi(t) dt},$$

$$\mathbb{E} \left[r^B \mid r^B < r^A \leq \infty \right] = \frac{\underbrace{p_{HH}}_{h^A=H} \int_{\hat{s}}^1 \underbrace{\Phi(t)}_{B \text{ wins}} r(t) dF^B(r(t)) + \underbrace{p_{LH}}_{h^A=L} \int_x^1 r(t) dF^B(r(t))}{p_{HH} \int_{\hat{s}}^1 \Phi(t) dF^B(r(t)) + p_{LH} F^B(\bar{r})}.$$

Note that when the positive weak equilibrium arises, $F^B(r(s))$ has a point mass of size $1 - F^B(\bar{r}^-)$ at \bar{r} or $r^A(\hat{s})$.

We show that $\mathbb{E} \left[r^A \mid r^A < r^B \leq \infty \right] = \mathbb{E} \left[r^B \mid r^B < r^A \leq \infty \right]$ in the benchmark case with the following conditions,

- The soft signal is uniformly distributed $s \sim U[0, 1]$.
- Each lender receives a perfect hard signal, $h^j = \theta_h$ for $j \in \{A, B\}$.
- $\bar{r} \rightarrow \infty$.

With the above simplifications, we have

$$r^A(s) = \frac{2}{s}, \quad F^B(r(s)) = 1 - \frac{s^2}{2q_s},$$

and

$$\mathbb{E} \left[r^A \mid r^A < r^B \leq \infty \right] = \frac{q_h \int_0^1 \frac{t^2}{2q_s} r(t) dt}{q_h \int_0^1 \frac{t^2}{2q_s} dt} = \frac{q_h \int_0^1 t \cdot r(t) \left(-\frac{t}{2q_s} \right) dt}{q_h \int_0^1 t \left(-\frac{t}{2q_s} \right) dt} = \mathbb{E} \left[r^B \mid r^B < r^A \leq \infty \right].$$

□

A.5 Proof of Proposition 4

Lemma 8. $s_B^{BE}(\eta)$ decreases in η .

Proof. Conditional on $\pi^B = 0$, we show that Bank A's quote $r^A(s; \eta)$ decreases in η for any s with $r^A(s; \eta) < \bar{r}$. Rewrite $r^A(s; \eta)$ as defined in Eq. (12) to incorporate η

$$\bar{r} > r^A(s; \eta) = \frac{\underbrace{\pi_B}_{=0} + p_{HH}^\eta \Phi(s, \eta) + p_{LH}^\eta}{p_{HH}^\eta \mu_{HH}^\eta \int_0^s t \phi(t, \eta) dt + p_{LH}^\eta \mu_{LH}^\eta q_s^\eta} - 1.$$

It is useful to rewrite $\int_0^{s_B^{BE}} t \phi(t, \eta) dt$ as a function of the direct soft signal z , whose distribution is irrelevant of η . Under MLRP, s strictly increases in z and we denote $z \equiv S^{-1}(s)$. The CDF of s , z then satisfies $\Phi(s) \equiv \Phi_z(S^{-1}(s)) = \int^{S^{-1}(s)} \phi_z(\nu) d\nu$. Taking derivative on both sides, the PDF of z satisfies

$$\phi(s) = \frac{\phi_z(S^{-1}(s))}{S'(z)}. \quad (50)$$

Using this term, we show that $\int_0^{s_B^{BE}} t \phi(t, \eta) dt$ is independent of η .

$$\int_0^{s(r)} t \phi(t) dt = \int_{\underline{z}}^{S^{-1}(s(r))} S(\nu) \frac{\phi_z(S^{-1}(t))}{S'(\nu)} S'(\nu) d\nu = \int_{\underline{z}}^{z(r)} S(\nu) \phi_z(\nu) d\nu.$$

Now we discuss the key terms' monotonicity in η . In the denominator, $p_{HH}^\eta \mu_{HH}^\eta \int_0^{s_B^{BE}} t \phi(t, \eta) dt$ is independent of η . To see this, let $z_B^{BE} \equiv S^{-1}(s_B^{BE})$, and we have

$$\begin{aligned} & p_{HH}^\eta \mu_{HH}^\eta \left(\int_{\underline{z}}^{z_B^{BE} \equiv S^{-1}(s_B^{BE})} S(\nu; \eta) \phi_z(\nu; \eta) d\nu \right) \\ &= \mathbb{P}(\theta_h^\eta = 1, h_A^\eta = h_B^\eta = H) \mathbb{P}(\theta_s^\eta = 1, z_A^\eta < z_B^{BE}) \\ &= \mathbb{P}(\theta_h^\eta = 1) \mathbb{P}(h_A^\eta = h_B^\eta = H \mid \theta_h^\eta = 1) \cdot \mathbb{P}(\theta_s^\eta = 1) \mathbb{P}(z_A^\eta < z_B^{BE} \mid \theta_s^\eta = 1) \\ &= \underbrace{\mathbb{P}(\theta_h^\eta = 1) \mathbb{P}(\theta_s^\eta = 1)}_{\frac{q_h}{\eta} q_s \eta = q_h q_s} \cdot \underbrace{\mathbb{P}(h_A^\eta = h_B^\eta = H \mid \theta_h^\eta = 1) \mathbb{P}(z_A^\eta < z_B^{BE} \mid \theta_s^\eta = 1)}_{\text{info technology, independent of } \eta}, \end{aligned}$$

which is independent of η . Note that this argument does not depend on the specific realizations of signals h_A^η, h_B^η, z_A . Hence, $p_{LH}^\eta \mu_{LH}^\eta q_s^\eta$, which is from another set of signal realizations $h_A^\eta = L, h_B^\eta = H$, is also independent of η .

In the numerator, $p_{HH}^\eta \Phi_s(s; \eta)$ decreases in η . To see this, as η goes up, the composition of hard versus soft fundamental changed— q_h^η goes down while q_s^η goes up, while the information technology or the conditional distribution of signals remain the same. Hence, it's less likely to receive favorable hard signals H and low soft signals (below a certain s), and so $p_{HH}^\eta \Phi_s(s; \eta)$ decreases in η . The last term $p_{LH}^\eta = \alpha(1 - \alpha)$ is independent of η .

Taken together, $r^A(s; \eta)$ decreases in η for any s with $r^A(s; \eta) < \bar{r}$ conditional on $\pi^B = 0$. This applies to the special case of s_B^{BE} , which is defined as

$$s_B^{BE} = \arg \sup_s \left\{ s : r^A(s, \eta; \pi^B = 0) \geq \bar{r} \right\}.$$

Specifically, for any $\eta_1 < \eta_2$ with $r^A(s_B^{BE}(\eta_1), \eta_1; \pi^B = 0) = \bar{r}$ and $r^A(s_B^{BE}(\eta_2), \eta_2; \pi^B = 0) = \bar{r}$, we have $r^A(s_B^{BE}(\eta_1), \eta_2; \pi^B = 0) < \bar{r}$ because $r^A(s; \eta)$ decreases in η . As $r^A(s; \eta)$ also decreases in s , thus $s_B^{BE}(\eta_2) < s_B^{BE}(\eta_1)$. Therefore, $s_B^{BE}(\eta)$ decreases in η . \square

Lemma 9. $s_A^{BE}(\eta)$ decreases in η .

Proof. Rewrite the definition equation of $s_A^{BE}(\eta)$ (22) to incorporate η

$$\pi^A(s_A^{BE}, \eta) = p_{HH}^\eta \frac{\int_0^{s_A^{BE}} t \phi(t; \eta) dt}{q_s} \left[\mu_{HH}^\eta s_A^{BE} (1 + \bar{r}) - 1 \right] + p_{HL}^\eta \left[\mu_{HL}^\eta s_A^{BE} (1 + \bar{r}) - 1 \right] = 0,$$

We first analyze the key terms' monotonicity in η . Note that

$$\begin{aligned} p_{HH}^\eta &= \frac{q_h}{\eta} \alpha^2 + \left(1 - \frac{q_h}{\eta} \right) (1 - \alpha)^2 = (1 - \alpha)^2 + \frac{q_h}{\eta} (2\alpha - 1), \\ \mu_{HH}^\eta &= \frac{\frac{q_h}{\eta} \alpha^2}{\frac{q_h}{\eta} \alpha^2 + \left(1 - \frac{q_h}{\eta} \right) (1 - \alpha)^2} = \frac{\alpha^2}{\alpha^2 + \left(\frac{\eta}{q_h} - 1 \right) (1 - \alpha)^2}, \\ p_{HL}^\eta \mu_{HL}^\eta &= \frac{q_h}{\eta} \alpha (1 - \alpha) \end{aligned}$$

all decrease in η . On the other hand, both $p_{HL}^\eta = \alpha(1 - \alpha)$ and

$$\frac{\int_0^{s_A^{BE}} t \phi(t) dt}{q_s^\eta} = \frac{\int_{\underline{z}^{BE} \equiv S^{-1}(s_A^{BE})} S(\nu) \phi_z(\nu) d\nu}{q_s^\eta} = \frac{\int_{\underline{z}^{BE}} \frac{q_s^\eta \tilde{f}(\nu)}{\phi_z(\nu)} \phi_z(\nu) d\nu}{q_s^\eta} = \int_{\underline{z}} \tilde{f}(\nu) d\nu \quad (51)$$

are independent of η , where Eq. (51) expresses $\frac{\int_0^{s_A^{BE}} t \phi(t) dt}{q_s^\eta}$ as a function of the direct soft signal z with $\phi_z(z)$ satisfying Eq. (50) and \tilde{f} defined as the PDF of z conditional on $\theta_s = 1$.

With the above observations, $\pi^A(s_A^{BE}, \eta)$ decreases in η :

$$0 > \frac{\partial \pi^A(s_A^{BE}, \eta)}{\partial \eta} = \underbrace{\frac{\partial p_{HH}^\eta}{\partial \eta}}_{-} \cdot \left(\int_{\underline{z}}^z f(\nu) d\nu \right) \underbrace{\left[\mu_{HH}^\eta s_A^{BE} (1 + \bar{r}) - 1 \right]}_{+} \\ + p_{HH}^\eta \cdot \left(\int_{\underline{z}}^z f(\nu) d\nu \right) \underbrace{\frac{\partial \mu_{HH}^\eta}{\partial \eta}}_{-} \cdot s_A^{BE} (1 + \bar{r}) + \underbrace{\frac{\partial (p_{HL}^\eta \mu_{HL}^\eta)}{\partial \eta}}_{-} s_A^{BE} (1 + \bar{r}).$$

In addition, $\pi^A(s_A^{BE}, \eta)$ increases in s_A^{BE} :

$$0 < \frac{\partial \pi^A(s_A^{BE}, \eta)}{\partial s_A^{BE}} = p_{HH}^\eta s_A^{BE} \phi(s_A^{BE}) \left[\mu_{HH}^\eta s_A^{BE} (1 + \bar{r}) - 1 \right] \\ + \left[p_{HH}^\eta \mu_{HH}^\eta \left(\frac{\int_0^{s_A^{BE}} t \phi(t) dt}{q_s^\eta} \right) + p_{HL}^\eta \mu_{HL}^\eta \right] (1 + \bar{r}).$$

By the implicit function theorem, we then have $\frac{\partial s_A^{BE}}{\partial \eta} = -\frac{\partial \pi^A(s_A^{BE}, \eta)}{\partial \eta} / \frac{\partial \pi^A(s_A^{BE}, \eta)}{\partial s_A^{BE}} > 0$. \square

Proof of Proposition 4

Proof. Recall from Lemma 2 that the equilibrium is positive-weak (zero-weak) if $s_B^{BE} < s_A^{BE}$ ($s_B^{BE} \geq s_A^{BE}$.) This combining with Lemma 8 and Lemma 9 complete the proof. \square

A.6 Derivation of Correlated Hard Signals

Another aspect of information technology advancement is that the lenders' hard information signals become more correlated. Formally, with probability ρ_h , lenders receive the same signal realization $h^c \in \{H, L\}$ and

$$\mathbb{P}(h^c = H | \theta_h = 1) = \mathbb{P}(h^c = L | \theta_h = 0) = \alpha;$$

with probability $1 - \rho_h$, each receives an independent hard signal according to Eq. (3).

With more correlated hard signals or a higher ρ_h , lenders are more likely to agree on the customer quality and so more likely to compete (the event of HH). In terms of inference, the posterior upon disagreement (that comes from the uncorrelated part of the assessment) is still the prior q_h .³⁰ Taken together, competition becomes fiercer, because lenders are more likely to compete but not more concerned about the winner's curse.

³⁰Upon competition (HH), lenders are less sure about a good quality borrower, i.e., $\mu_{HH}(\rho_h)$ decreases in ρ_h .

A.7 Information Acquisition

In this part, we first characterize lending profits and then provide a numerical illustration that the specialization equilibrium would arise.

A.7.1 Lending Profits

We characterize lending profits as a function of information acquisition, $\Pi_A(I_A^h, I_A^s, I_B^h, I_B^s)$ (we focus on Bank A due to symmetry.) We omit the case where there is an uninformed lender.

$\mathbf{I}_A^h = 1, \mathbf{I}_A^s = 1, \mathbf{I}_B^h = 1, \mathbf{I}_B^s = 0$ (**Specialization**). This is the equilibrium that we focus on—each lender has a hard information signal and only Bank A has a soft signal s . Bank A 's expected lending profit before signal realizations is thus

$$\Pi_A(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 0) = \int_x^1 \pi^A(r^A(s)|s) \phi(s) ds,$$

where $\pi^A(r^A(s)|s)$ is the profits for given signal realizations H and s and is given in Eq. (8). Using the equilibrium strategies in Proposition 1, we have

$$\pi^A(r^A(s)|s) = p_{HH} \cdot \frac{\int_0^{\min\{s, \bar{s}\}} (s-t) \phi(t) dt}{q_s} + (\pi^B + p_{LH}) \cdot \frac{s}{q_s} - p_{HL}, \text{ for } s \geq x.$$

The expression shows that Bank A earns the information rent from soft signal. Bank A observes s , while Bank B may only negatively update the prior q_s when winning the competition that $s^A \leq s$ (r); this is reflected in the terms $\frac{s}{q_s}$ and $\frac{\int_0^{\min\{s, \bar{s}\}} (s-t) \phi(t) dt}{q_s}$.

In this case, Bank B 's profit $\Pi_B(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 0) = \pi^B$ is given in Lemma 2. By symmetry, $\Pi_A(I_A^h = 1, I_A^s = 0, I_B^h = 1, I_B^s = 1) = \Pi_B(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 0) = \pi^B$.

$\mathbf{I}_A^h = 0, \mathbf{I}_A^s = 1, \mathbf{I}_B^h = 1, \mathbf{I}_B^s = 0$ (**Asymmetric technology**). In this case, Bank A only collects soft information while Bank B only collect hard information in industry a . This case is nested in the previous case of specialization ($I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 0$), by reformulating Bank A to have an uninformative hard signal, e.g.,

$$\mathbb{P}(h^A = H | \theta_h = 1) = \mathbb{P}(h^A = H | \theta_h = 0) = 1.$$

$\mathbf{I}_A^h = 1, \mathbf{I}_A^s = 0, \mathbf{I}_B^h = 1, \mathbf{I}_B^s = 0$ (**Hard information only**). In this case, both lenders only acquire hard information, i.e., investing in IT and data processing that are applicable to both industries. The credit competition corresponds to Broecker (1990) with two lenders. Lenders are symmetric

and the lending profit of, say Bank A , is

$$\Pi_A \left(I_A^h = 1, I_A^s = 0, I_B^h = 1, I_B^s = 0 \right) = \max \{ p_{LH} (\mu_{HH} q_s \bar{r} - 1), 0 \}.$$

The “max” operator arises because either both lenders withdraw with positive probability (zero profits), or both lenders make profits and neither has a point mass at \bar{r} , i.e., $F^j(\bar{r}^-) = 1$.

$\mathbf{I}_A^h = \mathbf{1}, \mathbf{I}_A^s = \mathbf{1}, \mathbf{I}_B^h = \mathbf{1}, \mathbf{I}_B^s = \mathbf{1}$ (**Acquire all information**). In this symmetric case, each lender invests in both information technologies and receives both the hard and soft signals. We characterize the credit market equilibrium based on Riordan (1993) who consider the competition between two lenders each with a continuous private signal. Here, each lender additionally has a binary signal that represents the hard information. Following the modeling of Riordan (1993), we work with the direct soft signal z . Specifically, let z and Z denote the realization and the random variable of the soft signal respectively, and let

$$\tilde{F}(z) \equiv \mathbb{P}(Z \leq z | \theta_s = 1), \quad \tilde{G}(z) \equiv \mathbb{P}(Z \leq z | \theta_s = 0)$$

denote the CDFs of Z conditional on the underlying state θ_s , with the corresponding PDFs denoted by \tilde{f} and \tilde{g} . Introduce $\mu(z) \equiv \mathbb{P}(\theta_s = g | S)$ as the posterior belief, which is s in our baseline model.

A lender only bids when the hard signal is H and the soft signal $z \geq x$. Let $R(z) \equiv r(z) + 1$ denote the equilibrium gross rate quote. Given competitor’s strategy $R(z)$, the lending profits from any R is then

$$\begin{aligned} \pi(R|z) = & \left[p_{HH} \mu_{HH} \mu(z) \tilde{F}(t(R)) + p_{HL} \mu_{HL} \mu(z) \right] R \\ & - p_{HH} \left[(1 - \mu(z)) \tilde{G}(t(R)) + \mu(z) \tilde{F}(t(R)) \right] - p_{HL}, \end{aligned} \quad (52)$$

where $t(R)$ the signal such that the other bank offers R . The first order condition w.r.t. R is

$$\begin{aligned} \frac{\partial \pi(R(t)|z)}{\partial R} = & \left[p_{HH} \mu_{HH} \mu(z) \tilde{F}(t) + p_{HL} \mu_{HL} \mu(z) \right] \\ & + \left\{ p_{HH} \mu_{HH} \mu(z) \tilde{f}(t) R(t) - p_{HH} \left[(1 - \mu(z)) \tilde{g}(t) + \mu(z) \tilde{f}(t) \right] \right\} \frac{dt}{dR}. \end{aligned}$$

The equilibrium strategy satisfies

$$\left. \frac{\partial \pi(R(t)|z)}{\partial t} \right|_{t=z} = 0$$

By symmetry, we have

$$\frac{dt}{dR} = \frac{1}{R'(t)}.$$

These two conditions imply

$$p_{HH}\mu_{HH}\tilde{f}(z)R(z) + (p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL})R'(z) = \frac{p_{HH}(1-\mu(z))\tilde{g}(z) + p_{HH}\mu(z)\tilde{f}(z)}{\mu(z)}, \quad (53)$$

or equivalently,

$$\frac{d\left\{[p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL}]R(z)\right\}}{dz} = \frac{p_{HH}(1-\mu(z))\tilde{g}(z) + p_{HH}\mu(z)\tilde{f}(z)}{\mu(z)}.$$

Integrating over z , we have

$$R(z) = \frac{\int_{\underline{z}}^z \frac{p_{HH}(1-\mu(t))\tilde{g}(t) + p_{HH}\mu(t)\tilde{f}(t)}{\mu(t)} dt + constant}{p_{HH}\mu_{HH}\tilde{F}(z) + p_{HL}\mu_{HL}}. \quad (54)$$

The unknown constant is pinned down by the boundary condition $\pi(\bar{r} + 1 | x) = 0$: Upon the threshold signal x , a lender quotes the maximum interest rate $\bar{r} + 1$ and makes zero profit,

$$0 = [p_{HH}\mu_{HH}\mu(x)\tilde{F}(x) + p_{HL}\mu_{HL}\mu(x)](\bar{r} + 1) - p_{HH}[(1-\mu(x))\tilde{G}(x) + \mu(x)\tilde{F}(x)] - p_{HL}. \quad (55)$$

Then a lender's lending profit is

$$\Pi_A(I_A^h = 1, I_A^s = 1, I_B^h = 1, I_B^s = 1) = \int_x^{\bar{z}} \pi(R(z) | z) [q_s \tilde{f}(z) + (1 - q_s) \tilde{g}(z)] dz,$$

where $R(z)$ is given by Eq. (54) and (55), profit $\pi(R(z), z)$ is given by Eq. .

A.7.2 Specialization Equilibrium

Figure 7 shows the region of information acquisition costs κ_h and κ_s to support the specialization equilibrium, so that one of the bank endogenously becomes the specialized bank in one industry by acquiring both soft and hard information while the other is non-specialized by acquiring the hard information only. In sum, we need κ_h to be sufficiently small while κ_s to lie in an intermediate range.

A.8 General Information Structure

In this extension, we focus on the well-behaved structure (i.e., smooth distribution of interest rates over $[\underline{r}, \bar{r}]$ and decreasing $r^A(z)$) and show that the lender strategies in Proposition 5 correspond to an equilibrium.

Proof of Proposition 5

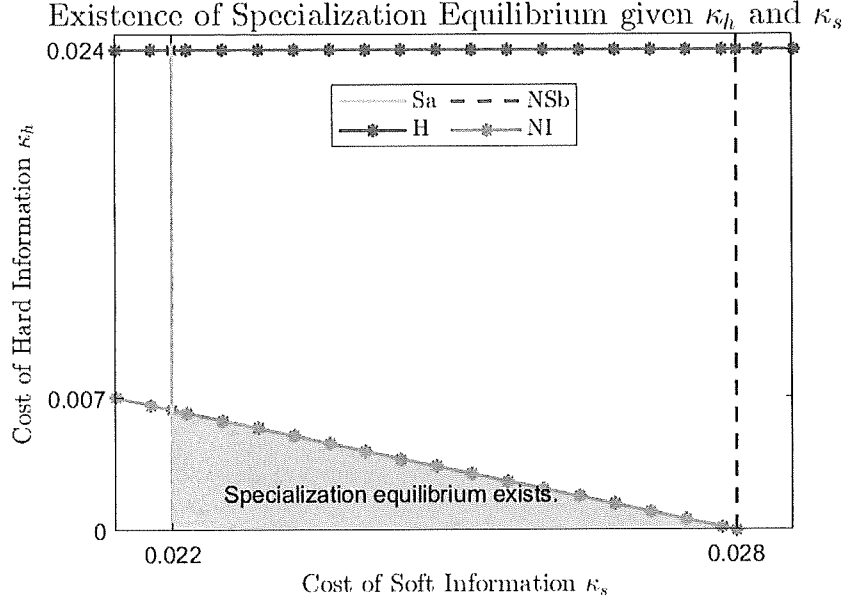


Figure 7: **Specialization Equilibrium** This plot depicts the incentive compatibility constraints where Bank A does not want to deviate from the specialization equilibrium. Parameters: $\bar{r} = 0.36$, $\rho_h = 0$, $q_h = 0.8$, $q_s = 0.9$, $\alpha_u = \alpha_d = \alpha = 0.7$, and $\tau = 1$.

Proof. Bank A's strategy

In the region of $z \in [\hat{z}, 1]$ that corresponds to $r^A(z) \in [\underline{r}, \bar{r}]$, $r^A(\cdot)$ is strictly decreasing so the inverse function $z^A(\cdot) \equiv r^{A(-1)}(\cdot)$ is properly defined. Bank B 's lending profit when quoting $r \in [\underline{r}, \bar{r}]$ is

$$\begin{aligned}
 \pi^B(r) &= \underbrace{\bar{p}_{HH}}_{h^A=H} \cdot \underbrace{\int_{\underline{z}}^{z^A(r)}}_{B \text{ wins}} \left[\underbrace{\mu_{HH}(t)}_{\text{repay}} (1+r) - 1 \right] \phi_z(t|HH) dt + \underbrace{\bar{p}_{LH}}_{h^A=L} \left[\underbrace{\bar{\mu}_{LH}}_{\text{repay}} (1+r) - 1 \right] \\
 &= (1+r) \left[\int_{\underline{z}}^{z^A(r)} p_{HH}(t) \mu_{HH}(t) dt + \bar{p}_{LH} \bar{\mu}_{LH} \right] - \int_{\underline{z}}^{z^A(r)} p_{HH}(t) dt - \bar{p}_{LH} \quad (56)
 \end{aligned}$$

Bank A 's equilibrium strategy $r^A(z)$ for $z \in [\hat{z}, 1]$ is such that Bank B is indifferent across $r \in [\underline{r}, \bar{r}]$. Hence,

$$r^A(z) = \frac{\overbrace{\pi^B + \int_{\underline{z}}^z p_{HH}(t) dt + \bar{p}_{LH}}^{B's \text{ lending amount}}}{\underbrace{\int_{\underline{z}}^z p_{HH}(t) \cdot \mu_{HH}(t) dt + \bar{p}_{LH} \bar{\mu}_{LH}}_{B's \text{ customers who repay}}} - 1, \quad \text{where } \hat{z} \leq z \leq \bar{z}. \quad (57)$$

In addition, $r^A(z) = \bar{r}$ for $z \in [z_x, \hat{z})$ and Bank A rejects the borrower when $z \in [z, z_x)$, where z_x satisfies

$$\pi^A \left(r^A(z_x) = \bar{r} \mid z_x \right) = 0.$$

This completes the proof of Bank A 's strategy in Proposition 5.

Bank B 's strategy

Bank A 's offered interest rate $r^A(z)$ upon $z \in [\hat{z}, \bar{z}]$ maximizes

$$\pi^A \left(r^A(z) \mid z \right) = \underbrace{p_{HH}(z)}_{h^B=H} \underbrace{[1 - F^B(r)]}_{A \text{ wins}} \left[\underbrace{\mu_{HH}(z)}_{\text{repay}} (1+r) - 1 \right] + \underbrace{p_{HL}(z)}_{h^B=L} \left[\underbrace{\mu_{HL}(z)}_{\text{repay}} (1+r) - 1 \right]$$

The FOC w.r.t. r is

$$\underbrace{\left[\frac{d[F^B(r)]}{dr} \right]}_{\Delta \text{winning prob}} \underbrace{p_{HH}(z) [\mu_{HH}(z) (1+r) - 1]}_{\text{profit upon winning}} + \underbrace{p_{HH}(z) [1 - F^B(r)] \mu_{HH}(z) + p_{HL}(z) \mu_{HL}(z)}_{\text{existing customer}} = 0.$$

Bank A 's optimal strategy $r^A(z)$ satisfies this first order condition,

$$0 = - \frac{d[F^B(r^A(z))]}{dr} p_{HH}(z) [\mu_{HH}(z) (1+r^A(z)) - 1] + p_{HH}(z) [1 - F^B(r^A(z))] \mu_{HH}(z) + p_{HL}(z) \mu_{HL}(z). \quad (58)$$

From Eq. (57) about $r^A(z)$, we derive the following key equation by taking derivatives w.r.t. z ,

$$\underbrace{\frac{dr^A(z)}{dz} \left[\int_{\underline{z}}^z p_{HH}(t) \mu_{HH}(t) dt + \bar{p}_{LH} \bar{\mu}_{LH} \right]}_{\text{B: } \uparrow \text{marginal customer return}} + \underbrace{p_{HH}(z) [(r^A(z) + 1) \mu_{HH}(z) - 1]}_{\text{B: } \uparrow \text{existing customer revenue}} = 0.$$

Plug this equation into the FOC (58), and we have

$$- \frac{d[F^B(r^A(z))]}{dz} \left[\int_{\underline{z}}^z p_{HH}(t) \mu_{HH}(t) dt + \bar{p}_{LH} \bar{\mu}_{LH} \right] = p_{HH}(z) [1 - F^B(r)] \mu_{HH}(z) + p_{HL}(z) \mu_{HL}(z),$$

which is equivalent to

$$\frac{d}{dz} \left\{ \frac{1 - F^B(r^A(z))}{\int_{\underline{z}}^z p_{HH}(t) \mu_{HH}(t) dt + \bar{p}_{LH} \bar{\mu}_{LH}} \right\} = \frac{p_{HL}(z) \mu_{HL}(z)}{\left[\int_{\underline{z}}^z p_{HH}(t) \mu_{HH}(t) dt + \bar{p}_{LH} \bar{\mu}_{LH} \right]^2}. \quad (59)$$

Since signals are independent conditional on the state being $\theta = 1$, the right-hand-side equals

$$\begin{aligned} & \frac{q\mathbb{P}(HL|\theta=1)\phi_z(z|\theta=1)}{\left[\int_{\underline{z}}^z q\mathbb{P}(HH|\theta=1)\phi_z(t|\theta=1)dt + \bar{p}_{LH}\bar{\mu}_{LH}\right]^2} \\ &= -\frac{\mathbb{P}(h^B=L|\theta=1)}{\mathbb{P}(h^B=H|\theta=1)}\frac{d}{dz}\left[\frac{1}{\int_{\underline{z}}^z q\mathbb{P}(HH|\theta=1)\phi_z(t|\theta=1)dt + \bar{p}_{LH}\bar{\mu}_{LH}}\right]. \end{aligned}$$

Then the solution $F^B(r^A(z))$ to the ODE (59) satisfies

$$\frac{1 - F^B(r^A(z))}{\int_{\underline{z}}^z \mu_{HH}(t)p_{HH}(t)dt + \bar{p}_{LH}\bar{\mu}_{LH}} = -\frac{\mathbb{P}(h^B=L|\theta=1)}{\mathbb{P}(h^B=H|\theta=1)}\left[\frac{1}{\int_{\underline{z}}^z \mu_{HH}(t)p_{HH}(t)dt + \bar{p}_{LH}\bar{\mu}_{LH}}\right] + Const.$$

Using the boundary condition $F^B(r^A(\bar{z}) = \underline{r}) = 0$, we solve for the constant

$$Const = \frac{1}{\mathbb{P}(\theta=1)}\frac{1}{\mathbb{P}(h^B=H|\theta=1)^2}.$$

Therefore,

$$\begin{aligned} F^B(r) &= \frac{1}{\mathbb{P}(h^B=H|\theta=1)} - \frac{\int_{\underline{z}}^{z^A(r)} \mu_{HH}(t)p_{HH}(t)dt + \bar{p}_{LH}\bar{\mu}_{LH}}{\mathbb{P}(\theta=1)\mathbb{P}(h^B=H|\theta=1)^2} \\ &= \frac{1}{\mathbb{P}(h^B=H|\theta=1)} - \frac{\mathbb{P}(\theta=1)\mathbb{P}(HH|\theta=1)\int_{\underline{z}}^{z^A(r)} \phi_z(t|\theta=1)dt + \mathbb{P}(\theta=1)\mathbb{P}(LH|\theta=1)}{\mathbb{P}(\theta=1)\mathbb{P}(h^B=H|\theta=1)^2} \\ &= \frac{\mathbb{P}(h^A=H|\theta=1)}{\mathbb{P}(h^B=H|\theta=1)}\left[1 - \int_{\underline{z}}^{z^A(r)} \phi_z(t|\theta=1)dt\right]. \end{aligned}$$

Bank B's profit π^B

Now we are left with one unknown variable π^B in Eq. (57). Similar as in the baseline model, the equilibrium could be positive-weak or zero-weak, depending on whether Bank A upon soft signal realization z_A^{BE} or Bank B breaks even when quoting \bar{r} . We define z_A^{BE} and z_B^{BE} as

$$\begin{aligned} 0 = \pi^A(\bar{r}|z_A^{BE}) &= p_{HH}(z_A^{BE})\frac{\mathbb{P}(h^A=H|\theta=1)}{\mathbb{P}(h^B=H|\theta=1)}\left[1 - \int_{\underline{z}}^{z_A^{BE}} \phi_z(t|\theta=1)dt\right] \cdot [\mu_{HH}(z_A^{BE})(1+\bar{r}) - 1] \\ &\quad + p_{HL}(z_A^{BE})[\mu_{HL}(z_A^{BE})(1+\bar{r}) - 1], \\ 0 = \pi^B(\bar{r}; z_B^{BE}) &= \int_{\underline{z}}^{z_B^{BE}} p_{HH}(t)\mu_{HH}(t)(1+\bar{r})dt - \int_{\underline{z}}^{z_B^{BE}} p_{HH}(t)dt + \bar{p}_{HL}[\bar{\mu}_{HL}(1+\bar{r}) - 1]. \end{aligned}$$

Equilibrium π^B is then

$$\pi^B = \max \left\{ \int_{\underline{z}}^{z_A^{BE}} p_{HH}(t) \mu_{HH}(t) (1 + \bar{r}) dt - \int_{\underline{z}}^{z_A^{BE}} p_{HH}(t) dt + \bar{p}_{HL} [\bar{\mu}_{HL} (1 + \bar{r}) - 1], 0 \right\}.$$

When $z_A^{BE} > z_B^{BE}$, equilibrium is positive weak with $\pi^B > 0$, and $\hat{z} = z_x = z_A^{BE}$; when $z_A^{BE} \leq z_B^{BE}$, equilibrium is zero weak with $\pi^B = 0$, and $z_B^{BE} = \hat{z} > z_x$. \square